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Computational Complexity Reduction of Orthogonal Precoding for Sidelobe Suppression of OFDM Signal

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***Abstract*—Spectrum sculpting (SS) is a precoding scheme for sidelobe suppression of orthogonal frequency division multiplexing (OFDM) signals and can shape a spectrum with deep notches at chosen frequencies. However, the SS method will degrade the error rate as the number of notched frequencies increases. Orthogonal precoding of the SS has both a notched spectrum and an ideal error rate, but the computational complexity of the precoder matrix is very large. This paper proposes a matrix decomposition of the precoder matrix of the orthogonal precoding to reduce the computational complexity. Numerical experiments show that the proposed method can drastically reduce the computational complexity.**

I. Introduction

The advantages of fast data transmission and robustness against multipath fading have led to orthogonal frequency division multiplexing (OFDM) being adopted in several telecommunications technologies. One of the drawbacks associated with the design of OFDM transmitters is that high out-of-band radiation is generated by the high sidelobes of the OFDM signal. A critical issue concerning OFDM-based cognitive radio systems is that unwanted in-band and out-of-band radiation interferes with the adjacent bands. Various methods of sidelobe suppression have been proposed [1]–[7].

Spectrum sculpting (SS) [6] is a precoding scheme for sidelobe suppression that can shape a spectrum with deep notches at chosen frequencies. However, the SS method will degrade the error rate as the number of notched frequencies increases. Orthogonal precoding of the SS [7] has both a notched spectrum and an ideal error rate; however, it requires a very large computational complexity for precoding and decoding.

To reduce the computational complexity, this paper proposes a matrix decomposition of the precoder in the orthogonal precoding. Numerical experiments show that the proposed method can drastically reduce the computational complexity.

II. Orthogonal Precoding of Spectrum Sculpting

In this paper, the OFDM signal is written as

$$s(t) = \sum_{i=0}^{\infty} s_i(t - iT), \tag{1}$$

where $T = T_s + T_g$, T_s is the OFDM symbol duration and T_g is the guard interval length. The i-th OFDM symbol $s_i(t)$ is written as

$$s_i(t) = \sum_{k\in\mathcal{K}} \bar{d}_{k,i} p_k(t) = \mathbf{p}^T(t)\bar{\mathbf{d}}_i, \tag{2}$$

where

$$p_k(t) = e^{j2\pi\frac{k}{T_s}t} I(t), \tag{3}$$

$$\mathbf{p}(t) = [p_{k_0}(t), \cdots, p_{k_{K-1}}(t)]^T, \tag{4}$$

the indicator function $I(t) = 1$ for $-T_g \le t < T_s$ and $I(t) = 0$ elsewhere, the $K \times 1$ vector $\bar{\mathbf{d}}_i = [\bar{d}_{0,i}, \cdots, \bar{d}_{K-1,i}]^T \in \mathbb{C}^K$ is the result of precoding the $D \times 1$ vector $\mathbf{d}_i = [d_{0,i}, \cdots, d_{D-1,i}]^T$ containing D information symbols in some finite symbol constellation, $K\,(\ge D)$ is the number of subcarriers, and $\mathcal{K} = \{k_0, \cdots, k_{K-1}\}$ are the subcarrier indices. The Fourier transform of (2) is

$$S_i(f) = \sum_{k\in\mathcal{K}} \bar{d}_{k,i} a_k(t) = \mathbf{a}^T(t)\bar{\mathbf{d}}_i, \tag{5}$$

where

$$a_k(f) = Te^{-j\pi(T_s - T_g)(f - \frac{k}{T_s})}\mathrm{sinc}(\pi T(f - \frac{k}{T_s})) \tag{6}$$

is the Fourier transform of $p_k(f)$,

$$\mathbf{a}(f) = [a_{k_0}(f), \cdots, a_{k_{K-1}}(f)]^T \tag{7}$$

and the cardinal sine is defined as $\mathrm{sinc}(x) = \sin(x)/x$. To render the power spectrum of $s(t)$ zero at the $M\,(\ll K)$ chosen frequencies in $\mathcal{M} = \{f_0, \cdots, f_{M-1}\}$, the SS scheme [6] satisfies the constraints

$$S_i(f_m) = 0,\ m = 0, 1, \cdots, M-1. \tag{8}$$

For (5), the constraints (8) can be cast in matrix form such as

$$\mathbf{A}\bar{\mathbf{d}}_i = \mathbf{0}, \tag{9}$$

where

$$\mathbf{A} = [\mathbf{a}(f_0), \mathbf{a}(f_1), \cdots, \mathbf{a}(f_{M-1})]^T. \tag{10}$$

Applying the orthogonal precoding [7] with $D = K - M$, the solution of (9) is determined as

$$\bar{\mathbf{d}}_i = \mathbf{G}_o \mathbf{d}_i, \tag{11}$$

where

$$\mathbf{G}_o = \mathbf{V}\begin{bmatrix}\mathbf{O}_{K\times D}\\ \mathbf{I}_D\end{bmatrix} = [\mathbf{v}_M\ \mathbf{v}_{M+1}\ \ldots\ \mathbf{v}_{K-1}], \tag{12}$$

$\mathbf{v}_M, \mathbf{v}_{M+1}, \cdots, \mathbf{v}_{K-1}$ are the last $D = K - M$ columns of a $K \times K$ unitary matrix $\mathbf{V} = [\mathbf{v}_0\ \mathbf{v}_1\ \ldots\ \mathbf{v}_{K-1}]$ obtained from the singular-value decomposition (SVD) that factorizes $\mathbf{A}$ as

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H, \tag{13}$$

$\mathbf{U}$ is an $M \times M$ unitary matrix, and $\mathbf{\Sigma}$ is a diagonal $M \times K$ matrix containing the singular values of $\mathbf{A}$ in non-increasing order along its diagonal. Because the precoder (12) satisfies $\mathbf{A}\mathbf{G}_o = \mathbf{O}$, the precoding (11) satisfies the constraint (9) (see Appendix).

The receiver performs the decoding that inverts the transmitter precoding (11) as

$$\mathbf{r}_i = \mathbf{G}_o^H \tilde{\mathbf{r}}_i, \tag{14}$$

where $\tilde{\mathbf{r}}_i$ is the i-th received OFDM symbol after the channel equalization. This decoding provides $\mathbf{r}_i = \mathbf{d}_i$ in the noiseless condition since $\mathbf{G}_o$ is unitary ($\mathbf{G}_o^H\mathbf{G}_o = \mathbf{I}_D$).

Ref. [7] shows that the orthogonal precoding has the ideal error rate performance and a sidelobe suppression performance identical to that of the precoding [6]. However, the precoding (11) and decoding (14) each require $KD = K(K-M) \simeq K^2$ multiplications. These are very large and thus must be reduced.

III. Analysis and Proposed Method

This paper proposes a matrix decomposition of the orthogonal precoder $\mathbf{G}_o$ to reduce the computational complexity in the orthogonal precoding.

Firstly, we consider the SVD of $\mathbf{V} - \mathbf{I}_K$ such that

$$\mathbf{V} - \mathbf{I}_K = \mathbf{X}\mathbf{Y}\mathbf{Z}^H, \tag{15}$$

where $\mathbf{X} = [\mathbf{x}_0\ \mathbf{x}_1\ \ldots\ \mathbf{x}_{K-1}]$ and $\mathbf{Z} = [\mathbf{z}_0\ \mathbf{z}_1\ \ldots\ \mathbf{z}_{K-1}]$, are $K \times K$ unitary matrices, $\mathbf{Y}$ is a diagonal $K \times K$ matrix containing the singular values of $\mathbf{V} - \mathbf{I}_K$ in non-increasing order along its diagonal, expressed as

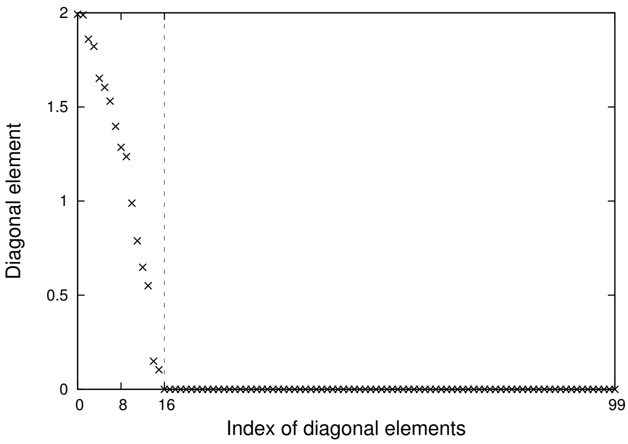
$$\mathbf{Y} = \mathrm{diag}\left(\sigma_0, \sigma_1, \cdots, \sigma_{K-1}\right), \tag{16}$$

and $\sigma_0 \geq \sigma_1 \geq \cdots \geq \sigma_{K-1}$ are the singular values of $\mathbf{V} - \mathbf{I}_K$.

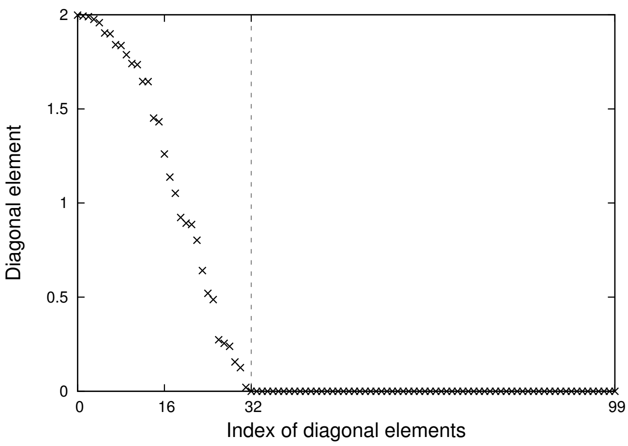
We analyzed $\mathbf{Y}$ expressing the singular values $\mathbf{V} - \mathbf{I}_K$ under the conditions in [7]. Figure 1 shows the first 100 diagonal elements of $\mathbf{Y}$, that is, the singular values $\sigma_0, \cdots, \sigma_{99}$. The results show that almost all diagonal elements can be considered as zeros, except for the first few values. The number of non-zero diagonal elements L is found to be $2M$ from these results. Although there are no guarantees that L is always equal to $2M$ or much smaller than K, we estimate that L is restricted by M. We have actually verified at least that $L = 2M \ll K$ is satisfied in all the cases described in [6] and [7].

Assuming that the $L\,(\leq \mathrm{rank}\{\mathbf{Y}\})$ singular values are not equal to zero, we can approximate the matrix $\mathbf{V} - \mathbf{I}_K$ by the Eckart–Young theorem, i.e.,

$$\mathbf{V} - \mathbf{I}_K \simeq \mathbf{X}\tilde{\mathbf{Y}}\mathbf{Z}^H, \tag{17}$$



(a) $M = 8$ ($\mathcal{M} = \{\pm 6100\pm 1, \pm 5100\pm 1\}$ kHz) and $K = 600$ ($\mathcal{K} = \{-300, \cdots, -1\} \cup \{1, \cdots, 300\}$).



(b) $M = 16$ ($\mathcal{M} = \{\pm 1500\pm 1, \pm 2400\pm 1, \pm 8100\pm 1, \pm 8800\pm 1\}$ kHz) and $K = 600$ ($\mathcal{K} = \{-500, \cdots, -201\} \cup \{201, \cdots, 500\}$)

Fig. 1. Singular values in $\mathbf{Y}$; $T_s = 1/15$ ms, and $T_g = 9T_s/128$. The conditions of Figs. 1(a) and 1(b) are based on those of Figs. 3(a) and 3(b) in [7], respectively.

where $\tilde{\mathbf{Y}}$ is a $K \times K$ diagonal matrix containing only the first L diagonal elements of $\mathbf{Y}$ and expressed as

$$\tilde{\mathbf{Y}} = \mathrm{diag}\left(\sigma_0, \sigma_1, \cdots, \sigma_{L-1}, 0, \cdots, 0\right). \tag{18}$$

From (17) and (18), we can obtain

$$\mathbf{V} \simeq \mathbf{I}_K + \mathbf{X}\tilde{\mathbf{Y}}\mathbf{Z}^H = \mathbf{I}_K + \mathbf{Q}\mathbf{R}^H, \tag{19}$$

where $\mathbf{Q}$ is the $K \times L$ matrix that consists of the first L columns of the matrix $\mathbf{X}\tilde{\mathbf{Y}}$, expressed as

$$\mathbf{Q} = [\sigma_0\mathbf{x}_0\ \sigma_1\mathbf{x}_1\ \ldots\ \sigma_{L-1}\mathbf{x}_{L-1}], \tag{20}$$

and $\mathbf{R}$ is the $K \times L$ matrix that consists of the first L columns of the matrix $\mathbf{Z}$, expressed as

$$\mathbf{R} = [\mathbf{z}_0\ \mathbf{z}_1\ \ldots\ \mathbf{z}_{L-1}] = [\mathbf{z}'_0\ \mathbf{z}'_1\ \ldots\ \mathbf{z}'_{K-1}]^H. \tag{21}$$

Combining (12) and (19), we finally obtain the decomposed

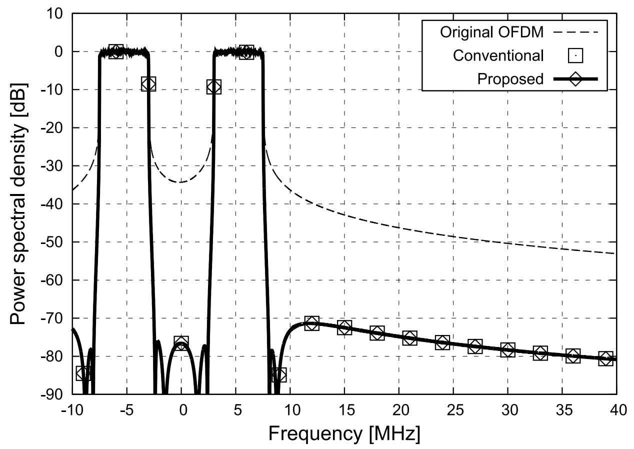


Fig. 2. Power spectral density of the original OFDM, conventional orthogonal precoding, and proposed method.

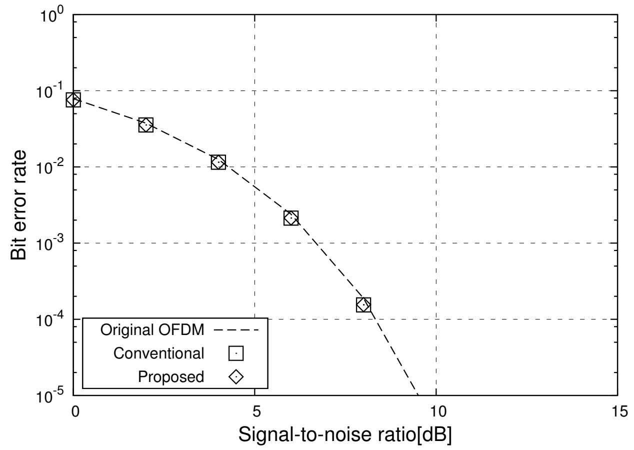


Fig. 3. QPSK bit error rate in AWGN channel.

approximation of the orthogonal precoder $\mathbf{G}_o$:

$$\mathbf{G}_o \simeq (\mathbf{I}_K + \mathbf{Q}\mathbf{R}^H) \begin{bmatrix} \mathbf{O}_{M\times D} \\ \mathbf{I}_D \end{bmatrix} = \mathbf{I}_D + \mathbf{Q}\mathbf{S} \tag{22}$$

where $\mathbf{S}$ is the $L \times D$ matrix composed of the last $D = K - M$ columns of $\mathbf{R}^H$, expressed as

$$\mathbf{S} = [\mathbf{z}'_M \ \mathbf{z}'_{M+1} \ \ldots \ \mathbf{z}'_{K-1}]. \tag{23}$$

For (22), the p3recoding (11) can be rewritten as

$$\bar{\mathbf{d}}_i \simeq \mathbf{d}_i + \mathbf{Q}\mathbf{S}\mathbf{d}_i, \tag{24}$$

and the decoding (14) can be rewritten as

$$\mathbf{r}_i \simeq \tilde{\mathbf{r}}_i + \mathbf{S}^H\mathbf{Q}^H\tilde{\mathbf{r}}_i. \tag{25}$$

The proposed precoding (24) and decoding (25) require $L(K+D) = 2M(2K-M)L$ multiplications instead of the $K(K-M)$ multiplications of the conventional orthogonal precoding if $L = 2M$.

TABLE I
COMPARISON OF COMPUTATIONAL COMPLEXITY IN MULTIPLICATIONS

precoding/decoding

Condition	Fig. 3(a) in [7]	Fig. 3(b) in [7]
Conventional [6]	$355,200$ (100%)	$1,937,600$ (100%)
Proposed	$19,072$ (5.4%)	$89,088$ (4.6%)

(Example: $K = 600, 600$, $M = 8, 16$, $L = 16, 32$, respectively)

IV. NUMERICAL EXPERIMENTS

To evaluate the performance of the proposed method, we conducted numerical experiments.

We firstly verified that the proposed method does not degrade the performance of the conventional orthogonal precoding under the conditions in Fig. 3(b) in [7] with $T_s = 1/15$ ms, $T_g = 9T_s/128$, $K = 600$ ($\mathcal{K} = \{-500, \cdots, -201\} \cup \{201, \cdots, 500\}$), $M = 16$ ($\mathcal{M} = \{\pm1500\pm1, \pm2400\pm1, \pm8100\pm1, \pm8800\pm1\}$ kHz), and $L = 2M$. Figure 2 shows the power spectral densities of the original OFDM, the conventional orthogonal precoding of the SS, and the proposed method. Figure 3 shows the bit error rates (BERs) in an additive white Gaussian noise (AWGN) channel. These show that the performance of the proposed method is identical to that of the conventional orthogonal precoding with QPSK modulation.

Next, we evaluated the computational complexity of the proposed method compared with the conventional orthogonal precoding under the conditions of Fig. 3(a)(b) in [7]. Table I shows the computational complexity in multiplications against that of the original OFDM. The conventional orthogonal precoding has an obvious enormous computational complexity. In contrast, the proposed method can drastically reduce the computational complexity.

V. CONCLUSIONS

This paper has proposed a matrix decomposition of the precoder matrix in the orthogonal precoding to reduce the computational complexity. Numerical experiments showed that the proposed method does not degrade the performances and can reduce the computational complexity drastically for both precoding and decoding, e.g., 4.6%, compared with the conventional orthogonal precoding.

APPENDIX

Here, we prove

$$\mathbf{A}\mathbf{G}_o = \mathbf{O}. \tag{26}$$

From (12), (13), and $\mathbf{V}$ being unitary ($\mathbf{V}^H\mathbf{V} = \mathbf{I}_K$), we obtain

$$\mathbf{A}\mathbf{G}_o = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H\mathbf{V} \begin{bmatrix} \mathbf{O}_{M\times D} \\ \mathbf{I}_D \end{bmatrix} = \mathbf{U}\mathbf{\Sigma} \begin{bmatrix} \mathbf{O}_{M\times D} \\ \mathbf{I}_D \end{bmatrix}. \tag{27}$$

Because $\mathrm{rank}\{\mathbf{A}\} = M$, the diagonal matrix $\mathbf{\Sigma}$ contains $D = K - M$ singular values equal to zero along its diagonal. Thus $\mathbf{\Sigma}$ is expressed as

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_M & \mathbf{O}_{M\times D} \\ \mathbf{O}_{D\times M} & \mathbf{O}_{D\times D} \end{bmatrix}, \tag{28}$$

where $\mathbf{\Sigma}_M$ is an $M \times M$ diagonal matrix, and

$$\mathbf{\Sigma} \begin{bmatrix} \mathbf{O}_{M \times D} \\ \mathbf{I}_{K-M} \end{bmatrix} = \mathbf{O}, \tag{29}$$

is satisfied trivially. For (27) and (29), we obtain (26).

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