Well-conditioned EFIE-TDS Surface Integral Equation

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***Abstract*—A well-conditioned coupled set of electric field integral equation (EFIE) and thin dielectric sheet (TDS) approximation surface integral equations for analyzing densely discretized composite structures with perfect electric conductor (PEC) and thin dielectric layer is proposed. Whereas TDS operator is well-posed, a EFIE operator is ill-posed when applied to densely discretized surfaces. This makes the coupled EFIE and TDS linear system ill-conditioned, and its iterative solution inefficient or even impossible. The proposed method regularizes the coupled set of EFIE-TDS using a Calderon multiplicative preconditioner (CMP) technique. The resulting linear system enables the efficient analysis of composite structures with PEC and thin dielectric layer. Numerical example validates the efficiency of the proposed method**

I. Introduction

Mixed structures with perfect electric conductor (PEC) and thin dielectric layer such as conformal antenna, frequency selective surface (FSS) radome and coated conductors are widely used. It is important to develop proper numerical techniques for the analysis of mixed PEC and thin dielectric structures. In general, volume integral equation with tetrahedral modeling or surface integral equation with triangle modeling are used [1]. But, it is more efficient to use thin dielectric sheet approximation for analysis of thin dielectric layer [2]. Additionally, electric field integral equation (EFIE) is combined with thin dielectric sheet (TDS) approximation surface integral equation to analyze both PEC and thin dielectric layer [3]. Coupled EFIE and TDS surface integral equation has fewer unknowns than volume integral equation or surface integral equation. While EFIE is ill-posed, TDS is well-posed because TDS is derived from volume integral equation. Therefore, MoM linear system obtained upon discretizing coupled EFIE and TDS surface integral equation is ill-posed and iterative number to solve matrix equation is large. Researches that apply Calderon identities to improve condition number of impedance matrix in method of moments (MoM) have been announced [4], [5]. Self-regularizing property of the EFIE operator, i.e., the fact that its square has a bounded spectrum makes MoM matrices well-conditioned. This paper proposed a well-conditioned coupled EFIE and TDS surface integral equation with Calderon identities.

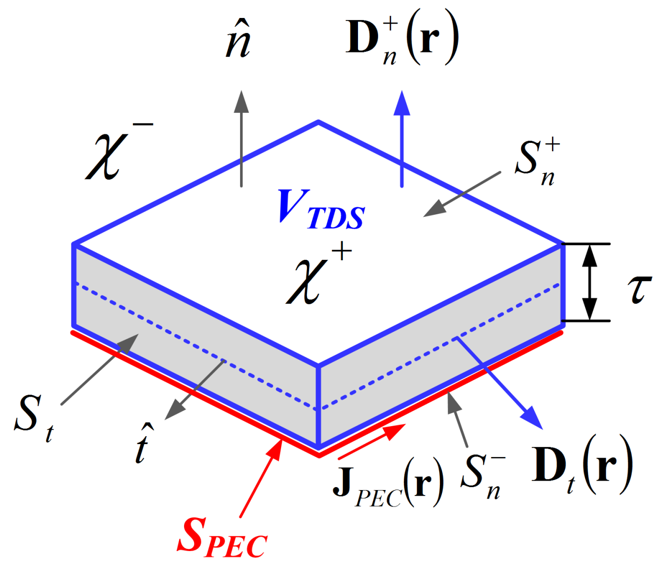

Fig. 1. Structure of thin dielectric layer and PEC

II. Formulation

A. EFIE-TDS surface integral equation

Consider a PEC enclosed by the surface S_{PEC} and a TDS with volume V_{TDS}, thickness τ and dielectric constant ϵ_r in free space. V_{TDS} is confined by the top, bottom and tangential surfaces S_n^+, S_n^- and S_t respectively. Enforcing electric field boundary condition and consistency conditions on S_{PEC} and in V_{TDS} yields [3]

$$\left[\mathcal{L}^S \mathbf{J}_p + \mathcal{L}^{V_t}\mathbf{D}_t + \mathcal{L}^{V_n}\mathbf{D}_n^+\right]_{tan} = \mathbf{E}_{tan}^{inc}(\mathbf{r}) \quad \forall \mathbf{r} \in S_{PEC} \quad (1)$$

$$\mathcal{L}^S \mathbf{J}_p + \mathcal{L}^{V_t \cdot I}\mathbf{D}_t + \mathcal{L}^{V_n \cdot I}\mathbf{D}_n^+ = \mathbf{E}^{inc}(\mathbf{r}) \quad \forall \mathbf{r} \in V_{TDS}. \quad (2)$$

Operators for obtaining scattered fields from surface currents on PEC surfaces and electric flux in TDS volume are

$$\mathcal{L}^S\mathbf{J}_p = i\omega\mu_0\int_S \mathbf{J}_p(\mathbf{r}')G(\mathbf{r},\mathbf{r}')dS' \\ -\frac{1}{i\omega\epsilon_0}\nabla\int_S \nabla'\cdot\mathbf{J}_p(\mathbf{r}')G(\mathbf{r},\mathbf{r}')dS' \tag{3}$$

$$\mathcal{L}^{V_t}\mathbf{D}_t = -\frac{k_0^2\tau}{\epsilon_0}\int_S \kappa(\mathbf{r}')\mathbf{D}_t(\mathbf{r}')g(\mathbf{r},\mathbf{r}')dS' \\ -\frac{\tau\nabla}{\epsilon_0}\left\{\int_C \Delta\kappa_t\hat{t}\cdot\mathbf{D}_t(\mathbf{r}')g(\mathbf{r},\mathbf{r}')dl' \right. \\ \left. -\int_{S_n^-}\Delta\kappa_n^-\nabla_t'\cdot\mathbf{D}_t(\mathbf{r}')g(\mathbf{r},\mathbf{r}')dS'\right\} \tag{4}$$

$$\mathcal{L}^{V_n}\mathbf{D}_n^+ = -\frac{k_0^2\tau}{\epsilon_0}\int_S \kappa(\mathbf{r}')\mathbf{D}_n^+(\mathbf{r}')g(\mathbf{r},\mathbf{r}')dS' \\ -\frac{\nabla}{\epsilon_0}\left\{\int_{S_n^+}\Delta\kappa_n^+\hat{n}\cdot\mathbf{D}_n^+(\mathbf{r}')g(\mathbf{r},\mathbf{r}')dS' \right. \\ \left. -\int_{S_n^-}\Delta\kappa_n^-\hat{n}\cdot\mathbf{D}_n^+(\mathbf{r}')g(\mathbf{r},\mathbf{r}')dS'\right\} \tag{5}$$

$$\mathcal{L}^{V_t\cdot I}\mathbf{D}_t = \frac{\mathbf{D}_t(\mathbf{r})}{\epsilon_0\epsilon_r} - \mathcal{L}^{V_t}\mathbf{D}_t \tag{6}$$

$$\mathcal{L}^{V_n\cdot I}\mathbf{D}_n^+ = \frac{\mathbf{D}_n^+(\mathbf{r})}{\epsilon_0\epsilon_r} - \mathcal{L}^{V_n}\mathbf{D}_n^+ \tag{7}$$

where

$$\kappa(\mathbf{r}) = 1 - \frac{1}{\epsilon_r(\mathbf{r})} \tag{8}$$

$$G(\mathbf{r},\mathbf{r}') = \frac{e^{-jk_0|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} \tag{9}$$

$$\begin{cases} \Delta\kappa_t = \kappa^+ - \kappa^-, & \mathbf{r}\in S_t \\ \Delta\kappa_n^+ = \kappa^+ - \kappa^-, & \mathbf{r}\in S_n^+ \\ \Delta\kappa_n^- = \kappa^+ - \kappa^-, & \mathbf{r}\in S_n^- \end{cases} \tag{10}$$

κ^+ and κ^- is contrast ration inside and outside dielectric domain. Arbitrary shapes of PEC and TDS structures can be analyzed with triangle modeling. To solve coupled EFIE and TDS by MoM, Rao-Wilton-Glisson (RWG) function, modified RWG function, and pulse function are used for electric surface current, tangential electric flux and normal electric flux, respectively.

B. Calderon multiplicative preconditioner (CMP) for EFIE-TDS surface integral equation

EFIE-TDS linear system tend to ill-posed due to property of EFIE operator. Unbounded property of EFIE can be improved using Calderon identity.

$$\hat{\mathbf{n}}(\mathbf{r})\times\mathcal{L}^S(\hat{\mathbf{n}}(\mathbf{r})\times\mathcal{L}^S) = -\frac{1}{4} + KK \tag{11}$$

where

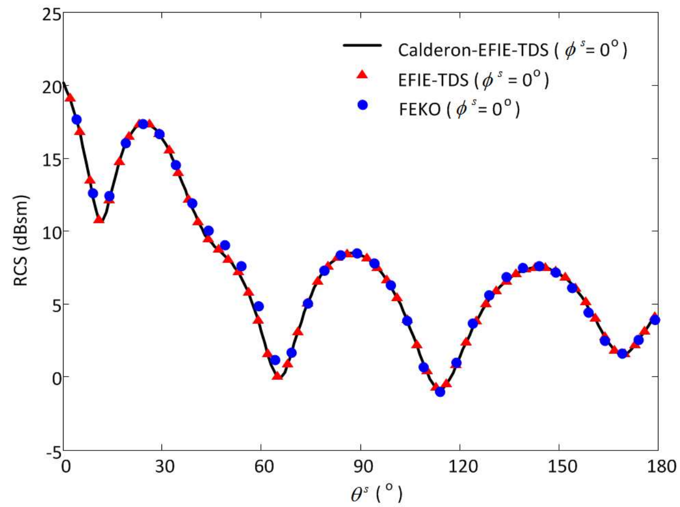

Fig. 2. RCS of a thin dielectric sphere shell and PEC sphere at $\phi^s = 0°$

$$K\mathbf{J} = \hat{\mathbf{n}}(\mathbf{r})\times\nabla\times\int_S \mathbf{J}(\mathbf{r}')G(\mathbf{r},\mathbf{r}')d\mathbf{r}' \tag{12}$$

K is a compact operator when acting on smooth surface. EFIE-TDS can be regularized using $\hat{\mathbf{n}}(\mathbf{r})\times\mathcal{L}^S(\hat{\mathbf{n}}(\mathbf{r})$ and the set of resulting equation is

$$\hat{\mathbf{n}}(\mathbf{r})\times\mathcal{L}^S(\hat{\mathbf{n}}(\mathbf{r})\times\mathcal{L}^S)\mathbf{J}_p + \hat{\mathbf{n}}(\mathbf{r})\times\mathcal{L}^S(\hat{\mathbf{n}}(\mathbf{r})\times\mathcal{L}^{V_t})\mathbf{D}_t \\ +\hat{\mathbf{n}}(\mathbf{r})\times\mathcal{L}^S(\hat{\mathbf{n}}(\mathbf{r})\times\mathcal{L}^{V_n})\mathbf{D}_n^+ = \\ \hat{\mathbf{n}}(\mathbf{r})\times\mathcal{L}^S(\hat{\mathbf{n}}(\mathbf{r})\times\mathbf{E}^{inc}(\mathbf{r}))\forall\mathbf{r}\in S_{PEC} \tag{13}$$

$$\mathcal{L}^S\mathbf{J}_p + \mathcal{L}^{V_t\cdot I}\mathbf{D}_t + \mathcal{L}^{V_n\cdot I}\mathbf{D}_n^+ = \mathbf{E}^{inc}(\mathbf{r}) \quad \forall\mathbf{r}\in V_{TDS} \tag{14}$$

The discretization of the operator products is by no means trivial. In this work, the CMP approach first [4] to discretize Caldern-preconditioned EFIEs is used to discretize the operator products. Buffa-Christiansen (BC) functions are used to make Gram matrix well-conditioned.

III. NUMERICAL EXAMPLES

The proposed method is applied to the analysis of PEC and thin dielectric sphere shell. Consider the two adjacent spheres, one PEC and the other thin dielectric sphere shell. PEC and thin dielectric sphere shell are centered about (-1.1 m, 0, 0) and (1.1 m, 0, 0), respectively. Radius of two spheres is 1 m. Dielectric constant and thickness are 2.6 and 0.05 m, respectively. It is illuminated by incident plane wave from $(\phi,\theta) = (0°, 180°)$ in 0.2 GHz. Bistatic radar cross section (RCS) at $\phi^s = 0°$ and $\phi^s = 90°$ are shown in Fig. 2 and 3. Results using EFIE-TDS with Calderon multiplicative precoidntioner agreed very well with EFIE-TDS and FEKO. Residual error is shown in Fig. 4. When GMRES for obtaining solution of matrix equation is used, iteration number of EFIE-TDS with Calderon multiplicative preconditioner is much less than that of EFIE-TDS. It is known that EFIE-TDS with Calderon multiplicative preoconditioner is well-conditioned.

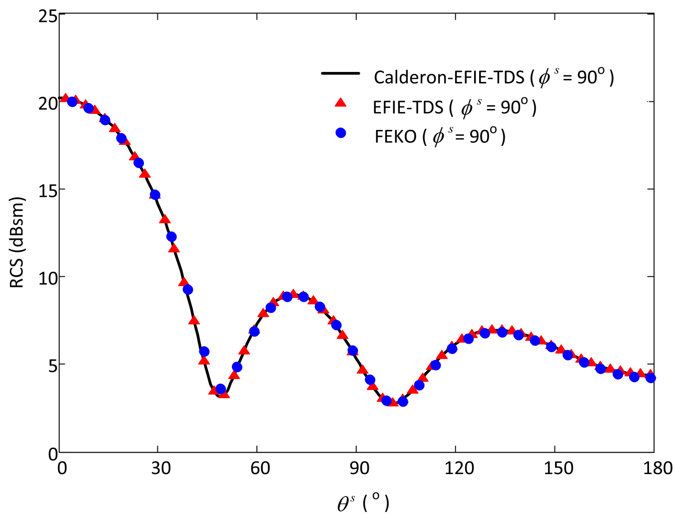

Fig. 3. RCS of a thin dielectric sphere shell and PEC sphere at $\phi^s = 90°$

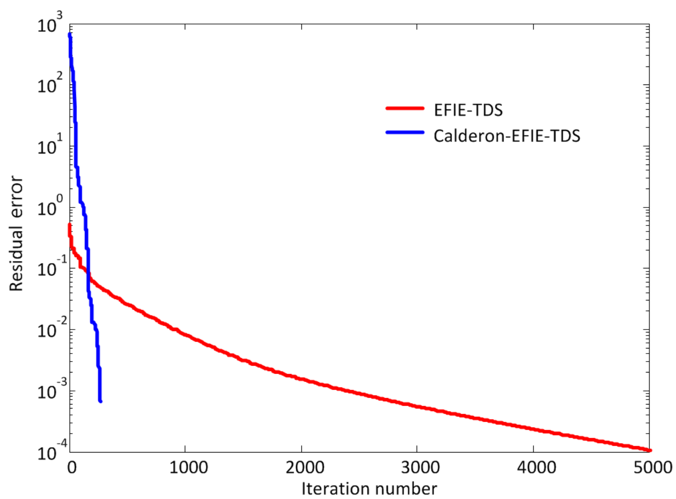

Fig. 4. Residual error obtained during the iterative solution

IV. Conclusion

This paper proposed a CMP-based regularizer for a coupled set of EFIE and TDS surface integral equation for composite structure with PEC and thin dielectric Just like in the original CMP, the preconditioner presented herein is multiplicative and easily integrated into available MoM codes. The numerical result obtained using this code confirmed the effectiveness of the proposed technique. This method can be applied to patch antenna and microwave circuit problems as well.

Acknowledgment

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