A New Approach to Solve the Partitioning Problem to Realize Highly Reliable Telecommunications Networks

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Abstract: This paper proposes a new algorithm to solve the partitioning problem to realize high reliable telecommunications networks. The partitioning problem is to check whether a telecommunication network can be divided into two subnetworks such that we can ensure the independence of two paths by assigning these paths in different subnetworks. We already proposed an algorithm to solve this partitioning problem. However, this algorithm is problematic because its execution time on computer increases exponentially with the size of telecommunications networks. Now, this paper proposes a new higher speed algorithm to solve the partition problem. The basic idea of the proposed algorithm is transforming a special type of configuration called degree-3 delta to simplify the network topology.

Keywords-- **Telecommunications Networks, Reliability, Partitioning Problem, Factoring Algorithm, Degree-3 delta-Star Transform Introduction**

1. Introduction

As effective way to achieve high reliability in telecommunications networks is to use “double homing”. This is achieved by having an edge router access two different core routes (a pair) so that communication continues even if one router in the pair fails.

However, as pointed out by refs. [1][2], double homing is effective only if there are two independent routes between any two pairs of routers, but such independence is not always easy to ensure because telecommunications networks are large and complex.

Our previous work proposed a simple approach to ensure such independence: partitioning the network into two subnetworks that do not share any equipment. That is, if two routes between two pairs of routers pass through completely different subnetworks, they do not have any common nodes or links and are thus independent[1][2].

The previous works [1][2] formulated this partitioning problem as a graph theoretical partitioning problem and proposed algorithms, for determining whether a network can be partitioned. Factoring algorithm is the fastest algorithm in previous works. (While some other researches [5]-[9] have addressed similar types of partitioning problems of graphs, we emphasize that they are quite different because, for example, those researchers [5]-[9] never considered double homing concepts.)

2. Preperation

First, we explain the mathematical and graph theoretical concepts and symbols.

The empty set is represented by ϕ.

For both sets S_1 and S_2, $S_1 - S_2$ is defined as the set of elements of S_1 not element of S_2. That is, $S_1 - S_2 = \{x \mid x \in S_1, x \notin S_2\}$.

A graph is a mathematical object consisting of nodes and links. Nodes are numbered 1, 2, … , N. A link connects two nodes. If a link connects node i and j, i and j are the end nodes of that link. If there is a link with end nodes i and j, i and j are considered to be directly connected by the link. If two nodes are directly connected by two links, these links are considered to be parallel links. If a node is the end node of d number of links, the degree of this node is d. If the degree of a node is d_0, this node is considered to be a degree-d_0 node. Two links directly connecting the same degree-2 node are considered to be series links.

We use G to indicate a graph; V is the set of all nodes in G, and E is the set of all links in G. That is, $G = (V, E)$.

$G_t = (V_t, E_t)$ is a subgraph of G if $V_t \subseteq V$, $E_t \subseteq E$, and the end nodes of every link of E_t is included in V_t. A path is a sequence of alternating nodes and links. If there is a path between two nodes, the two nodes are connected. If any two nodes in G are connected, G is a connected graph. Δ is a subgraph of G having three nodes and three links such that each link directly connects to each pair of two nodes of Δ.

We define additional concept, ‘node-pair’, for this graph. If a subset of V consists of exactly two nodes, this subset is considered to be a node-pair. $T(G)$ is the set of all node-pairs in G. We define T_0 as a specified subset of $T(G)$.

Next, we explain the operation that is being used in the algorithm.

Degree-1 reduction

If G has degree 1 node (a node not included in any node-pair of T_0), it is removed, and the link directly connecting to this degree-1 node is removed.

Parallel reduction

If G has parallel links, one of them is removed.

Series reduction

If G has series links for which the end nodes of one link are i and j, the end nodes of another link are j and k, and j is not included in any node-pair of T_0, those two links and j are removed. A link is then added that directly connects i and k.

Degree-3 delta-star transformation

If G has Δ having three nodes i, j, & k, and the degree of node i is three, the link whose end nodes are j, k is removed.This Δ is denoted by $\Delta(3)$.)

These reductions and transformation are illustrated in Figures 1, 2, 3 and 4.

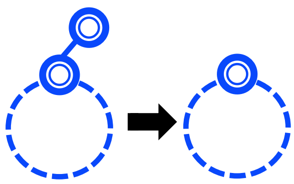

Figure 1. Degree-1 reduction.

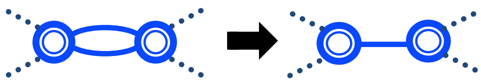

Figure 2. Parallel reduction.

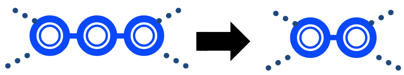

Figure 3. Series reduction.

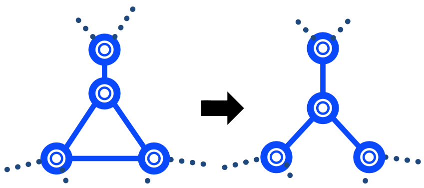

Figure 4. Degree-3 delta-star transformation.

Contract: A link in G is selected and removed; the two end nodes of this link are merged into one node.
Delete: A link in G is selected and removed.

Examples of these operations are illustrated in Figure. 5 and 6.

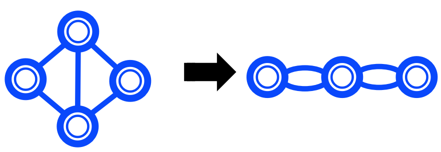

Figure 5. Contract

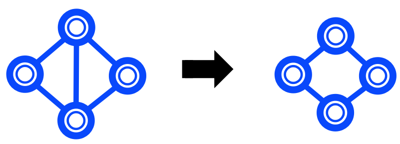

Figure 6. Delete

In factoring, a link in G is selected and used to generate two graphs. One (graph G^{+}) is generated by contracting this link, and the other (graph G^{-}) is generated by deleting this link.

Other graph theoretical concepts are due to [3].

3. Existing Research

3. 1 Structure of telecommunications networks

In telecommunications networks, communication from one user to another is done in six basic steps: a) the first user accesses an edge router, b) this edge router accesses a core router, c) this core router accesses another core router, d) this router access is repeated, e) the final core router accesses an edge router, and f) this edge router accesses the second user. .

In this process, steps a) and f) involve accessing a network, and steps b), c), d), and e) involve accessing a core network comprising core routers and links among them. Double homing is implemented in steps b) and e), in which an edge router can access two core routes (a pair) in step b), and the two core routers (a pair) can access the same edge router in step e), so traffic is not disrupted even if one router of the pair fails..

An example structure of a telecommunications networks is shown in Figure 7.

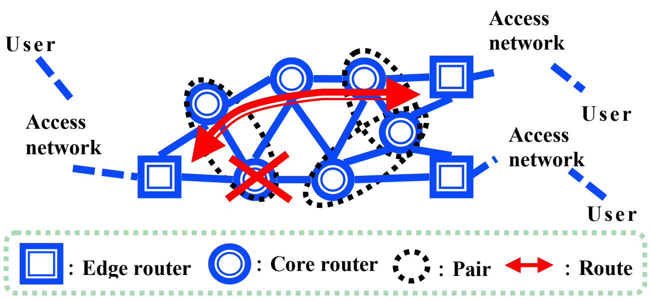

Fig. 7. Example structure of telecommunications network.

3. 2 Problem of double homing

As mentioned above, double homing is effective only if there are two independent routes between any two pairs of routes, where 'independent' means the two routes do not share any equipment (cable, multiplexer, or router). This is because the failure of a shared piece of equipment will cause both routes to fail.

As refs. [1][2] explained, such independence is difficult to ensure because a core network is typically very large and the number of routes is huge.

However refs. [1][2] emphasized that we can solve the problem by partitioning a telecommunication network into two subnetworks that do not share any equipment. If two routes between two pairs of routers pass through different subnetworks, they do not have any common nodes or links and are thus independent.

Now, the problem is reduced to effectively partitioning the telecommunication network. Refs. [1][2] already formulated this partitioning problem as a graph theoretical partitioning problem.

Next subsection describes the formulation.

3. 3 Formulation of the problem

We use graph G to represent telecommunication network. Links represent cables, and nodes represent multiplexers and routers. A node-pair represents a pair of routers used for double homing.

Refs. [1][2] formulated the partitioning problem as follows.

For a given G and T_0, determine the existence of graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ satisfying the four following conditions.

Condition 1. G_1 and G_2 are subgraphs of G.
Condition 2. $V_1 \cap V_2 = E_1 \cap E_2 = \phi$
Condition 3. G_1 and G_2 are connected graphs
Condition 4. For any node-pair $\{i, j\} \in T_0$, one of the following is true.
a) $i \in V_1$ and $j \in V_2$
b) $j \in V_1$ and $i \in V_2$

If G_1 and G_2 satisfying these conditions exist, G can be partitioned; otherwise, G cannot.

An example of a partitionable graph is shown in Fig. 8. The white circles and solid lines indicate nodes and links in G_1. The other circles and dotted lines indicate nodes and links in G_2.

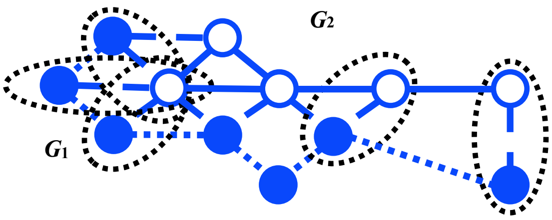

Figure 8. Partitionable graph.

3. 4 Existing algorithm for partitonalbe problem

The fastest algorithm [2] in previous works proposed for solving the partitioning problem uses the following theorem.

Theorem 1.

If G is reduced to G' by degree-1, series, or parallel reduction,
(1) If G' can be partitioned, G can be partitioned.
(2) If G' cannot be partitioned, G cannot be partitioned.

Theorem 2.
(1) If G can be partitioned, G^+ or G^- can be partitioned.
(2) If G cannot be partitioned, neither G^+ nor G^- can be partitioned.

Ref. [2] emphasized that we can solve the partitioning problem by factoring with reductions.

First, we apply reductions to G and, if the partitioning problem is solved for this reduced graph G', the partitioning problem for G is solved in accordance with Theorem 1. If the partitioning problem for G' is difficult to solve, G' is factored into G'^+ and G'^-. If the partitioning problem for G'^+ and G'^- is solved, the partitioning problem for G is solved in accordance with Theorem 2. These factorings with reductions are repeated until the graphs are sufficiently small so that the algorithm described in find-all-solutions algorithm can easily solve the partitioning problem.

Find-all-solutions algorithm is method to enumerate all solutions.

This algorithm is the fastest algorithm among previous works. However, we still have a problem because its execution time on computer increases exponentially with the size of telecommunications networks. Therefore, much faster algorithm is requested.

4. Proposed algorithm

We propose a new improved algorithm to realize much faster execution time. This new algorithm is based on the idea of adding degree-3 delta-star transformation (see Section 2.) to the algorithm of ref. [2], while degree-3 delta-star transformation have never been used in any previous works [1][2].

The following Theorem gives the base of our new algorithm.

Theorem 3.

If G is reduced to $G_{\Delta(3)}$ by applying a degree-3 delta-star transformation.
(1) If G can be partitioned, $G_{\Delta(3)}$ can be partitioned.
(2) If G cannot be partitioned, $G_{\Delta(3)}$ cannot be partitioned.

This theorem is derived in Appendix.

Now we can derive a new algorithm for solving the partitioning problem by adding the degree-3 delta-star transformation to the part of reductions in the factoring algorithm [2].

5. Numerical examples

We have implemented software to solve the partitioning problem by using our proposed algorithm.

Numerical examples show that this algorithm is faster than pervious algorithm. For example, the execution time of the proposed algorithm for Fig. 9 is 25923 seconds to solve partition problem, while previous algorithm of [2] needs 373698 seconds.

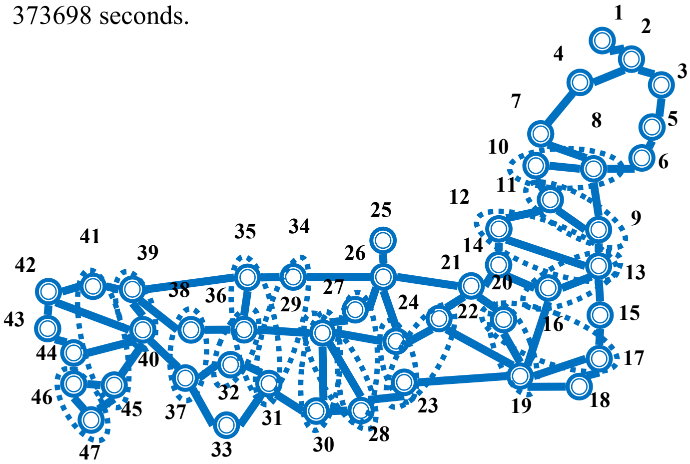

Figure 9. An example model

6. Conclusion

This paper has proposed a new faster algorithm to solve the problem of partitioning a telecommunication network into two independent subnetworks.

The key idea of our new algorithm is transforming a special type of configuration into another simpler configuration, where we call this transforming degree-3 delta-star transformation.

Future works are as follows.

(1) Improve the speed of the algorithm much faster.
(2) Apply the partitioning problem to real telecommunication networks

References

[1] M. Hayashi and H. Otuski, "End to end route diversifiationing by network partition", Proceedings of IEICE Autumn National Covention, A-2, 1992.

[2] T. Omura, R. Yoshioka, M. Hayashi, "A Factoring Algorithm for Solving the Problem of Partitioning Core Networks to Achieve Highly Reliable Telecommunications Networks", Workshop on Circuits and Systems, pp. 98-103, 2014.

[3] B. Bollobás, Modern Graph Theory (Graduate Texts in Mathematics), S. Axler and F. W. Gehring ed., Springer, Germany, 1998.

[4] H. Imai, The Operations Research Society of Japan, http://www.orsj.or.jp/~archive/pdf/bul/Vol.48_07_467.pdf, Apr. 2014. (in Japanese)

[5] Santo Fortunato, "Community detection in graphs", Physics Reports, Vol. 486, Issues 3–5, pp. 75–174, 2010.

[6] Sanjeev Arora, Satish Rao, and Umesh Vazirani, "Geometry, Flows, and Graph-Partitioning Algorithms", Communications of the ACM, Vol. 51 No. 10, pp. 96-105, 2008.

[7] Chris H.Q. Ding, "A Min-max Cult Algorithm for Graph Partitioning and Data Clustering", ICDM 2001, pp. 107 – 114, 2001.

[8] Charles J. Alpert, "Multilevel Circuit Partitioning", IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, Vol. 17, No. 8, pp. 655-667, 1998.

[9] Andrea Lancichinetti, "Detecting the overlapping and hierarchical community structure in complex networks", New Journal of Physics 11, 033015, 2009.

Appendix Proof for Theorem 3

Theorem 3 is proved.

Case1. It is easy to see that Theorem 3 is true when the number of the links of graph G is not more than 4.

Case2. We assume that the number of the links of graph G is more than 4.

Let G has $\varDelta(3)$. (See Section 2 as for $\varDelta(3)$.) It is true that G has another link never being included in $\varDelta(4)$, because G has more than four links. (We can assume that this new link is not one of a pair of the parallel links whose the other side link is included in $\varDelta(4)$. This is because, Theorem 1 guarantees that removing a link of a pair of parallel links never gives any effect to the result of partitioning problem.) Let m be 'the number of links – 4'.

The following induction logic derives that Theorem 3 is true for any $m = 0, 1, \dots$.

(a) We assume $m = 0$.

In this case, it is equivalent to case 1. Therefore, Theorem 3 is true.

(b) We assume that if $m = k$ then Theorem 3 is true.

Now, we focus on $m = k + 1$. We can pick up a link never being included in $\varDelta(3)$. Let G^+ be the graph obtained from G by contracting this link, and let G^- be the graph obtained from G by deleting this link.

<u>Step1</u>.

If G can be partitioned, G^+ or G^- can be partitioned by Theorem 2.

If G^+ can be partitioned then $G_{\square(3)}{}^+$ (the graph obtained from G^+ by applying delta-star transformation) can be partitioned; because the number of links of G^+ is k and the induction assumption is assumed.

Almost same logic derives that if G^- can be partitioned, $G_{\varDelta(3)}{}^-$ (the graph obtained from G^- by applying the delta-star transformation) can be partitioned.

Therefore, if G can be partitioned then $G_{\varDelta(3)}{}^+$ or $G_{\varDelta(3)}{}^-$ can be partitioned. By applying the contraposition of (2) of Theorem 2 to $G_{\varDelta(3)}{}^+$ or $G_{\varDelta(3)}{}^-$ can be partitioned', $G_{\varDelta(3)}$ can be partitioned. That is, (1) of Theorem 3 is true.

<u>Step 2.</u>

If G cannot be partitioned then G^+ and G^- cannot be partitioned, because of (2) of Theorem 2. By the induction assumption, $G_{\varDelta(3)}{}^+$ and $G_{\varDelta(3)}{}^-$ cannot be partitioned. By applying the contraposition of (1) of Theorem 2, $G_{\varDelta(3)}$ cannot be partitioned. That is, (2) of theorem 3 is true.

From Step 1 and 2, Theorem 3 is true if $m = k + 1$.

(c) From the logic of induction, Theorem 3 is true for any m.