Synthesis of a first-order passive complex coefficient bandpass filter including no transformers

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Abstract: In this paper, a method for expanding the freedom of the circuit design concerned with a first-order passive complex filter including no transformers is proposed. The proposed frequency transformation is obtained by arranging the conventional one for the complex filter using loose coupling transformers. By giving appropriate specifications to the conventional frequency transformation, we can exclude transformers. As an example, a complex bandpass filter obtained from a first-order prototype real lowpass filter is designed. The proposed circuit is composed of terminating resisters, capacitors and inductors only. The validity of the proposed method is confirmed through computer simulation.

1. Introduction

Recently, many complex coefficient filters (complex filters) have been proposed in the field of analog signal processing[1]-[6]. The complex filter as well as the real coefficient filter (real filter) is important for the applications to the orthogonal communication system, the low-IF radio system and so on.

Especially, passive complex filters can be realized without active components which limit their operating frequency. However, many of them include transformers[3]-[5] equipped with two or more separated windings. Its operating frequency tends to become relatively low due to its parasitic capacitance. In order to solve this problem, a method for designing passive complex filter using inductors, capacitors and terminating resisters only has been proposed [6]. It is true that this filter including no transformers, but its upper passband edge is fixed to be infinite. Therefore, the conventional filter [6] is a kind of a complex highpass filter. In other words, a complex bandpass filter including no transformers has not been proposed.

In this paper, we solve this problem by increasing its design parameters. The proposed method has adequate design parameters to obtain complex bandpass characteristics. A design example and simulation results are shown.

2. Proposed method

2.1 Frequency transformation

Figure 1 shows the proposed frequency transformation. In this figure, the frequency axis x of the real filter is converted to that of the complex filter. This frequency transformation is obtained by appropriately using the conventional one for the complex filter using loose coupling transformers [3]. The function $f(\omega)$ is defined as

$$x = f(\omega) = -\frac{1}{a\omega} - \frac{1}{b\omega - x_s} \quad , \tag{1}$$

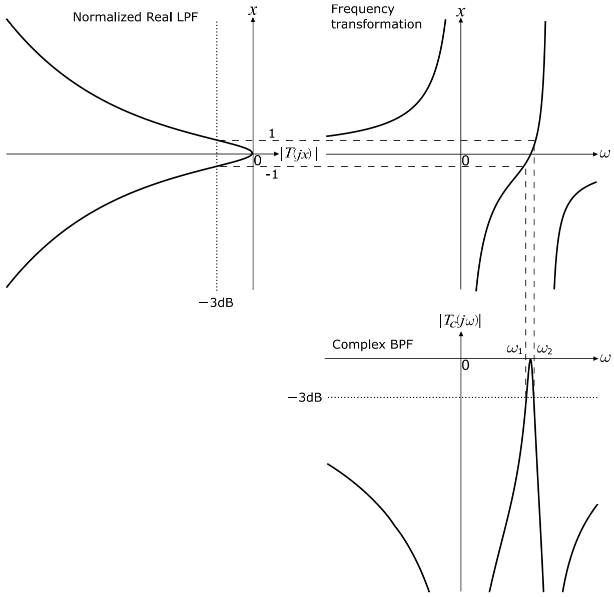
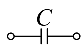
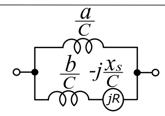
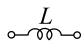
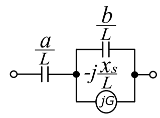


Figure 1. Frequency transformation[3].

Table 1: Element transformation.

Reference element	Transformed element
	
	

where $a > 0$, $b > 0$ and x_s is a real constant. From Fig.1, the following simultaneous equations can be obtained.

$$1 = \frac{1}{a\omega_2} - \frac{1}{b\omega_2 - x_s} \tag{2}$$

$$-1 = \frac{1}{a\omega_1} - \frac{1}{b\omega_1 - x_s} \tag{3}$$

From this frequency transformation, the inductor L included in the normalized real filter gets transformed to

$$Z_L(jx) = jxL = \frac{1}{j\omega\frac{a}{L}} + \frac{1}{j\omega\frac{b}{L} - j\frac{x_s}{L}} \quad . \tag{4}$$

The admittance Y_c of the capacitor C becomes

$$Y_c(jx) = jxC = \frac{1}{j\omega\frac{a}{C}} + \frac{1}{j\omega\frac{b}{C} - j\frac{x_s}{C}} \quad . \tag{5}$$

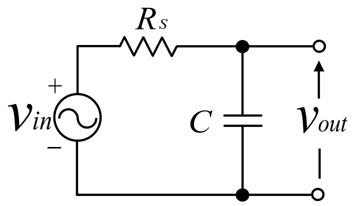


Figure 2. Prototype real filter.

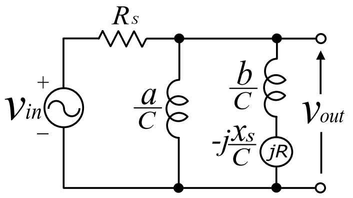


Figure 3. Prototype complex filter.

In these equations, j is an imaginary unit. Table 1 summarizes the element transformations given by Eqs. (4) and (5). As a result, the proposed circuit becomes a complex bandpass filter. From Eq. (1), it is found that the proposed transformation has three parameters(a, b, x_s).

In order to decide these parameters, it is necessary to prepare three conditions. Because the proposed filter has a complex bandpass response as shown in Fig.1, we have two conditions related to passband edges. The other is a condition for excluding the transformer described in Sect. 2.4.

Figure 2 shows a prototype real lowpass filter. Through the proposed frequency transformation, the prototype lowpass filter becomes the prototype complex filter shown in Fig.3.

2.2 Realization of imaginary resister

Figure 4 shows an imaginary resister. The relationship between v and i is given by

$$v = jRi \quad . \tag{6}$$

Decomposing voltage v and current i into their real and imaginary signal paths given by

$$\left.\begin{array}{rcl} v &=& v_r + jv_i \\ i &=& i_r + ji_i \end{array}\right\} \quad , \tag{7}$$

where subscripts r and i denote the real and the imaginary signal paths, respectively. Substituting the above equations into Eq. (6), we have

$$v_r + jv_i = -Ri_i + jRi_r \quad . \tag{8}$$

From the above equation, we have

$$\left.\begin{array}{rcl} v_r &=& -Ri_i \\ v_i &=& Ri_r \end{array}\right\} \quad . \tag{9}$$

We introduce the following v'_i and i'_i given by

$$\left.\begin{array}{rcl} v'_i &=& R_0 i_i \\ i'_i &=& \dfrac{1}{R_0} v_i \end{array}\right\} \quad , \tag{10}$$

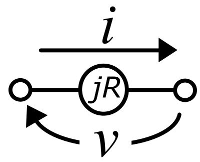


Figure 4. Imaginary resister.

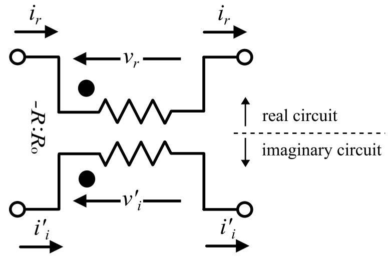


Figure 5. Ideal transformer.

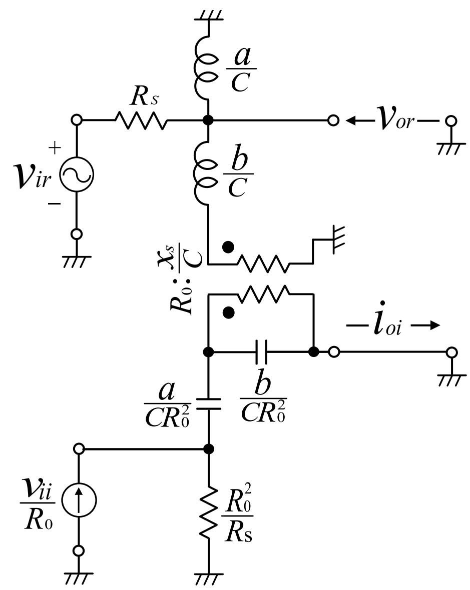


Figure 6. Realization with an ideal transformer.

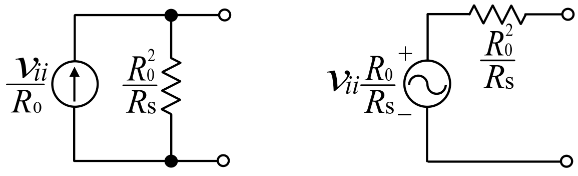
where R_0 is a positive constant. From the above equation, we have

$$\left.\begin{array}{rcl} i_i &=& \dfrac{1}{R_0} v'_i \\ v_i &=& R_0 i'_i \end{array}\right\} \quad . \tag{11}$$

Substituting this into Eq.(9) yields

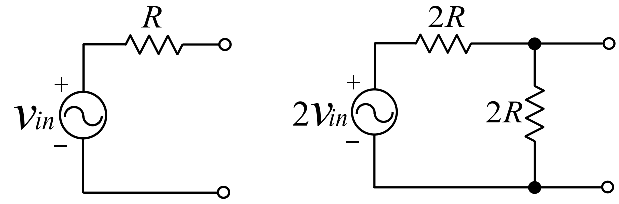
$$\begin{pmatrix} v_r \\ i_r \end{pmatrix} = \begin{pmatrix} \dfrac{-R}{R_0} & 0 \\ 0 & \dfrac{-R_0}{R} \end{pmatrix} \begin{pmatrix} v'_i \\ -i'_i \end{pmatrix} \quad . \tag{12}$$

This expresses a two-port circuit written in an F-matrix form. This indicates an ideal transformer whose turn ratio is $-R : R_0$. Figure 5 shows the ideal transformer which simulates the imaginary resister shown in Fig.4. Equation (11)



(a) Current source. (b) Voltage source.

Figure 7. Conversion a current source to a voltage source.



(a) Prototype circuit. (b) Equivalent circuit.

Figure 8. Separation of the terminating resister.

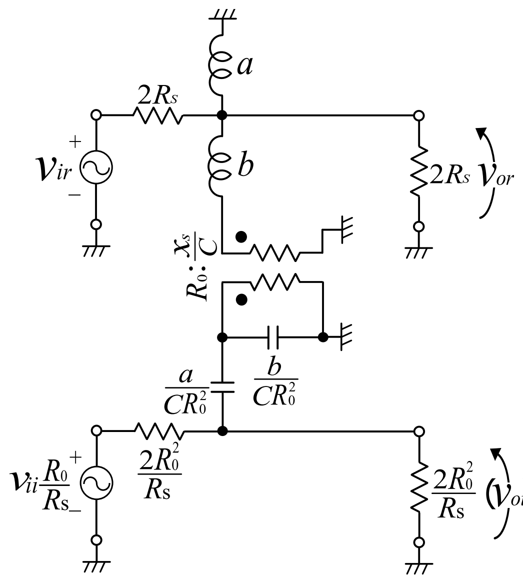


Figure 9. Equivalent circuit of Fig.6.

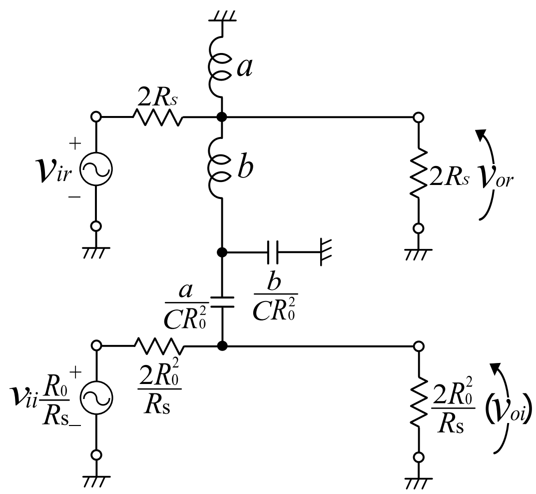


Figure 10. Proposed circuit.

indicates that the relationship between voltage and current are interchanged in the imaginary circuit. Therefore, the imaginary circuit becomes a dual circuit of the real circuit. Consequently, the circuit shown in Fig.3 can be equivalently realized by the circuit shown in Fig.6.

In this circuit v_{ir} and v_{or} are the input and the output signals of the real circuit, respectively, and v_{ii} and i_{oi} represent those of the imaginary circuit. Since the imaginary circuit is dual of the real one, the input signal source of the imaginary circuit becomes the current source v_{ii}/R_0.

2.3 Arrangements

The current source v_{ii}/R_o with internal resister R_0^2/R_s in Fig.6 can be equivalently converted into a voltage source $v_{ii}R_0/R_s$ with internal resister R_o^2/R_s. This equivalent conversion is shown in Fig.7.

Moreover, we can add terminating resisters to the output side of the circuit as shown in Fig.6 without affecting its frequency response. This equivalent conversion is shown in Fig.8. According to Thevenin's theorem, it is obvious that the circuit Figs.6 and 9 are equivalent to each other.

2.4 Excluding the ideal transformer

The turn ratio of the ideal transformer included in Fig.9 is given by the value of the imaginary resister. The turn ratio of the ideal transformer becomes $1 : 1$ when the value of the imaginary resister is $-j$. In this case, we can exclude the ideal transformer. Its condition is given by

$$x_s = CR_0 \quad . \tag{13}$$

The resulting circuit shown in Fig.10.

3. Design example

As a design example, a complex bandpass filter whose passband edges are $\omega_1 = 9.5$ and $\omega_2 = 10.5$ rad/s is designed. We set the element values of the normalized real lowpass filter shown in Fig.2 at

$$\left.\begin{array}{rcl} R_s &=& 1 \\ C &=& 1 \end{array}\right\} \quad . \tag{14}$$

Usually, $R_0 = 1$. Solving the simultaneous equations given by Eqs. (2), (3) and (13), we have

$$\left.\begin{array}{rcl} a &=& 0.022089 \\ b &=& 0.077308 \\ x_s &=& 1 \end{array}\right\} \quad . \tag{15}$$

Figure 11 shows the simulation results. In this figure, the passband area of the conventional filter is on 9.5 rad/s $-\infty$. From this figure, it is confirmed that the proposed circuit has a complex bandpass response. The image rejection ratio of the proposed circuit is 4.4dB higher than conventional one between 9.5-10.5 rad/s.

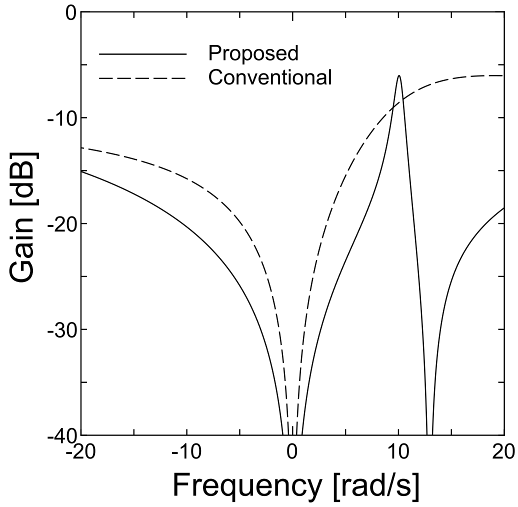


Figure 11. Simulation result.

4. Conclusion

In this paper, a synthesis of a first-order passive complex filter using no transformers has been proposed. From the simulation result, it is confirmed that the proposed circuit has the complex bandpass response.

The further investigation is required to confirm the validity of the proposed method through experiment.

References

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Appendix

A Solution of proposed frequency transformation

We set the element values of the complex bandpass filter shown in Fig.9 at

$$\left.\begin{array}{rcl} R_0 &=& 1 \\ C &=& 1 \end{array}\right\} \tag{16}$$

In order to exclude the ideal transformer, we obtain the following equation from Eq.(13).

$$x_s = CR_0 = 1 \tag{17}$$

Solving the simultaneous equations given by Eqs.(2), (3) and (17) for a and b, we have

$$\left.\begin{array}{rcl} a &=& \dfrac{\omega_D^2 \pm \sqrt{\omega_D^4 + 8\omega_2\omega_G^2\omega_D}}{4\omega_2\omega_G^2} \\ b &=& \dfrac{\omega_A^2 \pm \sqrt{\omega_A^4 - \omega_1\omega_G^2\omega_A}}{\omega_A\omega_G^2} \\ \omega_G &=& \sqrt{\omega_1\omega_2} \\ \omega_A &=& \dfrac{\omega_1+\omega_2}{2} \\ \omega_D &=& \omega_2 - \omega_1 \end{array}\right\} \tag{18}$$

Also, solving Eq.(2),for b we have

$$b = \frac{1}{\omega_2(a\omega_2+1)} \quad . \tag{19}$$

From Eq.(18), if $\omega_2 > 0$, $\omega_D > 0$ and $\omega_G > 0$, the following condition is satisfied.

$$a = \frac{\omega_D^2 + \sqrt{\omega_D^4 + 8\omega_2\omega_G^2\omega_D}}{4\omega_2\omega_G^2} > 0 \tag{20}$$

In addition, the condition of $\omega_2 > 0$, $\omega_D > 0$ and $\omega_G > 0$ are given by the following equation.

$$\omega_2 > \omega_1 > 0 \tag{21}$$

From Eq.(19), if $\omega_2 > 0$ and $a > 0$, b has a positive solution. Therefore, it can be concluded that both of a and b become positive when the condition of inequality (21) is satisfied.