A Consideration of Noise Cancellation by using PCA-ICA Method with Delay Estimation

Toshiaki Yokoda[1], Mohammad Reza Alsharif[1], Mahdi Khosravy[1]
[1]Department of Information Engineering, University of the Ryukyus
Senbaru 1, Nishihara, Okinawa, Japan
E-mail : k078521@eve.u-ryukyu.ac.jp, asharif@ie.u-ryukyu.ac.jp, mahdikhosravy@yahoo.com

Abstract: The noise cancellation technologies are useful for speech recognition and other applications. There are some kind of methods for cancellation of background noise. In this paper, The desired signals are separated from background noise by using proposed PCA-ICA method (Principal Component Analysis, Independent Component Analysis). The proposed PCA-ICA method requires several number of observed signals that is, the same as the number of sources. The noise signal can be removed in the same way as BSS (Blind Source Separation). We have done experiments (two-observation one-source one-noise, with delay) and evaluated the results.

1. Introduction

Usually, some different speech signals that are recorded with PCM are mutually independent. Independent component analysis (ICA) uses only the fact that "Source signals are mutually independent". On the other hand, Principal component analysis (PCA) calculates the correlation between two sources and searching the decorrelation transform. These two methods perform different result for blind source separation because of difference of statistics. Figure.1 shows the histogram of Gaussian noise and Audio signal. Audio signals are mostly super-gaussian distribution, and there simultaneous distribution is shown in figure.2 for two audio signals. We can recognize the two orthogonal axes in this scatter plot. PCA can not recognize these two orthogonal axes, but it has important function for decorrelation process. Our research uses PCA-ICA method for separation of the audio source signal from background noises.

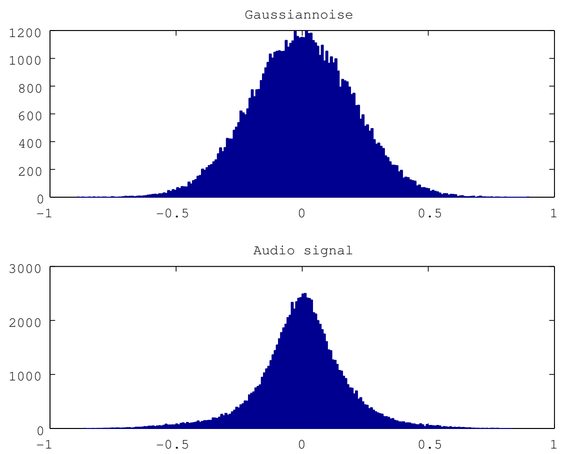

Figure 1. Histogram of Gaussian noise and audio signal.

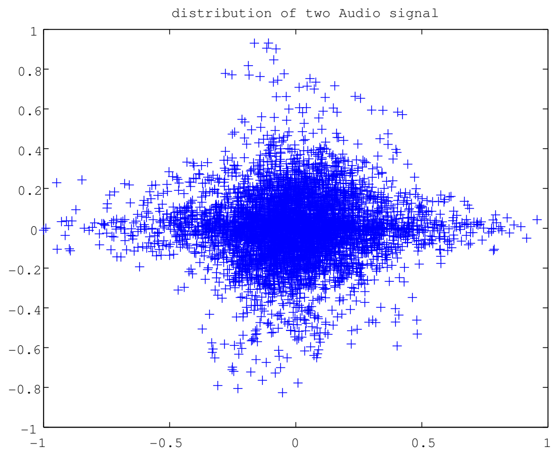

Figure 2. 5000-sample scatter plot of two audio signals.

2. Problem Definition

We have two source signals that are statistically independent and their number is the same as the number of observation signals. There are different attenuation when signals transmit to each microphones from each sources. Usually, we call the transfer matrix as "Mixing matrix" $\mathbf{A} = \left((\mathbf{a_{11}}, \mathbf{a_{21}})^{\mathbf{T}}, (\mathbf{a_{12}}, \mathbf{a_{22}})^{\mathbf{T}}\right)$. Then, observation signals $\mathbf{x} = (\mathbf{x_1}, \mathbf{x_2})^{\mathbf{T}}$ are represented as

$$\mathbf{x} = \mathbf{A}\mathbf{s}. \tag{1}$$

Where $\mathbf{s} = (\mathbf{s_1}, \mathbf{s_2})^{\mathbf{T}}$ is source signals. We can find "Unmixing matrix" $\mathbf{W} = \left((\mathbf{w_{11}}, \mathbf{w_{21}})^{\mathbf{T}}, (\mathbf{w_{12}}, \mathbf{w_{22}})^{\mathbf{T}}\right)$ easily if we have the information about matrix $\mathbf{A}$.

$$\mathbf{y} = \mathbf{W}\mathbf{x} = \mathbf{W}\mathbf{A}\mathbf{s} = \mathbf{A}^{-1}\mathbf{A}\mathbf{s} = \mathbf{s} \tag{2}$$

Where $\mathbf{y} = (\mathbf{y_1}, \mathbf{y_2})^{\mathbf{T}}$ However, BSS problem has no information about mixing matrix $\mathbf{A}$ and source signals. And the problem assumes only that source signals are mutually independent. The method are already established to solving the BSS problems [1].

In our research, we defined the new problem as the observation signals include delay. The position of microphones and sources are symmetric as shown in Figure.3. Now, we define d_1 as distance between desired signal source and microphone 1, and also between noise source and microphone 2. d_2 is distance between desired signal source and microphone 2, and also between noise source and microphone 1.

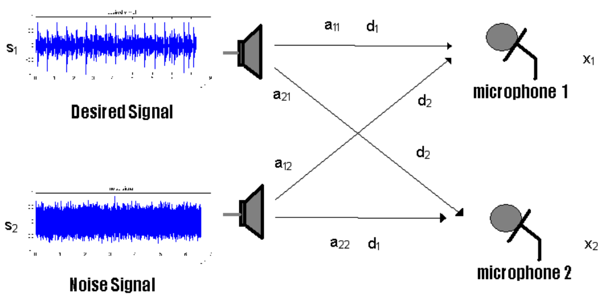

Figure 3. The problem of Blind Source Separation with delay.

The difference of samples between coming signals from two sources to each microphones is D. For example, at sampling frequency = 11025Hz, d_1 = 2m and d_2 = 3.5m, the difference of samples D should be about 50-samples. In this problem, the observation signals $\mathbf{x(t)} = (\mathbf{x_1(t)}, \mathbf{x_2(t)})^{\mathbf{T}}$ are represented as below.

$$x_1(t) = a_{11}s_1(t) + a_{12}s_2(t - D) \tag{3}$$

$$x_2(t) = a_{21}s_1(t - D) + a_{22}s_2(t) \tag{4}$$

Also this formula can be represented as follows

$$\mathbf{x} = \mathbf{A}\mathbf{s} = \begin{bmatrix} a_{11} & a_{12}z^{-D} \\ a_{21}z^{-D} & a_{22} \end{bmatrix} \begin{bmatrix} \mathbf{s_1} \\ \mathbf{s_2} \end{bmatrix} \tag{5}$$

Figure.4 and Figure.5 show an example of the source signals and observation signals.

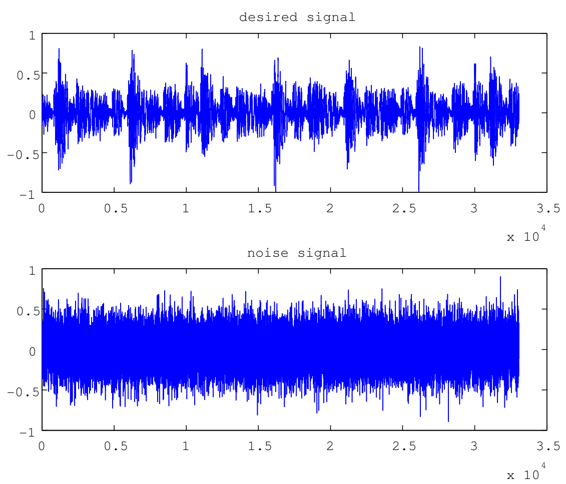

Figure 4. Desired signal and noise signal

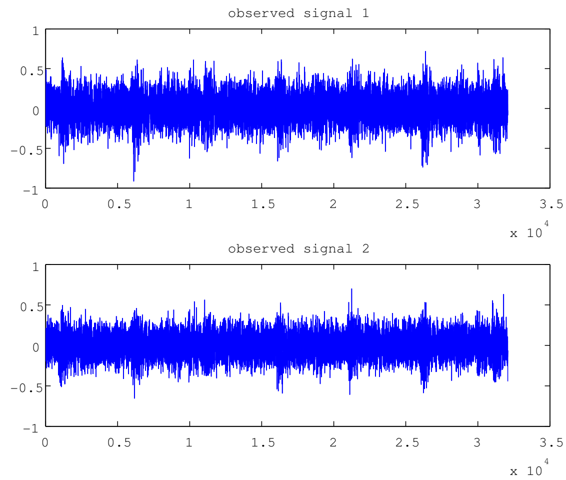

Figure 5. Two observed signals.

surement. The corss-correlation is defined as

$$R_{x_1x_2}(\tau) = \sum_t x_1(t)x_2(t - \tau) \tag{6}$$

In this problem, $R_{x_1x_2}$ has two peaks as shown in Figure.6. These peaks imply samples of delay. Assume these peaks can be measured is τ_1, τ_2. Then, we generate delayed observed signals $x_2^{\tau_1}(t) = x_2(t + \tau_1)$ and $x_2^{\tau_2}(t) = x_2(t + \tau_2)$. Finally, $\mathbf{x}_{\tau_1} = (\mathbf{x_1}, \mathbf{x_2^{\tau_1}})^{\mathbf{T}}$ and $\mathbf{x}_{\tau_2} = (\mathbf{x_1}, \mathbf{x_2^{\tau_2}})^{\mathbf{T}}$ will be used for unmixing procedures.

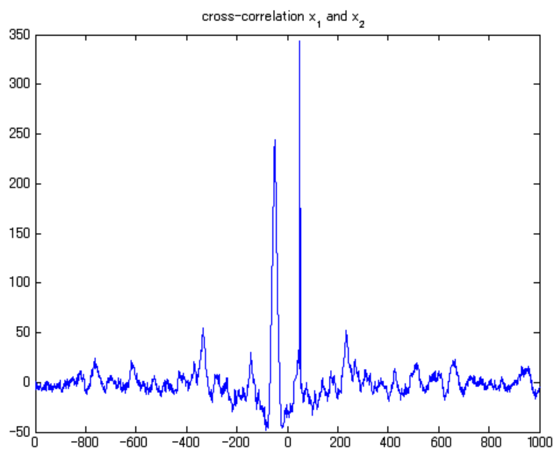

Figure 6. Cross-correlation between x_1 and x_2

3. Proposed Method

The proposed method is constructed by three steps for solving the defined problem.

3.1 Delay Estimation and Synchronization

The first step measures the number of samples of delay between two observed signals by using cross-correlation mea-

3.2 Decorrelation with PCA

+ Second step makes whitening matrix for $\mathbf{x}_{\tau_1}$ and $\mathbf{x}_{\tau_2}$ using PCA. Here eigenvector $\mathbf{e} = (\mathbf{e_1}, \mathbf{e_2})$ of covariance matrix $C_{\mathbf{x}}^{\tau_1} = E[\mathbf{x}_{\tau_1}\mathbf{x}_{\tau_1}^{\mathbf{T}}]$, and $\mathbf{\Lambda} = \mathbf{diag}(\lambda_1, \lambda_2)$ is diagonal matrix of eigenvalue of $C_x^{\tau_1}$. The matrix $\mathbf{V}$:

$$\mathbf{V} = \mathbf{\Lambda}^{-\frac{1}{2}}\mathbf{e}^{\mathbf{T}} \tag{7}$$

is "Whitening matrix" that can make uncorrelated signal. $\mathbf{V}$ is used for the determining of initial value of $\mathbf{W}$.

3.3 Searching for W with ICA

Third step searches unmixing matrix $\mathbf{W}$ classically with Kullback-Leibler divergence [2] is defined below.

$$KL(\mathbf{W}) = -\mathbf{H}(\mathbf{Y};\mathbf{W}) + \sum_{\mathbf{i}} \mathbf{H}(\mathbf{Y_i};\mathbf{W}) \tag{8}$$

Here, $H(\mathbf{Y};\mathbf{W})$ is entropy of joint probability, $H(Y_i;\mathbf{W})$ is entropy of the marginal probability. $\mathbf{y_1}$ and $\mathbf{y_2}$ are mutually independent means that $KL(\mathbf{W})$ is reaching to zero. We used steepest descent method for searching optimum $\mathbf{W}$. ϕ is partial derivatives of approximated probability density function,

$$\phi(\mathbf{y}) = -\left(\frac{\partial \log \mathbf{p}(\mathbf{y_1})}{\partial \mathbf{y_1}}, \frac{\partial \log \mathbf{p}(\mathbf{y_2})}{\partial \mathbf{y_2}}\right)^{\mathbf{T}} \tag{9}$$

and the $\mathbf{W}$ will be converged by update rule below.

$$\mathbf{W_{t+1}} = \mathbf{W_t} + \mu(\mathbf{I} - \phi(\mathbf{y})\mathbf{y^T})\mathbf{W_t} \tag{10}$$

If $\mathbf{W}$ is unmixing matrix, unmixed signals $\mathbf{y_1}, \mathbf{y_2}$ should be reached into source signals. Because if coefficients of $a_{11}w_{11}$ and $-a_{21}w_{12}$ is almost same, and $\mathbf{x_1}$, $\mathbf{x_2^{\tau_1}}$ are exactly matching to $\mathbf{s_1}$. Then $\mathbf{s_1}$ of unmixed signals $\mathbf{y}$ is cancelated. The remaining output $\mathbf{y_1}$ or $\mathbf{y_2}$ reaches to the one of source signals. The same procedure also will be performed for $\mathbf{x_2^{\tau_1}}$.

3.4 Block Diagram

Figure.7 shows the block diagram of proposed method.

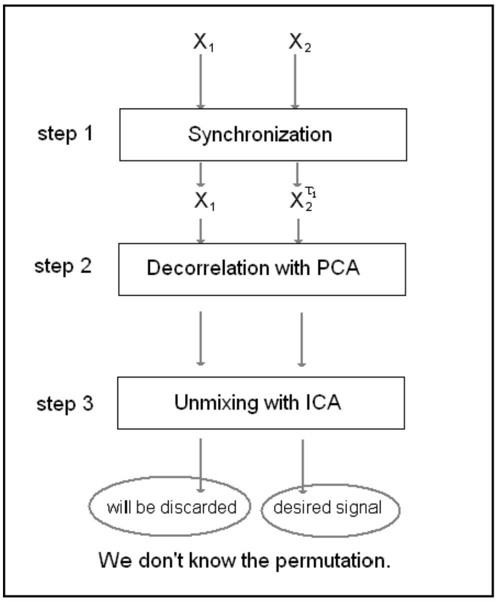

Figure 7. Block diagram of proposed method.

We must pay attention to the permutation problem. The two signals are permuted by step2 and step3.

4. Experiment and Evaluation

We compared our proposal method and the classical method for blind source separation with delay. A music source and Gaussian-noise source are used for this experiment (See Figure.4). In this experiment, we define the delay D = 50-samples. The separation by our proposal method with delay estimation exhibits better result than the classical method without delay estimation.

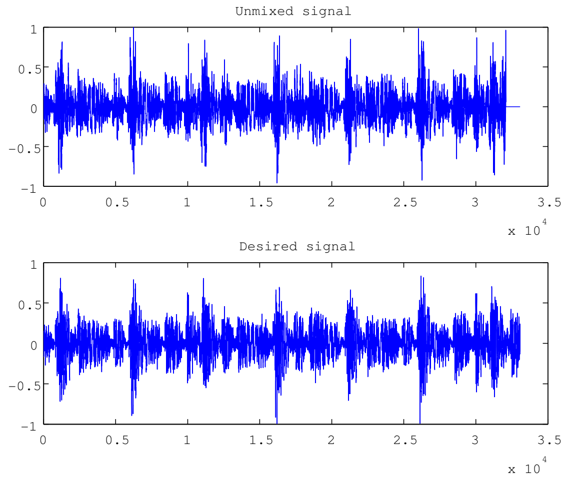

Figure 8. Result: BSS by our proposal method.

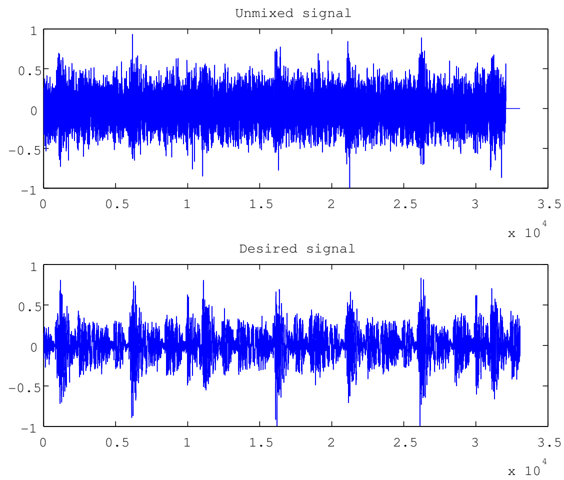

Figure 9. Result: BSS by classical method.

Figure.8 shows the result of unmixing by the proposed method and source signal. Figure.9 is the result of the classical method. There are obviously differences between two methods results.

4.1 Asymmetric Position

On the another situation that the positions of microphones and sources are asymmetric, It will also work well. Now we define the asymmetric transfer function,

$$x_1(t) = a_{11}s_1(t - D_1) + a_{12}s_2(t - D_2) \quad (11)$$
$$x_2(t) = a_{21}s_1(t - D_3) + a_{22}s_2(t - D_4) \quad (12)$$

Here, $D_1 \neq D_2 \neq D_3 \neq D_4$. In this situation, we can find two peaks from cross-correlation (Figure.10) as same as symmetrical situation. The noise cancellation also has been

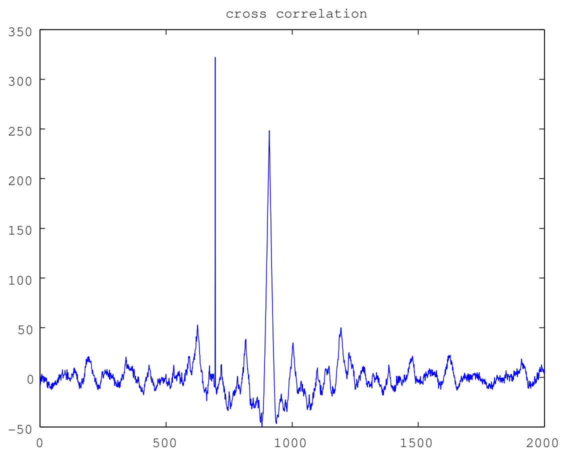

Figure 10. Cross-correlation in asymmetrical situation.

performed in this situation, and a satisfactory result obtained the same as symmetric one.

4.2 Function of PCA

In this research, we discovered that whitening process by PCA has very important function in the case of mixture with delay makes high correlation between the observation signals. Figure.11 shows scatter plot of real situation. Assume the

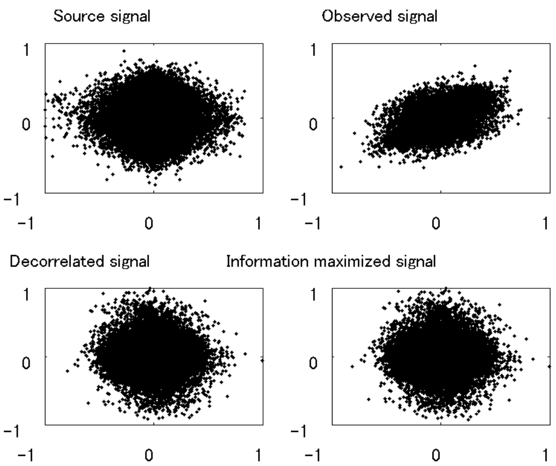

Figure 11. Four scatter plot (Sourcesignal, Observation, Decorrelation, Unmixing with ICA)

$D_i = 0$, the observation signals have only one peaks in cross-correlation. we can not find sources without ICA in this way.

Figure.12 shows scatter plot without delay. However, the de-

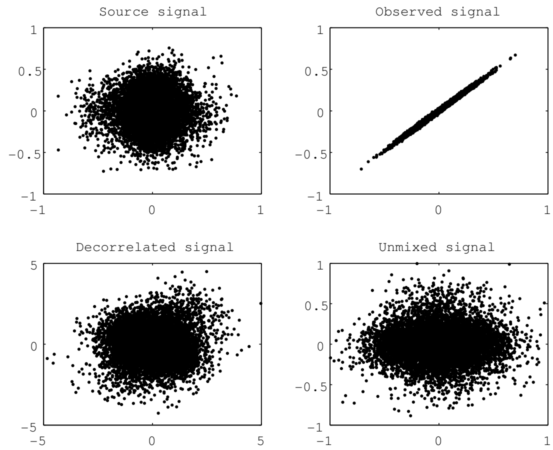

Figure 12. Four scatter plot (Sourcesignal, Observation, Decorrelation, Unmixing with ICA), without delay

lay should be included in real situation. Then the decorrelation process becomes useful method.

5. Conclusion

Blind source separation is difficult for convoluted observation signals [3] [4], and also for our proposed method. The classical method can not separate the signals that are convoluted mixture and or delay. On the other hand, our method that can estimate delay is more useful for the signals include delay.

References

[1] A. Hyvärinen, J.Karhunen, and E. Oja, "Independent Component Analysis", *John Wiley and Sons*, 2001.

[2] A. J. Bell and T. J. Sejnowski, "An information-maximization approach to blind separation and blind deconvolution", *Neural Computation*, vol.7, 1995.

[3] Shoko Araki, Shoji Makino, Robert Aichner, Tsuyoki Nishikawa and Hiroshi Saruwatari, "Subband-based Blind Separation for Convolutive Mixtures of Speech", *IEICE trans. Fundamentals*, Vol.E88-A, No.12, December 2005.

[4] Shoko Araki, Ryo Mukai, Shoji Makino, Tsuyoki Nishikawa and Hiroshi Saruwatari, "The Fundamental Limitation of Frequency Domain Blind Source Separation for Convolutive Mixtures of Speech", *IEEE Transactions on speech and audio processing*, Vol.11, No 2, March 2003.