Closed form of Spectral Efficiency for Non Orthogonal Multiple Access in Nakagami Fading Environment

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Abstract: In this paper, an original analysis on the uplink spectral efficiency of non-orthogonal multiple access (NOMA) is introduced. Based on our accurate approximation, a closed-form expression of the spectral efficiency is proposed. Unlike the literature, the proposed expression yields accurate values, validated by Monte-Carlo simulation, of the spectral efficiency when the channel gains are Nakagami distributed and the random number of active users are taken into account.

1. Introduction

In the era of future mobile communications, the trend of internet of things is growing. This trend inevitably introduces the massive growth of traffic volume and diversity of devices. The recent radio access technology, orthogonal frequency division multiple access (OFDMA), is therefore not the best choice in terms of spectral efficiency and power utilization [1]. In accordance with this issue, a new radio access, namely non orthogonal multiple access (NOMA) e.g. [1]-[6], is presented. Based on the principle of NOMA, individual users are allowed to share the same spectrum with power multiplexing. On the receiver side, the known successive interference cancellation (SIC) method [7] is employed to extract the desired signal from stronger interferences by cancelling (subtracting) them with superposition coding. From the results in [3], NOMA provides more 30% throughput than that of OFDMA.

In the terms of research challenge, there are many aspects waiting to be revealed on this issue. For instance, [3]-[4] focus on the spectral efficiency estimation in which all parameters are however set constantly. Also, there is some other work; for example, the analysis on the outage probability [5]and the rate optimization problem [6] on the performance of NOMA. Nevertheless, some random-nature parameters therein the work, such as channel gains and number of active users, are fixed due to the sake of simplicity.

In contrast to the literature, here we focus on the exact evaluation of the uplink spectral efficiency in Nakagami fading channel whose actual channel gains are random. Although, this lets us into the difficulty of computation and complexity of the resultant expression of spectral efficiency, we fortunately have an efficient method to cope with it. This makes our work outstanding from the other.

This paper is organized as follows. Section II illustrates the system model. Section III shows the original analysis on NOMA uplink spectral efficiency in Nakagami fading. Sequentially, the exact closed form is presented. Section IV illustrates the extension of our work to the general case in which the number of actual users is random. Section V demonstrates the numerical and simulation results. Section VI draws the conclusion of this work.

2. System Model

This section reveals the system model used entirely in this paper. In the scenario of a modern (5G) cellular network,a base station, recalled as eNodeB [2], serves its user equipments (UEs). Based on multi-carrier scheme and NOMA technique, each UE is able to transmit its information through multiple subcarriers as well as to share the same resources (same subcarriers) with one another both in frequency and time domains. Thus, the receiver at the eNodeB is required to operate SIC process in order to distinguish the desired signals from interferences. Hence, the signal received at the eNodeB can be expressed as

$$S_n = \sum_{i=1}^{N}\sum_{l=1}^{L} P_{i,l}\left|h_{i,l}\right|^2 + N_0 \qquad (1)$$

where N is the total number of UEs in the cell. L is the maximum number of subcarriers to which individual UE is allowed. $P_{i,l}$ and $\left|h_{i,l}\right|^2$ are the power and the gain of the signal from UE i on particular subcarrier l. N_0 is the power spectral of zero-mean additive Gaussian white noise (AWGN). Hint that the signal (channel) gain $h_{i,l}$; $i \in \{1,2,..,N\}, l \in 1,2,..,L\}$ models Nakagami fading. It is known that the corresponding power gain $\left|h_{i,l}\right|^2$ becomes a unit-mean gamma random variable with probability density function [8]

$$f_{|h_n|^2}(z) = \frac{z^{m-1}}{\Gamma(m)} m^m e^{-mz} \qquad (2)$$

where $0.5 \le m < \infty$ is Nakagami fading index and $\Gamma(m) = (m-1)!$ is the gamma function. When $m = 1$, the probability density function is exponentially distributed $f_{|h_n|^2}(z) = e^{-z}$ which models Rayleigh fading (non-line-of-sight fading). At this point, it is seen that Nakagami fading model covers both non-line-of-sight (Rayleigh) and line-of-sight (Rician) cases. Moreover, the dominant signal (line-of-sight signal) turns to be stronger when m goes larger.

In the next section, the famous Shannon formula is used to evaluate the spectral efficiency of uplink NOMA in Nakagami fading. Then, a new closed-form expression is proposed and extended to the general case of random users in Section 4.

2. Uplink NOMA Spectral Efficiency

With the system model declared above, the uplink spectral efficiency, in bps/Hz, of each individual mobile UE *n* is [3]

$$C_n = \sum_{l=1}^{L} \log_2\left(1 + \frac{P_{n,l}\left|h_{n,l}\right|^2}{\sum_{i=1,i\neq n}^{N} P_{i,l}\left|h_{i,l}\right|^2 + N_0}\right) \tag{3}$$

where $\sum_{i=1,i\neq n}^{N} P_{i,l}\left|h_{i,l}\right|^2$ is considered as the interferences of the desired signal $P_{n,l}\left|h_{n,l}\right|^2$ on subcarrier *l*. Due to the fact that the power gains, $\left|h_{n,l}\right|^2$; $i \in \{1,2,..,N\}, l \in 1,2,..,L\}$, are random variables, therefore the spectral efficiency becomes a function of multi random variables with the average

$$\begin{aligned} C_{n,avg} &= \sum_{l=1}^{L} E\left[\log_2(1+SINR_l)\right] \\ &= \sum_{l=1}^{L}\left\{\int_0^\infty \log_2(1+z) f_{SINR_l}(z)dz\right\} = \log_2 e \sum_{l=1}^{L}\left\{\int_0^\infty \ln(1+z) f_{SINR_l}(z)dz\right\} \end{aligned} \tag{4}$$

where SINR$_l$ (signal to interference plus noise ratio) is equal to $P_{n,l}\left|h_{n,l}\right|^2 / \sum_{i=1,i\neq n}^{N} P_{i,l}\left|h_{i,l}\right|^2 + N_0$. Unfortunately, the complexity is on the integration of the probability density function of SINR, $f_{SINR_l}(z)$. To solve this mathematical difficulty, we propose an efficient method to calculate the spectral efficiency by the following lemma.

Lemma 1: Let *u* be a unit-mean gamma random variable having a probability density function $f(u) = \frac{u^{m-1}}{\Gamma(m)} m^m e^{-mu}$ and *v* be any non-negative random variable and independent from *u*. Denote *z=u/v* in which $u = \left|h_{n,l}\right|^2$ and $v = \frac{1}{P_{n,l}\left|h_{n,l}\right|^2}\left(\sum_{i=1,i\neq n}^{N} P_{i,l}\left|h_{i,l}\right|^2 + N_0\right)$, then

$$E\left[\ln(1+z)\middle|v\right] = \int_0^\infty \frac{1}{z}\left[1 - \frac{1}{(1+z)^m}\right] MGF(mz)dz \tag{5}$$

where $MGF(mz) = E\left[e^{-zmv}\right]$ is the moment generating function (MGF) [9] of *v*.

Proof: See the proof in [10].
Recall [11], then the MGF can be solved in a simpler form,

$$MGF(mz) = e^{-zmN_0/P_{n,l}} \prod_{i=1,i\neq n}^{N}\left(1 + \frac{P_{i,l}}{P_{n,l}} z\right)^{-1}. \tag{6}$$

Replace $MGF(mz)$, derived from (6), in (5) and (4) respectively. As a result, the uplink spectral efficiency of NOMA in Nakagami fading is presented in a closed-form expression as

$$C_{n,avg} = \log_2 e \sum_{l=1}^{L}\left\{\int_0^\infty \frac{e^{-zmN_0/P_{n,l}}}{z}\left[1 - \frac{1}{(1+z)^m}\right]\prod_{i=1,i\neq n}^{N}\left(1 + \frac{P_{i,l}}{P_{n,l}} z\right)^{-1} dz\right\} \tag{7}$$

Note that this closed-form expression is for individual UE, letting say UE *n*. Assume all *N* UEs are active simultaneously, then the total spectral efficiency is as $C_{tot} = NC_n$. In practice, the number of active UEs is however random and distributed from 0,1,2,..,*N*. In the next section, we therefore take this issue into account and recalculate the proposed expression.

4. Random Number of Active UEs

Here, the number of active UEs is reasonably assumed to be binomially distributed. The probability mass function, when the number of active UEs is *k*, can be defined as

$$P(k) = \binom{N}{k} p^k (1-p)^{N-k}, k \in \{0,1,2,..,N\} \tag{8}$$

with the active probability p. When the number of interfering UEs is random, we need to recalculate $MGF(mz)$. Assume all transmitted uplink powers are identical. Recall (6) and apply Lemma 2 which is

Lemma 2: Let $V(\cdot)$ be any function and *X* be a binomial random variable with parameters (*N*, *p*). Then, $E\left[V^X(\cdot)\right] = \left[1 - p + pV(\cdot)\right]^N$.

Proof: $MGF(X) = E\left[e^{tX}\right] = \sum_{k=0}^{N} e^{tk}\binom{N}{k} p^k (1-p)^{N-k}$

$= \sum_{k=0}^{N}\binom{N}{k}\left(pe^t\right)^k (1-p)^{N-k} = \left[1 - p + pe^t\right]^N$ where *t* is a constant.

Replace e^t with any function $V(\cdot)$, thus Lemma 2 is derived.

Apply Lemma 2 to (7). The resultant spectral efficiency is

$$C_{n,avg} = \log_2 e \sum_{l=1}^{L}\left\{\int_0^\infty \frac{e^{-zmN_0/P_{n,l}}}{z}\left[1 - \frac{1}{(1+z)^m}\right]\left[1 - p + p\left(\frac{1}{1+z}\right)\right]^{N-1} dz\right\}. \tag{9}$$

Finally, the total spectral efficiency becomes $C_{tot} = NpC_n$.

5. Numerical and Simulation Results

This section presents the numerical results of NOMA uplink spectral efficiency calculated from the proposed closed-form expression in (7). The simulation, so-called Monte Carlo simulation, of (3) is used to validate our proposed expression. To accomplish such reliable results, we average out over 2,000,000 samples of the power gains. Figure 1 illustrates the impact of channel conditions on the spectral efficiency. Assume that there are 5 active UEs, each of which occupies 3 subcarriers, in the cell and their power transmissions are in the same level. Hint that signal-to-noise ratio (SNR) is defined as $SNR = P_{n,l} / N_0$ in decibel (dB). In this work, the channel conditions are represented by the integer Nakagami fading index *m*. It is known that, when *m*=1, the channel acts as Rayleigh model in which the line-of-sight between transmitters and receivers disappears. Otherwise, when *m* is larger, the dominant line-of-sight becomes more obvious and the spectral efficiency increases. Furthermore, we can see that the solid lines, representing numerical results, are matched to the cross symbols, belonging to the simulation. This validates the expression in (7).

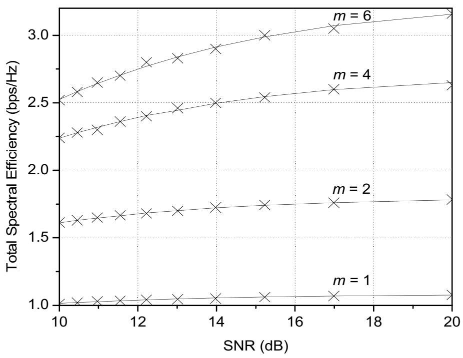

Fig. 1 *Total NOMA Spectral efficiency at various fading index; N=5, L=3, and p=1.*

In Figure 2, the total spectral efficiency versus SNR with varying active probability in (9) is concerned. Here the total number of UEs is *N*=5. Each UE use 3 subcarriers to convey its information (*L*=3). We set the active probability (*p*) varies from 0.1, 0.3, 0.5, 0.7, and 1.0. It is found that large active probability (*p*>0.5) urges UEs to transmit their signals and thus interfere one another. This leads to the drop of the overall spectral efficiency.

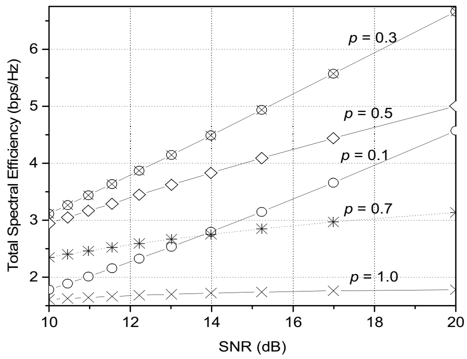

Fig. 2 *Total NOMA spectral efficiency at various active probability; N=5,m=2, and L=3*

6. Conclusion

This paper presents an analytical method to achieve a new accurate closed-form expression of uplink NOMA spectral efficiency in Nakagami fading. In contrast to the previous work, the impacts of random channel gains and number of actives users are considered. The proposed expression yields the results that benefit us to evaluate the exact average value of the spectral efficiency at different system parameters; for instance, SNR, active probability, number of employed subcarriers.

7. References

[1] Andrews J. G.*et al.*, 'What will 5G be?',*IEEE Journal on Selected Areas in Commun.*, 2014, 32, (6), pp. 1065-1082.

[2] Docomo 5G white paper, '5G radio access: requirements, concept and technologies',*NTT Docomo Inc.*, 2014.

[3] Saito Y.*et al.*, 'Non-orthogonal multiple access (NOMA) for cellular future radio access',*IEEE Proceeding of VTC Spring*, June 2013, pp.1-5

[4] BenjebbourA.*et al.*, 'Concept and practical considerations of non-orthogonal multiple access (NOMA) for future radio access',*IEEE Proceeding of ISPACS*, November 2013, pp. 770-774

[5] Z. Ding *et al.*, 'On the performance of non-orthgonal multiple access in 5G systems with randomly deployed users',*IEEE Trans. on Signal Processing Lett.*, 2014, 21, (12), pp. 1501-1505

[6] Timotheou S.*et al.*, 'Fairness for non-orthogonal multiple access in 5G systems',*IEEE Trans. on Signal Processing Lett*., 2015, 22, (10), pp. 1647-1651.

[7] Hasna M.O.*et al.*, 'Performance analysis of mobile cellular systems with successive co-channel interference cancellation',*IEEE Trans. on Wireless Commun.*, 2003, 2, (1), pp. 29-40

[8] Proakis J.G.*et al.*, 'Digital Communications', (Mc-GrawHill, New York, 2008, fifth edition)

[9] Ross S.M., 'Introduction to probability models', (Elsevier Academic Press, California, 2007, ninth edition)

[10] Sedtheetorn P.*et al.*, 'Theoretical analysis on bit error rate of VSG CDMA in Nakagami fading',*IEEE Trans. on Information Theory*, 2011, 57, (6), pp. 3405-3410

[11] SedtheetornP.*et al*., 'Accurate packet error rate analysis of variable spreading gain-code division multiaccess and multicode division multiaccess wireless communication networks', *IET Trans. on Commun*., 2011,5, (16), pp. 2407-2417