The modified Newton-Raphson algorithm using Region of Interest in EIT

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Abstract: Electrical impedance tomography (EIT) reconstructs internal resistivity distribution using the various reconstruction methods. In reconstruction methods for EIT, the modified Newton-Raphson (mNR) method is known as nonlinear least-square method which has relatively good performance. But mNR does not obtain a satisfactory image due to ill-conditioning of Hessian matrix. To solve this problem, many methods have been developed and researched. This paper proposes a new mNR, which improves a reconstruction image in point of shape and resistivity, using assumed value and region of interest (ROI).

1. Introduction

The Electrical impedance tomography (EIT) visualizes a resistivity distribution using the various reconstruction algorithms by non-invasive technique in the cross-section of the domain of interest[1-2]. EIT system consists of two part. One is hardware part that current is injected into object and voltage is measured. The other is software part that estimates internal resistivity distribution using measured voltage. Reconstruction algorithm has mathematical model, forward problem and inverse problem. Forward problem is process that inject current into electrode and then measure voltage. We used the Maxwell equations and the complete electrode model in forward problem. In general, the forward problem cannot be solved analytically. So, we have to resort to the numerical method. In this paper, the finite element method (FEM) is used to obtain the numerical solution. Inverse problem is process that estimate resistivity distribution using injected current and measured voltage. The two process reconstruct estimated resistivity distribution image in monitor. EIT has problems those are ill-posedness from low sensitivity of external measured voltage comply with change of internal resistivity. Various methods have been proposed to solve these ill-posedness problems and are still matter of concern.

Yorkey developed a modified Newton-Raphson (mNR) method for EIT image reconstruction and compared it with other existing methods. He concluded that the mathematical inverse problem can be solved adequately only by an iterative method. Several tests are presented that the mNR gave good performance. mNR reveals relatively good performance in terms of convergence rate and residual error compared to that of other methods[3].

However, the mNR method cannot give us satisfactory reconstruction images due to large modeling error, poor signal to noise ratios and ill-conditioned characteristics. Those is related with condition number that the ratio between the maximum and minimum eigenvalues of the matrix in calculation. Many methods have been proposed to solve this problem improving the conditioning the matrix.

We proposed a algorithm using ROI(Region of Interest) and assumed resistivity in order to improve reconstruction image in inverse problem. The ROI is determined by approximated region which is considered as object region in domain. And assumed resistivity is definded in base of ROI. The ROI and assumed value are obtained by previous resistivity in iterative operation but initial ROI and assumed value are obtained by the mNR.

2. Modified Newton-Raphson algorithm

The inverse problem of EIT maps the boundary voltages from experiments to resistivity image. The object function may be chosen to minimize the error in the least-square sense. The object function is follow as

$$\Phi(\rho) = \frac{1}{2}[U - V(\rho)]^T [U - V(\rho)] \tag{1}$$

where, U is the vector of measured voltage and $V(\rho)$ is the calculated boundary voltage vector that must be matched U .

To find ρ which minimizes the above object function, its derivative is set to zero as

$$\Phi'(\rho) = -[V'(\rho)]^T [U - V(\rho)] \tag{2}$$

where, $[V'] = J$ is the Jacobian matrix. The solution of the (2) uses the Newton-Raphson linearization about a resistivity vector ρ^k at k th iteration as

$$\Phi'(\rho^{k+1}) \approx \Phi'(\rho^k) + \Phi''(\rho^k)(\rho^{k+1} - \rho^k) = 0 \tag{3}$$

The term Φ'' is called the Hessian matrix, expressed as

$$\Phi'' = [V']^T [V'] + [V''][V - U] \otimes [V']^T [V'] \tag{4}$$

where $\otimes$ is the Kronecker matrix product. Since V'' is difficult to calculate and relatively small, the second term in the above (4) is usually omitted. Therefore the Hessian matrix is modified as

$$\Phi'' \approx [V']^T [V'] \tag{5}$$

Thus, the iterative equation for updating the resistivity vector based on the above object function is expressed as

$$\rho^{k+1} = \rho^k + H^{-1} J^T \left(U - V\left(\rho^k\right) \right) \tag{6}$$

where, J and $H = J^T J$ are the Jacobian and Hessian matrix, respectively. The Hessian matrix is known as ill-conditioned that degrades the performance of the image reconstruction method. To solve this problem, the object function that should be minimized is regularized as

$$\Phi(\rho) = \frac{1}{2}[U - V(\rho)]^T [U - V(\rho)] + \frac{1}{2}\alpha (R\rho)^T (R\rho) \tag{7}$$

Where, R is the regularization matrix, α is the regularization parameter.

The iterative equation for updating the resistivity vector based on the above regularized object function (7) is expressed as

$$\rho^{k+1} = \rho^k + \left(H + \alpha R^T R\right)^{-1} \left\{ J^T \left[U - V\left(\rho^k\right) \right] - \alpha R^T R \rho^k \right\} \tag{8}$$

The regularization parameter α is chosen a posteriori. The regularization matrix R can have several structures. One popular conventional method is smoothing type structure where a row of R is formed by counting all neighboring elements of the element concerning that row as

$$R_i = \left(0, \cdots 0, e_1, 0, \cdots, -\sum_1^k e_k, \cdots, 0, 0, \cdots 0, e_k, 0, \cdots, 0 \right) \tag{9}$$

where R_i is i th row of matrix R, k is the number of neighboring element, the summation notation is located i th column and $e_1, \cdots, e_k$ is -1.

Hence, it is filled with zeros except at those positions that reflect a neighboring element, which get -1. The diagonal entry is filled with the negative sum of all neighboring elements[4].

3. Proposed algorithm

The above mNR method shows poor spatial resolution in reconstruction images. In this paper, we will discuss the image reconstruction algorithm which has good performance in good shape and resistivity distribution.

We propose the new algorithm for better performance as using ROI and assumed resistivity. The ROI is approximated region which is considered as object region in domain. If the resistivity in domain is two phase(object and background), assumed resistivity can be guessed simply as a object and a background are divided by threshold. The object function is as follows:

$$\Phi(\rho) = \frac{1}{2}[U - V(\rho)]^T [U - V(\rho)] + \frac{1}{2}\alpha(\rho - \rho^*)^T W(\rho - \rho^*) \tag{10}$$

And the iterative solution is written as:

$$\rho^{k+1} = \rho^k + (H + \alpha W)^{-1} \left\{ J^T \left[U - V\left(\rho^k\right) \right] - \alpha W\left(\rho^k - \rho^*\right) \right\} \tag{11}$$

where, W is a diagonal weighing matrix. ρ^* is a assumed resistivity vector.

W and ρ^* are obtained from the previous resistivity every iteration. The sequence of finding them is as follow.

First, we classify two regions which are ROI and background region with previous ρ by proper criterion. In this paper, the criterion is calculated as follows:

$$\mu_i = \frac{\rho_i}{\rho_{avg}} \tag{12}$$

where ρ_{avg} is average value of resistivity from image obtained by previous ρ, ρ_i and μ_i are each i th element of ρ, μ. The initial ρ is made of a first iteration result of the mNR.

Second, if the criterion μ_i is larger than threshold value μ_{TH} which is scalar and posteriori, then μ_i is included in ROI, else is included in background region(In this paper, we assume a object resistiviy is larger than background resistivity).

Third, W_i in ROI has smaller value than 1 and W_i in background region has the 1 of value.

$$W_i = \begin{cases} W_i < 1, & \text{for ROI} \\ W_i = 1, & \text{for background region} \end{cases} \tag{13}$$

Where W_i is i th element of W. We use the diagonalized matrix.

$$W = diag(W) \tag{14}$$

Hence elements of ROI are more sensitive to change of voltages and elements of background region are affected from previous resistivity. After all, the condition number of Hessian matrix is reduced by this process.

Fourth, ρ^* is saturated by each average resistivity of ROI and background region made of previous resistivity.

$$\rho_i^* = \begin{cases} \text{average of ROI}, & \text{in ROI} \\ \text{average of Background region}, & \text{else} \end{cases} \tag{15}$$

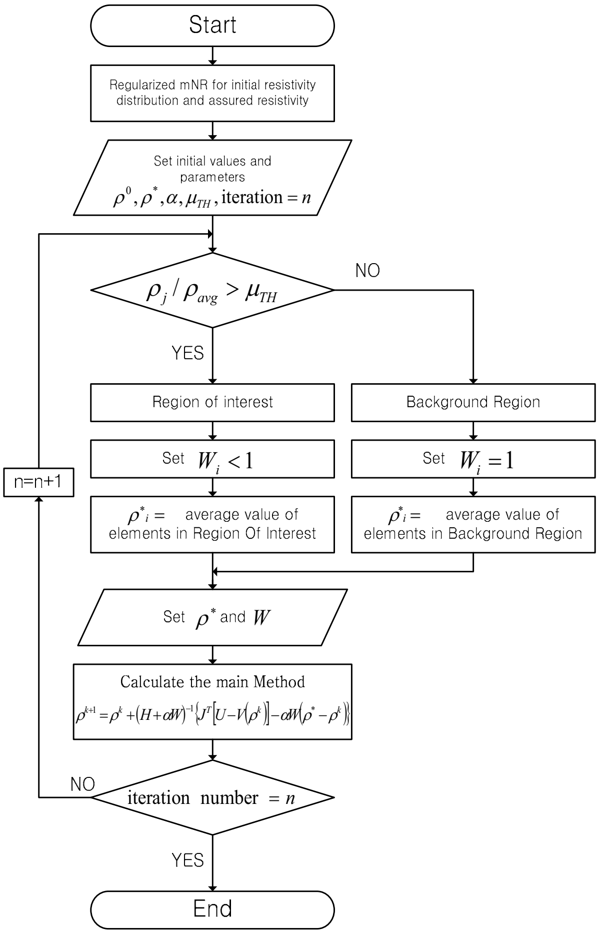


Fig 1. Flow chart of proposed method

4. Computer simulation

The proposed algorithm has been tested by comparing its results for numerical simulations with the mNR algorithm.

4.1 The true image for simulation

The resistivity values of the objet and the background are set to 600 Ωcm and 300 Ωcm, respectively. We simulated for follow case. Fig. 2 shows true resistivity distributions. Case 1 is that a target exists at the centre of phantom. Case 2 is that a target exists near the boundary of the phantom. The color bar represents resistivity of domain. The parameters in this simulation are $\alpha = 1 \times 10^{-6}$, $\mu_{TH} = 1$.

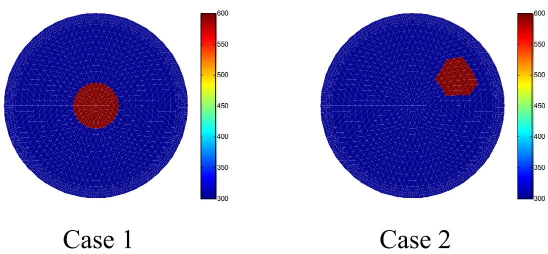


Fig. 2 True images

4.2 Reconstructed images

Fig. 3 and 4 are the reconstruction images for the true images. Rows and columns are algorithms and iteration number, respectively. We display 1st and 10th iteration reconstruction images for each algorithm.

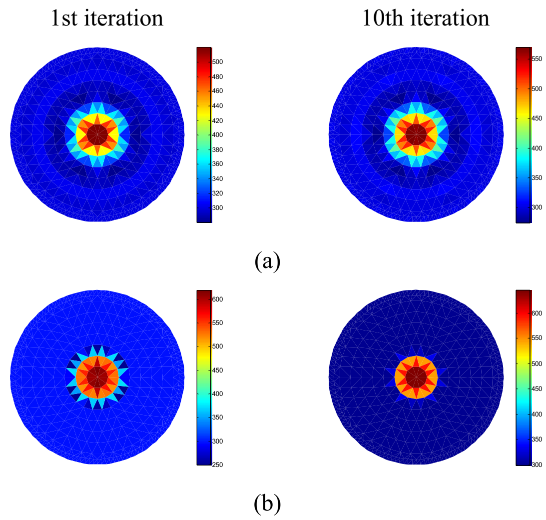


Fig. 3 Reconstructed images for Case 1;
(a) mNR (b) proposed algorithm.

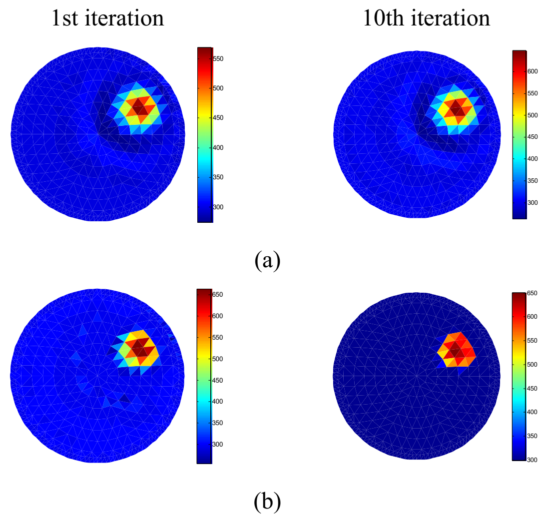


Fig. 4 Reconstructed images for Case 2;
(a) mNR (b) proposed algorithm.

In the mNR, 10th reconstruction image is similar to 1st iteration. Though initial convergence performance is good, the resistivity distribution is too smooth. The proposed algorithm initially reconstructs shape of true image and similarly estimate true image after several iterations.

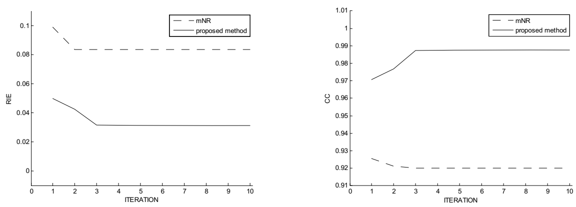
For comparison and evaluation, we define relative image error (IE) and correlation coefficient (CC) as follow

$$IE = \frac{\|\rho - \rho_{true}\|}{\|\rho_{true}\|} \qquad (16)$$

$$CC = \frac{\sum_{i=1}^{M}(\rho(i)-\overline{\rho})(\rho(i)_{true}-\overline{\rho}_{true})}{\sqrt{\sum_{i=1}^{M}(\rho(i)-\overline{\rho})^2 \sum_{i=1}^{M}(\rho(i)_{true}-\overline{\rho}_{true})^2}} \quad (17)$$

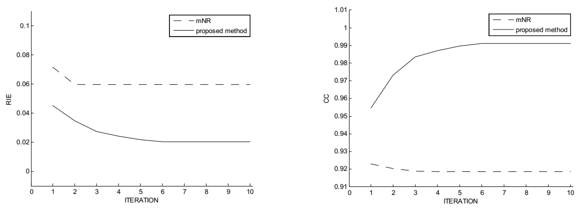
where, ρ and ρ_{true} are estimated resistivity and true resistivity respectively, $\rho(i)$ and $\rho(i)_{true}$ are element of ρ and ρ_{true}, $\overline{\rho}$ and $\overline{\rho}_{true}$ are average of ρ and ρ_{true}.

The smaller IE and the closer CC to 1 give the better reconstruction performance.



IE CC

(a) Case 1



IE CC

(b) Case 2

Fig. 5 IE and CC for each Cases.
Dot line and solid line are
mNR, proposed algorithm, respectively.

5. Conclusion

It is very difficult to image reconstruction due to limit data in EIT. In this paper, We proposed a algorithm that is based on the Newton-Rapshon algorithm, using ROI and assumed resistivity. The ROI and assumed resistivity are estimated from reconstructed image in previous iterative operation. The proposed algorithm shows a improved image quality and resistivity distribution. We has verified the proposed algorithm as comparing with mNR in computer simulation.

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