Effective Capacity of Cognitive Cooperative Relay Networks over α-μ Fading Channels

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***Abstract*—In this paper, the effective capacity, a link-layer model supporting Quality of Service (QoS) metrics such as user data rate, packet loss rate and service delay, is analyzed. We consider an underlay cognitive cooperative relay network (CCRN) subject to independent non-identically distributed α-μ fading. An analytical expression of the effective capacity is derived by means of Fox H-functions and Meijer G-functions along with the Mellin transform of the product of two H-functions. The network performance is analyzed against the QoS exponent and the peak interference power constraint imposed by primary users.**

I. Introduction

Cognitive radio networks (CRNs) have attracted considerable attention as a technology that offers the potential of tremendous improvements of spectral efficiency. In particular, the underlay spectrum access paradigm performs favourably in scenarios where the cognitive or secondary users (SUs) concurrently access the spectrum with the licensed or primary users (PUs). In this case, SU transmitters (SU-Txs) are allowed to send their signals such that the interference caused to the PU receivers (PU-Rxs) is kept below a given threshold. In addition, cooperative relaying is another advanced radio transmission technology that has been shown to provide spatial diversity gains which in turn increases radio coverage and link reliability. Due to the fact that the SU transmit power is constrained by the tolerable interference power of the PU receiver, the combination of CRNs with cooperative relaying has emerged which is referred to as cognitive cooperative relay network (CCRN).

In order to analyse the performance of CRNs and CCRNs, outage probability, outage capacity, and ergodic capacity are frequently used. However, channel capacity based on the classical Shannon capacity is not sufficient when it comes to assessing the effective transmission data rate for delay-sensitive services. The notion of effective capacity has therefore been adopted to analyse the performance of diverse services with different delay quality of service (QoS) requirements. In particular, effective capacity is considered as a link-layer model that relates to other QoS metrics including user data rate, packet loss rate, and service delay. By definition, the effective capacity is regarded as the maximum constant data rate that can be maintained while satisfying a given packet loss rate target with a specific delay. In [1], the effective capacity of CRNs under Rayleigh fading is analyzed. In [2], from a cross-layer design perspective and assuming Rayleigh fading, the authors analyze the effective capacity and propose a scheme aiming at maximizing the supported arrival-rate subject to a given statistical delay QoS constraint in CRNs with cooperative transmission.

As for the fading channel, we focus on the α-μ fading model [3] which is a rather general fading model comprising other well-known fading distributions as special cases such as one-sided Gaussian, Rayleigh, Weibull, Nakagami-m, and negative exponential distribution. Here, parameter α accounts for the non-linearity of the channel while parameter μ captures the effect of clusters of multipath wave propagation. Some studies, for example, [4], [5], and [6] have adopted the α-μ fading model to analyze the outage probability and outage capacity of CRNs. Specifically, for the case of α-μ fading and identical fading parameters, analytical expressions for the minimum outage probability and delay-limited capacity are derived for a CRN in [4]. In [5], for non-identical parameter μ, the performance of a CRN subject to the interference power constraint of the PU is assessed in terms of outage probability, amount of fading, and approximate ergodic capacity. In [7], expressions for the outage probability and symbol error probability of a single decode-and-forward (DF) relaying network are derived while cognitive radio aspects are not considered.

In this paper, in order to relate the physical layer characteristics of the wireless channel to the data link layer in CCRNs, we advance and complement our work on outage probability [6] and ergodic capacity [8], to provide an effective capacity analysis of an underlay CCRN with DF relay over α-μ fading channels. Given the probability density function (PDF) of the signal-to-noise ratio (SNR) for the considered CCRN and the case that the ratios of independent α-μ random variables [9] have non-identical μ parameters, an analytical expression for the effective capacity is derived by means of Fox H-functions and Meijer G-functions along with the Mellin transform of the product of two H-functions. The obtained analytical expressions are used to investigate the effect of fading parameters and QoS exponent on the effective capacity of the CCRN.

The remainder of this paper is organized as follows. Section II introduces the system model and describes the characteristics of the α-μ fading channel. In Section III, the performance analysis of the system in terms of effective capacity is provided. In particular, based on the PDF of the instantaneous SNR at the SU-Tx obtained in [8], an analytical expression of the effective capacity is derived. Numerical results for some example scenarios are presented in Section IV. Finally, a summary of this work is given in Section V.

Notation: Throughout this paper, the following notations are used. The symbol ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ stands for the hypergeometric function [10, eq. (3.194.5)]. The notation $(.)_n$ is the Pochhammer symbol [10], [11] such that $(a)_n = a(a+1)\ldots(a+n-1) = \frac{\Gamma(a+n)}{\Gamma(a)}$. The expression $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$ defines the gamma function. The symbols $\mathbb{E}[\cdot]$ and $\mathbb{V}[\cdot]$ denote expectation and variance, respectively. The notations $G_{p,q}^{m,n}(\cdot\,|\,\cdot)$ and $H_{p,q}^{m,n}(\cdot\,|\,\cdot)$ stand for the Meijer G-function and the Fox H-function [12], respectively.

II. System Model

Consider a CCRN as shown in Fig. 1 where the SU-Tx communicates with the SU-Rx only through the help of a DF secondary relay (SR). Given the maximum transmit power limits in the secondary network due to the PU interference power constraint, it is assumed that a direct link between SU-Tx and SU-Rx does not exist. Furthermore, it is assumed that the CCRN uses the underlay spectrum access paradigm and that the interference of the PU-Txs to the secondary network can be lumped into the additive zero-mean white Gaussian noise [13], [14].

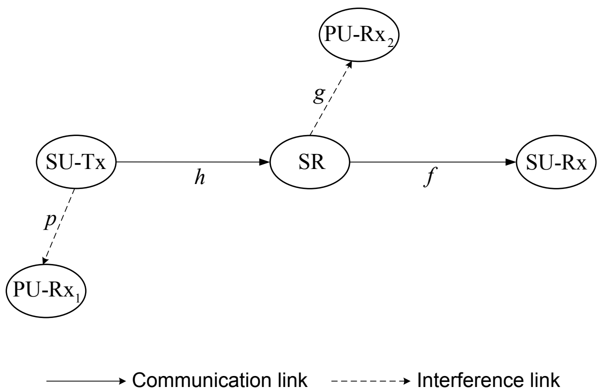

Fig. 1. System model of a relay-based spectrum sharing network.

Let us denote the channel coefficients for the links SU-Tx→SR, SU-Tx→PU-Rx$_1$, SR→SU-Rx, and SR→PU-Rx$_2$ as W, X, Y and Z, respectively, forming a set of channel coefficients as $\mathcal{S} = \{W, X, Y, Z\}$. In case of α-μ fading, the PDF of $s \in \mathcal{S}$ follows the α-μ distribution [3], [15] which is given as

$$f_s(x) = \frac{\alpha\mu^\mu x^{\alpha\mu-1}}{\hat{x}^{\alpha\mu}\Gamma(\mu)} \exp\left(-\mu\frac{x^\alpha}{\hat{x}^\alpha}\right) \quad (1)$$

where $\hat{x} = (\mathbb{E}[s^\alpha])^{\frac{1}{\alpha}}$ is the α-th root mean value, $\alpha > 0$ is an arbitrary fading parameter, and $\mu > 0$ is calculated as $\mu = \mathbb{E}^2[s^\alpha]/\mathbb{V}[s^\alpha]$. Further, the k-th moment of s is given as

$$\mathbb{E}[s^k] = \hat{x}^k \frac{\Gamma\left(\mu + \frac{k}{\alpha}\right)}{\mu^{\frac{k}{\alpha}}\Gamma(\mu)} \quad (2)$$

Similarly, let us denote $h = |W|^2$, $p = |X|^2$, $f = |Y|^2$, $g = |Z|^2$ as the instantaneous channel power gains of the respective communication and interference links of the underlay CCRN and form a corresponding set as $\mathcal{K} = \{h, p, f, g\}$. Then, the PDF of $k \in \mathcal{K}$ can be expressed according to [16] by

$$f_k(x) = \frac{\alpha_k x^{\frac{\alpha_k\mu_k}{2}-1}}{2\Omega_k^{\frac{\alpha_k\mu_k}{2}}\Gamma(\mu_k)} \exp\left[-\left(\frac{x}{\Omega_k}\right)^{\frac{\alpha_k}{2}}\right] \quad (3)$$

where

$$\Omega_k = \frac{\Gamma(\mu_k)}{\Gamma(\mu_k + \frac{2}{\alpha_k})} \quad (4)$$

In the sequel, we assume that perfect channel state information (CSI) is available at SU-Tx and SU-Rx and that the SR is provided with full CSI about respective channel power gains. For example, CSI from the SR transmitter to PU-Rx may be obtained by a band manager that mediates between primary and secondary network [17]. Given this setting, the secondary nodes can adapt their transmit powers subject to the CSIs in such a way that the interference power constraints at the PU-Rxs are respected. More specifically, the SNRs at SR and SU-Rx can be formulated as

$$\gamma_1 = \frac{P(h,p)h}{N_0}; \qquad P(h,p)p \le Q_1 \quad (5)$$

$$\gamma_2 = \frac{P(f,g)f}{N_0}; \qquad P(f,g)g \le Q_2 \quad (6)$$

where $P(h,p)$ and $P(f,g)$ are the instantaneous transmission powers at the SU-Tx and the SR, respectively. Furthermore, Q_1 and Q_2 denote the peak interference powers allowed at the PU-Rx$_1$ and the PU-Rx$_2$, respectively, and N_0 is the noise power. Assuming that SU-TX and SR transmit with their maximum instantaneous powers Q_1/p and Q_2/g, respectively, the expressions (5) and (6) can be rewritten for a homogenous network with $Q_1 = Q_2 = Q$ as

$$\gamma_1 = \frac{hQ}{pN_0} \quad (7)$$

$$\gamma_2 = \frac{fQ}{gN_0} \quad (8)$$

III. Performance Analysis

Hereinafter, based on the PDF of the instantaneous SNR at the SU-Rx, an expression of the effective capacity for the considered CCRN is derived.

A. PDF of the instantaneous SNR at SU-Rx

For the system topology shown in Fig.1, the PDF $f_{\gamma_s}(\gamma)$ of the instantaneous SNR γ_s at the SU-Rx has been derived and utilized in our previous works [6], [8]. As such, we refer the interested readers to these works for a detailed derivation of the PDF and provide here only the final result. According to [6], [8], assuming $\alpha_h = \alpha_p = \alpha_1$ and $\alpha_f = \alpha_g = \alpha_2$, the PDF $f_{\gamma_s}(\gamma)$ of the instantaneous SNR

$$\gamma_s = \min\{\gamma_1, \gamma_2\} \tag{9}$$

at the SU-Rx of the considered system can be obtained as

$$\begin{aligned} f_{\gamma_s}(\gamma) = \frac{1}{2k_1k_2\gamma} \Big\{ & A_1\gamma^{\frac{\alpha_1\mu_h}{2}}(\beta_1)^{-\mu_p-\mu_h}\alpha_1\mu_h \\ & \times\Big[k_2 - A_2\gamma^{\frac{\alpha_2\mu_f}{2}} \\ & \times {}_2F_1\left(\mu_f, \mu_g+\mu_f, 1+\mu_f, -A_4\gamma^{\alpha_2/2}\right)\Big] \\ & +A_2\gamma^{\frac{\alpha_2\mu_f}{2}}(\beta_2)^{-\mu_g-\mu_f}\alpha_2\mu_f \\ & \times\Big[k_1 - A_1\gamma^{\frac{\alpha_1\mu_h}{2}} \\ & \times {}_2F_1\left(\mu_h, \mu_p+\mu_h, 1+\mu_h, -A_3\gamma^{\alpha_1/2}\right)\Big]\Big\} \end{aligned} \tag{10}$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ stands for the hypergeometric function [10, eq. (3.194.5)] and

$$A_1 = \left(\frac{N_0\Omega_p}{Q\Omega_h}\right)^{\alpha_1\mu_h/2} \tag{11}$$

$$A_2 = \left(\frac{N_0\Omega_g}{Q\Omega_f}\right)^{\alpha_2\mu_f/2} \tag{12}$$

$$A_3 = \left(\frac{N_0\Omega_p}{Q\Omega_h}\right)^{\alpha_1/2} \tag{13}$$

$$A_4 = \left(\frac{N_0\Omega_g}{Q\Omega_f}\right)^{\alpha_2/2} \tag{14}$$

For brevity, we have introduced the following terms:

$$\beta_1 = 1 + A_3\gamma^{\alpha_1/2} \tag{15}$$

$$\beta_2 = 1 + A_4\gamma^{\alpha_2/2} \tag{16}$$

$$k_1 = \mu_h\mathcal{B}(\mu_h, \mu_p) \tag{17}$$

$$k_2 = \mu_f\mathcal{B}(\mu_f, \mu_g) \tag{18}$$

where $\mathcal{B}(\cdot,\cdot)$ denotes the Beta function.

B. Effective Capacity

According to [18], effective capacity constitutes a dual concept to bandwidth. In this context, let us assume that the secondary network must satisfy a statistical delay QoS constraint which may be related to the queue length. It has been shown that the probability for the queue length of the transmit buffer exceeding a certain threshold, x, decays exponentially as a function of x [19]. Then, the so-called delay QoS exponent θ can be defined

$$\theta = -\lim_{x\to\infty}\frac{\ln\left(\Pr\{q(\infty) > x\}\right)}{x} \tag{19}$$

where $q(n)$ indicates the transmit buffer length at time n and $\Pr\{\cdot\}$ stands for probability. For $\theta \to 0$, the system is able to tolerate infinite long packet delay while for $\theta \to \infty$ no delay is allowed in the system.

Let $\{R[n], n = 1, 2, \ldots\}$ be a stochastic service process which is assumed to be stationary and ergodic. Given that the capacity function $\Lambda(-\theta)$ exists such that

$$\Lambda(-\theta) = \lim_{N\to\infty}\frac{1}{N}\ln\left(\mathbb{E}\left\{e^{-\theta\sum_{n=1}^{N}R[n]}\right\}\right) \tag{20}$$

the effective capacity is given by [18]

$$\begin{aligned} E_c(\theta) &= -\frac{\Lambda(-\theta)}{\theta} \\ &= -\lim_{N\to\infty}\frac{1}{N\theta}\ln\left(\mathbb{E}\left\{e^{-\theta\sum_{n=1}^{N}R[n]}\right\}\right) \end{aligned} \tag{21}$$

where $\mathbb{E}\{\cdot\}$ denotes the expectation operator, θ is the QoS exponent interpreted as the delay constraint, and $R[n]$ is the data rate of the relay channel. The effective capacity in (21) gives the maximal data rate that can be supported by the channel under the delay constraint θ.

For the case of block-fading channels, the effective capacity given in (21) can be simplified because the elements of the sequence $\{R[n], n = 1, 2, \ldots\}$ are then uncorrelated, yielding

$$E_c(\theta) = -\frac{1}{\theta}\ln\left(\mathbb{E}\left\{e^{-\theta R[n]}\right\}\right) \tag{22}$$

For the sake of simplicity, we omit the discrete time index n in the sequel. It should also be noted that the data rate of the relay can then be related to the SNRs in the first and second hop as

$$R = \frac{1}{2}T_f B \min\{\ln(1+\gamma_1), \ln(1+\gamma_2)\} \tag{23}$$

Defining a new random variable from (9) as $\gamma_i = \min\left\{\frac{h}{p}, \frac{f}{g}\right\}$, the effective capacity in (22), in its integral form can be written as

$$E_c(\theta) = -\frac{1}{\theta}\ln\left[\int_0^\infty\left(1+\frac{\gamma Q}{N_0}\right)^{-\eta} f_{\gamma_i}(\gamma)d\gamma\right] \tag{24}$$

where $\eta = \frac{\theta T_f B}{2}$ and $f_{\gamma_i}(\gamma)$ can be obtained from (10) after replacing, respectively, A_1, A_2, A_3, and A_4 by

$$A'_1 = \left(\frac{\Omega_p}{\Omega_h}\right)^{\frac{\alpha_1\mu_h}{2}} \quad (25)$$

$$A'_2 = \left(\frac{\Omega_g}{\Omega_f}\right)^{\frac{\alpha_2\mu_f}{2}} \quad (26)$$

$$A'_3 = \left(\frac{\Omega_p}{\Omega_h}\right)^{\alpha_1/2} \quad (27)$$

$$A'_4 = \left(\frac{\Omega_g}{\Omega_f}\right)^{\alpha_2/2} \quad (28)$$

as

$$\begin{aligned} f_{\gamma_i}(\gamma) = \frac{1}{2k_1k_2\gamma}\Big\{ & A'_1\gamma^{\frac{\alpha_1\mu_h}{2}}(\beta'_1)^{-\mu_p-\mu_h}\alpha_1\mu_h \\ & \times\Big[k_2 - A'_2\gamma^{\frac{\alpha_2\mu_f}{2}} \\ & \times {}_2F_1\left(\mu_f, \mu_g+\mu_f, 1+\mu_f, -A'_4\gamma^{\alpha_2/2}\right)\Big] \\ & + A'_2\gamma^{\frac{\alpha_2\mu_f}{2}}(\beta'_2)^{-\mu_g-\mu_f}\alpha_2\mu_f \\ & \times\Big[k_1 - A'_1\gamma^{\frac{\alpha_1\mu_h}{2}} \\ & \times {}_2F_1\left(\mu_h, \mu_p+\mu_h, 1+\mu_h, -A'_3\gamma^{\alpha_1/2}\right)\Big]\Big\} \end{aligned} \quad (29)$$

where $\beta'_1 = 1 + A'_3\gamma^{\alpha_1/2}$, $\beta'_2 = 1 + A'_4\gamma^{\alpha_2/2}$.

Next, let us rewrite (29) as

$$f_{\gamma_i}(\gamma) = \jmath_1 + \jmath_2 \quad (30)$$

where

$$\begin{aligned} \jmath_1 = & \frac{\alpha_1\mu_h A'_1}{2k_1}\gamma^{\frac{\alpha_1\mu_h}{2}-1}(1+A'_3\gamma^{\alpha_1/2})^{-\mu_p-\mu_h} \\ & -\frac{\alpha_1\mu_h A'_1 A'_2}{2k_1k_2}\gamma^{\frac{\alpha_1\mu_h}{2}+\frac{\alpha_2\mu_f}{2}-1}(1+A'_3\gamma^{\alpha_1/2})^{-\mu_p-\mu_h} \\ & \times\sum_{n=0}^{\infty}\frac{(\mu_f)_n(\mu_g+\mu_f)_n(-1)^n}{(1+\mu_f)_n n!}\gamma^{n\alpha_2/2}{A'_4}^n \end{aligned} \quad (31)$$

$$\begin{aligned} \jmath_2 = & \frac{\alpha_2\mu_f A'_2}{2k_2}\gamma^{\frac{\alpha_2\mu_f}{2}-1}(1+A'_4\gamma^{\alpha_2/2})^{-\mu_g-\mu_f} \\ & -\frac{\alpha_2\mu_f A'_1 A'_2}{2k_1k_2}\gamma^{\frac{\alpha_2\mu_f}{2}+\frac{\alpha_1\mu_h}{2}-1}(1+A'_4\gamma^{\alpha_2/2})^{-\mu_g-\mu_f} \\ & \times\sum_{n=0}^{\infty}\frac{(\mu_h)_n(\mu_h+\mu_p)_n(-1)^n}{(1+\mu_h)_n n!}\gamma^{n\alpha_1/2}{A'_3}^n \end{aligned} \quad (32)$$

As a result, the effective capacity in (24) can be rewritten as

$$E_c(\theta) = -\frac{1}{\theta}\ln\Bigg[\underbrace{\int_0^{\infty}\left(1+\frac{\gamma Q}{N_0}\right)^{-\eta}\jmath_1 d\gamma}_{I_1} + \underbrace{\int_0^{\infty}\left(1+\frac{\gamma Q}{N_0}\right)^{-\eta}\jmath_2 d\gamma}_{I_2}\Bigg] \quad (33)$$

After rearranging terms in (31), (32), and (33), the integrals I_1 and I_2 can be rewritten as

$$\begin{aligned} I_1 = & \frac{\alpha_1\mu_h A'_1}{2k_1}I_{11} - \frac{\alpha_1\mu_h A'_1 A'_2}{2k_1k_2} \\ & \times\sum_{n=0}^{\infty}\frac{(\mu_f)_n(\mu_g+\mu_f)_n(-1)^n}{(1+\mu_f)_n n!}{A'_4}^n I_{12} \end{aligned} \quad (34)$$

$$\begin{aligned} I_2 = & \frac{\alpha_2\mu_f A'_2}{2k_2}I_{21} - \frac{\alpha_2\mu_f A'_1 A'_2}{2k_1k_2} \\ & \times\sum_{n=0}^{\infty}\frac{(\mu_h)_n(\mu_h+\mu_p)_n(-1)^n}{(1+\mu_h)_n n!}{A'_3}^n I_{22} \end{aligned} \quad (35)$$

where

$$I_{11} = \int_0^{\infty}\gamma^{\frac{\alpha_1\mu_h}{2}-1}\left(1+A'_3\gamma^{\alpha_1/2}\right)^{-\mu_p-\mu_h}\left(1+\frac{\gamma Q}{N_0}\right)^{-\eta}d\gamma \quad (36)$$

$$I_{21} = \int_0^{\infty}\gamma^{\frac{\alpha_2\mu_f}{2}-1}\left(1+A'_4\gamma^{\alpha_2/2}\right)^{-\mu_g-\mu_f}\left(1+\frac{\gamma Q}{N_0}\right)^{-\eta}d\gamma \quad (37)$$

$$\begin{aligned} I_{12} = & \int_0^{\infty}\gamma^{\frac{\alpha_1\mu_h}{2}+\frac{\alpha_2\mu_f}{2}+\frac{n\alpha_2}{2}-1}\left(1+A'_3\gamma^{\alpha_1/2}\right)^{-\mu_p-\mu_h} \\ & \times\left(1+\frac{\gamma Q}{N_0}\right)^{-\eta}d\gamma \end{aligned} \quad (38)$$

$$\begin{aligned} I_{22} = & \int_0^{\infty}\gamma^{\frac{\alpha_1\mu_h}{2}+\frac{\alpha_2\mu_f}{2}+\frac{n\alpha_1}{2}-1}\left(1+A'_4\gamma^{\alpha_2/2}\right)^{-\mu_g-\mu_f} \\ & \times\left(1+\frac{\gamma Q}{N_0}\right)^{-\eta}d\gamma \end{aligned} \quad (39)$$

To solve (36), (37), (38), and (39), we make use of the Meijer G-function to transform the expressions $\left(1+\frac{\gamma Q}{N_0}\right)^{-\eta}$, $\left(1+A'_3\gamma^{\alpha_1/2}\right)^{-\mu_p-\mu_h}$, and $\left(1+A'_4\gamma^{\alpha_2/2}\right)^{-\mu_g-\mu_f}$ into Fox H-functions. Specifically, using [20, eq. (8.3.2.21)], we obtain

$$\begin{aligned} \left(1+A'_3\gamma^{\alpha_1/2}\right)^{-\mu_p-\mu_h} = & \frac{1}{\Gamma(\mu_p+\mu_h)} \\ & \times H_{1,1}^{1,1}\left(\begin{smallmatrix}(1-\mu_p-\mu_h,1)\\(0,1)\end{smallmatrix}\middle| A'_3\gamma^{\alpha_1/2}\right) \end{aligned} \quad (40)$$

$$\begin{aligned} \left(1+A'_4\gamma^{\alpha_2/2}\right)^{-\mu_g-\mu_f} = & \frac{1}{\Gamma(\mu_g+\mu_f)} \\ & \times H_{1,1}^{1,1}\left(\begin{smallmatrix}(1-\mu_g-\mu_f,1)\\(0,1)\end{smallmatrix}\middle| A'_4\gamma^{\alpha_2/2}\right) \end{aligned} \quad (41)$$

$$\left(1+\frac{\gamma Q}{N_0}\right)^{-\eta} = \frac{1}{\Gamma(\eta)}H_{1,1}^{1,1}\left(\begin{smallmatrix}(1-\eta,1)\\(0,1)\end{smallmatrix}\middle|\frac{\gamma Q}{N_0}\right) \quad (42)$$

Then, (36), (37), (38), and (39) can be solved with the help of the Mellin transform of the product of two H-functions [12, eq. (2.6.8)] [21] giving

$$I_{11} = H^{2,2}_{2,2}\left(\left.\begin{matrix}(1-\frac{\alpha_1\mu_h}{2},\frac{\alpha_1}{2}),(1-\mu_p-\mu_h,1)\\(0,1),(\eta-\frac{\alpha_1\mu_h}{2},\frac{\alpha_1}{2}),(0,1)\end{matrix}\right| A_3'\left(\frac{Q}{N_0}\right)^{\frac{-\alpha_1}{2}}\right) \times\frac{\left(\frac{Q}{N_0}\right)^{-\frac{\alpha_1\mu_h}{2}}}{\Gamma(\mu_p+\mu_h)\Gamma(\eta)} \quad (43)$$

$$I_{21} = H^{2,2}_{2,2}\left(\left.\begin{matrix}(1-\frac{\alpha_2\mu_f}{2},\frac{\alpha_2}{2}),(1-\mu_g-\mu_f,1)\\(0,1),(\eta-\frac{\alpha_2\mu_f}{2},\frac{\alpha_2}{2}),(0,1)\end{matrix}\right| A_4'\left(\frac{Q}{N_0}\right)^{\frac{-\alpha_2}{2}}\right) \times\frac{\left(\frac{Q}{N_0}\right)^{-\frac{\alpha_2\mu_f}{2}}}{\Gamma(\mu_g+\mu_f)\Gamma(\eta)} \quad (44)$$

$$I_{12} = H^{2,2}_{2,2}\left(\left.\begin{matrix}(1-s_1,\frac{\alpha_1}{2}),(1-\mu_p-\mu_h,1)\\(0,1),(\eta-s_1,\frac{\alpha_1}{2}),(0,1)\end{matrix}\right| A_3'\left(\frac{Q}{N_0}\right)^{-\frac{\alpha_1}{2}}\right) \times\frac{\left(\frac{Q}{N_0}\right)^{-s_1}}{\Gamma(\mu_p+\mu_h)\Gamma(\eta)} \quad (45)$$

$$I_{22} = H^{2,2}_{2,2}\left(\left.\begin{matrix}(1-s_2,\frac{\alpha_2}{2}),(1-\mu_g-\mu_f,1)\\(0,1),(\eta-s_2,\frac{\alpha_2}{2}),(0,1)\end{matrix}\right| A_4'\left(\frac{Q}{N_0}\right)^{-\frac{\alpha_2}{2}}\right) \times\frac{\left(\frac{Q}{N_0}\right)^{-s_2}}{\Gamma(\mu_g+\mu_f)\Gamma(\eta)} \quad (46)$$

where $s_1 = \frac{\alpha_1\mu_h}{2}+\frac{\alpha_2\mu_f}{2}+\frac{n\alpha_2}{2}$ and $s_2 = \frac{\alpha_1\mu_h}{2}+\frac{\alpha_2\mu_f}{2}+\frac{n\alpha_1}{2}$. Substituting the obtained expressions (43), (44), (45), and (46) into (34) and (35), and the latter two results then into (33), the effective capacity of the analyzed network is given by (47).

IV. Numerical Results

In this section, the effective capacity in nats/s/Hz of the considered CCRN is numerically evaluated. In the sequel, we assume $T_f B = 1$.

Fig. 2 shows the effective capacity of the secondary network as a function of the delay constraint θ in Rayleigh ($\alpha = 2, \mu = 1$) and Nakagami-m ($\alpha = 2, \mu = m$) fading channels where m is the fading severity parameter. For the Rayleigh fading channel, the effective capacity is compared for peak interference power to noise ratios of $Q/N_0 = 0$ dB and $Q/N_0 = 2$ dB. It is interesting to note that the effective capacity of the secondary network under Rayleigh fading can alternatively be obtained using [1, eq. (17)]. As can be seen from the figure, an increase of the delay constraint θ causes the effective capacity for all the considered cases to approach zero. This indicates that there is no information received at the secondary user for the case where no delay is allowed in the system. The effective capacity is higher for Nakagami-m fading compared to Rayleigh fading with $Q/N_0 = 0$ dB. However, as the peak interference power constraint at the PU-Rx increases, Rayleigh fading with $Q/N_0 = 2$ dB offers much higher effective capacity than the rest of the analyzed scenarios.

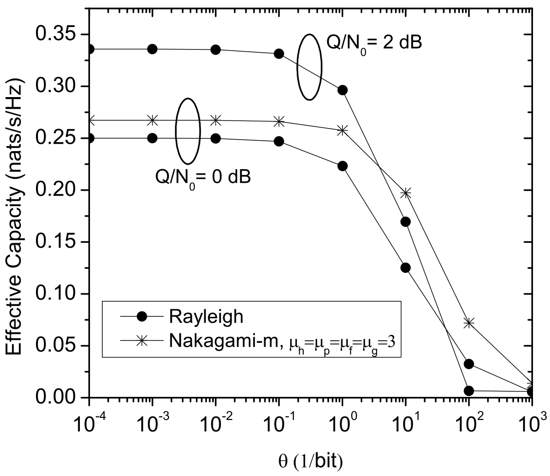

Fig. 2. Effective capacity versus delay constraint θ for an underlay cognitive radio network employing DF relaying over α-μ fading channels.

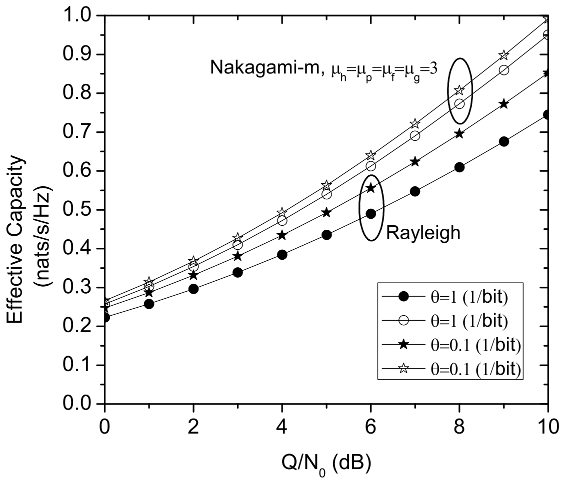

Fig. 3. Effective capacity versus peak interference power to noise ratio Q/N_0.

Fig. 3 plots the effective capacity as a function of the peak interference power to noise ratio Q/N_0 that is allowed at the PU-Rx. As can be seen from the figure, the system always performs better over the Nakagami-m fading in terms of effective capacity than over Rayleigh fading for all the analyzed scenarios.

V. Summary

In this paper, we have examined the effective capacity of an underlay CCRN with DF relaying subject to the peak interference power constraint of the primary network. An analytical expression for the effective capacity of the CCRN over α-μ fading channels has been derived by means of Fox H-functions and Meijer G-functions along with the Mellin transform of the product of two H-functions. The obtained

$$
\begin{aligned}
E_c(\theta) \;=\; & -\frac{1}{\theta}\ln\Bigg[\frac{\alpha_1\mu_h A_1'}{2k_1}\frac{\left(\frac{Q}{N_0}\right)^{-\frac{\alpha_1\mu_h}{2}}}{\Gamma(\mu_p+\mu_h)\Gamma(\eta)} H_{2,2}^{2,2}\left(\left.\begin{smallmatrix}(1-\frac{\alpha_1\mu_h}{2},\frac{\alpha_1}{2}),(1-\mu_p-\mu_h,1)\\(0,1),(\eta-\frac{\alpha_1\mu_h}{2},\frac{\alpha_1}{2}),(0,1)\end{smallmatrix}\right| A_3'\left(\frac{Q}{N_0}\right)^{\frac{-\alpha_1}{2}}\right) - \frac{\alpha_1\mu_h A_1' A_2'}{2k_1k_2} \\
& \times \sum_{n=0}^{\infty}\frac{(\mu_f)_n(\mu_g+\mu_f)_n(-1)^n}{(1+\mu_f)_n\, n!}{A_4'}^n H_{2,2}^{2,2}\left(\left.\begin{smallmatrix}(1-s_1,\frac{\alpha_1}{2}),(1-\mu_p-\mu_h,1)\\(0,1),(\eta-s_1,\frac{\alpha_1}{2}),(0,1)\end{smallmatrix}\right| A_3'\left(\frac{Q}{N_0}\right)^{-\frac{\alpha_1}{2}}\right)\frac{\left(\frac{Q}{N_0}\right)^{-(\frac{\alpha_1\mu_h}{2}+\frac{\alpha_2\mu_f}{2}+\frac{n\alpha_2}{2})}}{\Gamma(\mu_p+\mu_h)\Gamma(\eta)} + \frac{\alpha_2\mu_f A_2'}{2k_2} \\
& \times \frac{\left(\frac{Q}{N_0}\right)^{-\frac{\alpha_2\mu_f}{2}}}{\Gamma(\mu_g+\mu_f)\Gamma(\eta)} H_{2,2}^{2,2}\left(\left.\begin{smallmatrix}(1-\frac{\alpha_2\mu_f}{2},\frac{\alpha_2}{2}),(1-\mu_g-\mu_f,1)\\(0,1),(\eta-\frac{\alpha_2\mu_f}{2},\frac{\alpha_2}{2}),(0,1)\end{smallmatrix}\right| A_4'\left(\frac{Q}{N_0}\right)^{\frac{-\alpha_2}{2}}\right) - \frac{\alpha_2\mu_f A_1' A_2'}{2k_1k_2}\sum_{n=0}^{\infty}\frac{(\mu_h)_n(\mu_h+\mu_p)_n(-1)^n}{(1+\mu_h)_n\, n!}{A_3'}^n \\
& \times H_{2,2}^{2,2}\left(\left.\begin{smallmatrix}(1-s_2,\frac{\alpha_2}{2}),(1-\mu_g-\mu_f,1)\\(0,1),(\eta-s_2,\frac{\alpha_2}{2}),(0,1)\end{smallmatrix}\right| A_4'\left(\frac{Q}{N_0}\right)^{-\frac{\alpha_2}{2}}\right)\frac{\left(\frac{Q}{N_0}\right)^{-(\frac{\alpha_1\mu_h}{2}+\frac{\alpha_2\mu_f}{2}+\frac{n\alpha_1}{2})}}{\Gamma(\mu_g+\mu_f)\Gamma(\eta)}\Bigg] \qquad (47)
\end{aligned}
$$

effective capacity expression can relate the physical layer characteristics of the wireless channel with the data link layer of the CCRN as a function of the delay constraint, θ, and for different α-μ parameter settings. Numerical and simulation examples have illustrated the impact of the QoS exponent and the peak interference power constraint on the effective capacity for the CCRN under different fading channel conditions.

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