Quantitative Analysis of the Amplitude Difference Between Near and Far Field Directivity

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***Abstract* - In this short paper we briefly revise the criteria defining near and far field. After an extension of the radian sphere notion to finite size sources, we generalize the amplitude and phase criteria for arbitrary source structures. Next we identify with a representative example the wavelength distance where the near field has reached the shape of the far field directivity, then we quantify the difference between near and far field directivity in function of the position to the source.**

***Index Terms* — radian, directivity, amplitude, phase**

I. Introduction

For any time harmonic electromagnetic source excited at frequency f with an accepted average active power P, the complex Poynting vector $\overline{S}=\overline{E}\times\overline{H}^{*}/2$ describes how this power P is distributed in the electric field $\overline{E}$ and magnetic field $\overline{H}$. Only the real part of this vector oriented radially away from the source corresponds to an active power radiated outwards, while its imaginary and transverse components measure mainly reactive power oscillating in space instead of being radiated. In what follows we propose to review and quantify the notions of near and far field in relation to the Poynting vector.

II. The Radian Shell

Several regions can be defined by the field properties at a distance r from a reference point $\overline{0}$, usually the source feed or the phase center. In the case of infinitesimally small sources such as the Hertz dipole or the Fitzgerald loop, the electric and magnetic field terms decaying in 1/r become larger than the $1/r^2$ and $1/r^3$ ones from a distance $r = \lambda/2\pi = 1/k$ [1]. This distance defines the so-called radian sphere, inside which the reactive power and stored energy, exclusively due to $1/r^2$ and $1/r^3$ terms, dominate the radiated power. This distance criteria does not depend on the source dimension, as it is infinitesimal. Still it remains valid for non infinitesimal field sources, as shown hereafter.

For a collection of equivalent electric $\overline{J}$ and magnetic $\overline{M}$ currents distributed on the outer surface S of a finite size source, possibly made of metal and dielectric parts, the $\overline{E}$ and $\overline{H}$ fields at a location $\overline{r}$ outside S are given by (1), where the individual contributions of the tangential and radial components of both currents flowing at every location $\overline{r}'$ on S are integrated [2]. In (2) $\overline{J}_t=\overline{J}\times\hat{R}$ and $\overline{M}_t=\overline{M}\times\hat{R}$ are the components of the total currents $\overline{J}$ and $\overline{M}$ transverse to the unit vector $\hat{R}=\overline{R}/R$, while $\overline{J}_r=(\overline{J}.\hat{R})\hat{R}$ is the radial component. The magnetic field $\overline{M}(\overline{r})$ has a similar and dual expression.

The distance where the $1/R^2$ and $1/R^3$ terms are similar or larger in magnitude than the 1/R terms is again given by the condition $kR \leq 1$, as can be derived from (2). The distance R must now be understood as the minimum value of $R(\overline{r}')=|\overline{r}-\overline{r}'|$ for all possible locations $\overline{r}'$ on S. In other words, the radiated fields become dominant outside a radian shell surface englobing S at a distance R=1/k from it.

$$\overline{E}(\overline{r})=\oiint_S \eta\left[\overline{J}_t(\overline{r}')f_{Jt}-\overline{J}_r(\overline{r}')f_{Jr}\right]dS'-\oiint_S \overline{M}_t(\overline{r}')f_{Mt}dS' \quad (1)$$

where :

$$f_{Jt}=k^2\left[\frac{j}{kR}+\frac{1}{(kR)^2}-\frac{j}{(kR)^3}\right]\frac{e^{-jkR}}{4\pi}$$
$$f_{Jr}=k^2\left[\frac{2}{(kR)^2}+\frac{2j}{(kR)^3}\right]\frac{e^{-jkR}}{4\pi} \quad (2)$$
$$f_{Mt}=k^2\left[\frac{j}{kR}+\frac{1}{(kR)^2}\right]\frac{e^{-jkR}}{4\pi}$$

III. Phase And Amplitude Accuracy

The directivity of an electromagnetic source is defined by (3). We choose to name "near field directivity" the right hand side term inside the limit for $r\to\infty$:

$$D(\theta,\phi)=\lim_{r\to\infty}\frac{4\pi r^2\,\mathrm{Re}\left[\overline{S}(\theta,\phi)\cdot\hat{r}\right]/2}{P} \quad (3)$$

For an infinitesimal dipole source [3]:

$$\frac{1}{2}\mathrm{Re}\left[\overline{S}\right]=\frac{\eta}{8}|Idl|^2\frac{\sin^2\theta}{(\lambda r)^2}\hat{r} \quad (4)$$

With other words, the near and far field directivity of an infinitesimal dipole are identical from $r \geq 0$ to ∞.

For finite size sources, there is a difference between the

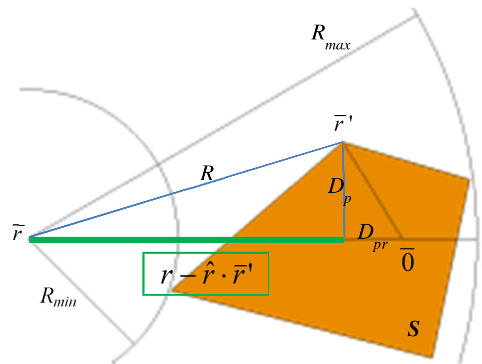

Fig. 1. Geometry for far field approximations on R

near field directivity and its far field version, computed with the electric and magnetic fields given by (1) and (2), but where only the 1/R terms are retained and $\bar{R}$ is replaced by $\bar{r}$ (or $r - \hat{r}\cdot\bar{r}'$) for the amplitude (or phase) terms, respectively.

In Fig. 1 we consider an observation point $\bar{r}$ located at a distance r from $\bar{0}$ and a section S of the source containing the axis $\bar{0}\bar{r}$. When R is approximated by r, for the amplitude terms, the largest error is measured by $D_r = \max(r-R_{min};R_{max}-r)$ for all possible sections S of the source around the axis $\bar{0}\bar{r}$. Reducing the amplitude error requires to increase r/D_r. When R is approximated by $r - \hat{r}\cdot\bar{r}'$ the phase error given by (5) is maximum very close to where $\bar{r}'$ lies at the largest possible distance D_p, from $\bar{0}\bar{r}$ and where $D_{pr} = \hat{r}\cdot\bar{r}'$. From Fig. 1 it can be derived that :

$$k\left[R-\left(r-\hat{r}\cdot\bar{r}'\right)\right] \le 2\pi / n \Leftrightarrow r/\lambda \ge \frac{n}{2}(D_p/\lambda)^2 + D_{pr}/\lambda \quad (5)$$

The well known and widely used rule $r > 2D^2/\lambda$ is a special case of (5) for n=16, where D_{pr}/λ can be neglected and when D_p can be replaced by $D/2$, namely when the reference point $\bar{0}$ is centered in the section S of largest dimension D [3,4]. We show hereafter how to relate r/D_r and n to the difference between the near field and far field directivity.

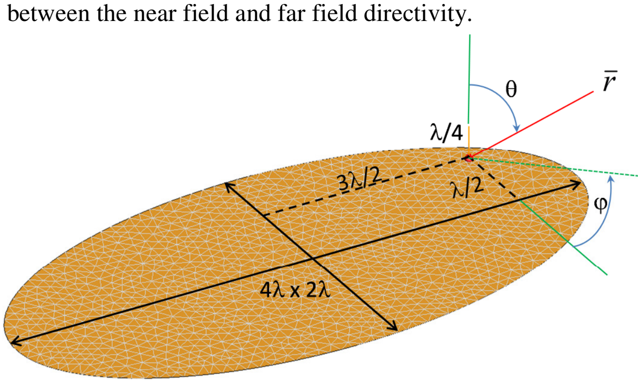

Fig. 2. λ/4 monopole on elliptical ground plane

Before doing so, we emphasize that the two different dimensions D_r and D_p governing the amplitude and phase error depend on the observation point $\bar{r}(r,\theta,\varphi)$, especially and significantly for a non symmetrical source. In the case of the $\lambda/4$ monopole excentered on an elliptic ground plane shown in Fig. 2 we find for example at $\bar{r} = (4\lambda, 63°, 0°)$ that D_r/λ=1,15 and D_p/λ=3,5 while for $\bar{r} = (4\lambda, -75°, 90°)$ D_r/λ=2,85 and D_p/λ=1,5.

This asymmetrical and elongated configuration has been intentionnally selected to avoid error cancellations due to symmetries and to enhance the positional dependence of the amplitude accuracy. Computations have been performed with FEKO™, using double precision and a high density $\lambda/15$ mesh. What follows is a brief summary of many simulations. A first distance to consider is when all the main lobes of $D(\theta,\varphi)$ are present in $S_r(r,\theta,\varphi)$. It occurs as soon as $n > 2$ and $r/D_r > 2{,}5$, but the maxima and minima can be shifted many degrees away from their far field position while also exhibiting large amplitude errors. The condition $n \ge 2$ is met from $r/\lambda \ge 25$ along φ=0° but already from $r/\lambda \ge 8$ along φ=90°, as illustrated in Fig. 3.

Once the near field directivity pattern is established, one can measure its relative difference with the far field one for various $\bar{r}(r,\theta,\varphi)$. In Fig. 4 a summary is presented for the seven main peaks along φ=0° and φ=90°. The (θ,φ) location of those seven peaks is mentioned in the graph legend, and four of them are pointed to with arrows in Fig. 3. A relative accuracy of 10% (0,5dB) on the directivity amplitude requires r/λ between 10 to 20 along φ=90° (red curves) while r/λ between 1 and 10 suffices along φ=0° (green curves). This example illustrates the fact that the difference between near and far field directivity depends on the observation location for an asymmetrical source structure. It is also worth mentioning that the corresponding angular deviation of the peaks can reach up to 10°. Bringing this angular deviation down to 1° requires r/λ beyond 100.

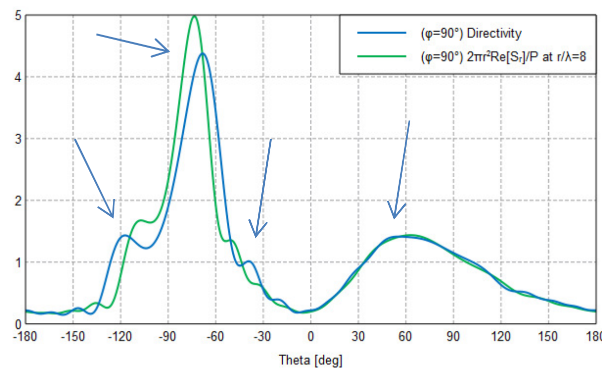

Fig. 3. Near to far field formation

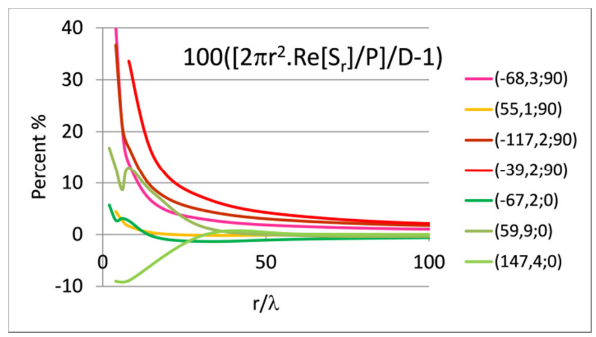

Fig. 4. Amplitude accuracy

IV. Conclusion

We first showed that the radian sphere of an infinitesimal source can be extended to the concept of radian shell around any antenna. Next we showed that the classical criteria used to define near and far field could be refined in presence of non symmetrical sources. Finally a quantitative assessment of the difference between near and far field has been presented in the challenging case of a monopole excentered on an elongated elliptical ground plane.

References

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