# Analytical Evaluation of Reflection Characteristics of Metal Plate Loading FSR with Diagonal Incident Wave

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*Abstract*—In this study, we present a simple equation to calculate the reflection phase of a metal plate loading Frequency Selective Reflector (FSR) with a diagonal incident wave. In addition, we demonstrated that a simple equation described these waves based on the results of measurement of the metal plate loading FSR and calculation results using the method of moments (MoM).

### I. INTRODUCTION

A metal plate loading Frequency Selective Reflector (FSR) can be modeled by an equivalent circuit. It is confirmed that the reflection phase can be calculated using the method of moments (MoM) and that an equivalent modelling circuit yielded a simple equation[1].

In this study, we present a simple equation to calculate the reflection phase of a metal plate loading FSR with a diagonal incident wave. In addition, we show the validity of the simple equation by comparing the result calculated using MoM with the result of measurements of a metal place loading FSR.

# II. SIMPLE EQUATION OF THE REFLECTION PHASE OF A METAL PLATE LOADING FSR

When a plane wave of infinite size with the uniform amplitude distribution is diagonally incident on a metal plate loading FSR, the tangent line component of the electromagnetic field propagating in the z direction can be modelled by an equivalent circuit, as seen in Fig. 1. Where B is the normalized susceptance,  $\beta$  is the propagation constant of the dielectric slab, and  $\ell$  is the dielectric thickness. The reflection phase  $\Phi$  is given as follows.

$$\Phi = 2\phi \pm \pi \tag{1}$$

Here,  $\phi$  is given by the following equation.

$$\phi = \tan^{-1} \frac{1}{B - \alpha_m \cot \beta \ell} \tag{2}$$

When  $m_1$  is the TM wave component and  $m_2$  is the TE wave component,  $\alpha_1$  and  $\alpha_2$  are as follows.

$$\alpha_1 = \frac{\epsilon_r \cos \theta}{\sqrt{\epsilon_r - \sin^2 \theta}}, \alpha_2 = \frac{\sqrt{\epsilon_r - \sin^2 \theta}}{\cos \theta}$$
(3)

Here,  $\beta$  is as follows.

$$\beta = \frac{2\pi}{\lambda} \sqrt{\epsilon_r - \sin^2 \theta} \tag{4}$$



Fig. 1. Equivalent circuit of metal plate loading FSR

# III. DERIVATION OF NORMALIZED SUSCEPTANCE

It is necessary to calculate the normalized susceptance B to use the simple equation. The following two conditions apply.

- (i) B is not affected by the metal plate
- (ii) The half-infinite domain of the upper part of the FSR is  $\epsilon_0$ , and the half-infinite domain of the bottom is  $\epsilon_0 \epsilon_r$ , as shown in Fig. 2

The analysis model of Fig. 2 is expressed by the equivalent circuit in Fig. 3. B is calculated using this equivalent circuit as follows.

$$B = -(1 + \alpha_m) \tan \Phi_T \tag{5}$$

Here,  $\Phi_T$  is the transmission phase; it is calculated by simulation.



Fig. 2. Analysis model



Fig. 3. Equivalent circuit of the analysis model

#### IV. METHOD FOR MEASUREMENT

Fig. 4 shows the constitution of a device which measures reflectance. The device is constructed using a network analyzer, a horn reflector antenna, and a corner reflector. The electric wave generated by the network analyzer becomes a plane wave (because of the horn reflector antenna) and is emitted.

Fig. 5 shows the state of the reflection in the corner reflector for this plane wave. The wave reflected at point X is reflected to Y and returns to the wave's



Fig. 4. Design of a device for measuring reflectance Central axis







Fig. 6. Analysis model of patch-type metal plate loading FSR



Fig. 7. Vertically incident wave



Fig. 8. TM wave at an incidence angle of 30 degrees

surface. The electric field distribution returning on the left side of the central axis is identical to the electric field distribution incident on the left side, because the electric field distribution on the plane of incidence is symmetric about the central axis. Therefore, the electric field distribution is equal to electric field distribution multiplied by the reflection coefficient of the sample at an incidence angle  $\theta$ . In addition, the electric field distribution returning on the right side is similar. The electric field distribution multiplied by the reflection coefficient at incidence angle  $\theta$  of the planar sample with an electric field distribution identical to that of the incident plane wave returns to the horn reflector antenna.

Therefore, the reflection phase is determined from the ratio of the reception electric field of a sample, and the sample is replaced by a metal plate.

# V. MODEL COMPARISON

We measured the fabricated patch-type metal plate loading FSR. We compared both the simple equation and the MoM calculation result to the measured value. Fig. 6 shows the analysis model of the patch-type metal plate loading FSR. The design parameters of the FSR are as follows.

p = 9.5 mmr = 4.5 mmw = 0.3 mm

$$\sigma = 0.3 \text{ mm}$$

$$g = 0.5 \text{ mm}$$

$$\epsilon_r = 2.65$$

We fixed the measurement frequency range at 10 GHz to 15 GHz and measured the reflection phase at incidence angles of zero degrees, 30 degrees, 45 degrees, and 60 degrees. We removed the reflection wave before the electric wave hit the corner reflector with the time domain function of the network analyzer and measured the response.

Figs. 7 to 13 show a comparison between the results for the measured value, the simple equation, and the MoM calculation result for the patch-type metal plate loading FSR. The results agreed at most frequencies. The ripple seen in the results for measurement depends on multiple reflections between the feeding part of the horn reflector antenna and the measured object which is located at the corner reflector. The wave surface at the horn reflector antenna aperture is a plane wave for



Fig. 9. TE wave at incidence angle of 30 degrees















Fig. 13. TE wave at incidence angle of 60 degrees

geometrical optics, but ripples occur because of the phase distribution in the wave.

In Figs. 11 and 13, the simple equation does not match the MoM and measured values after a certain frequency. Because the simple equation and MoM do not consider higher modes, we estimate that the result beyond the frequency that the grating lobe produces will not match. The frequency that the grating lobe produces for every incidence angle is shown in Table 1. The point at which the result does not match is the as the frequency that the grating lobe produces, as indicated by the vertical bar in Fig. 11 and Fig. 13.

 TABLE I

 FREQUENCY PRODUCED BY THE GRATING LOBE

0 [deg]	19.2 [GHz]
30 [deg]	14.8 [GHz]
45 [deg]	13.5 [GHz]
60 [deg]	12.7 [GHz]

## VI. CONCLUSION

We presented a simple equation to calculate the reflection phase of a metal plate loading FSR with a diagonal incident wave. In addition, we showed a method of measurement for reflectance and a device design to easily measure a diagonal incident wave. Finally, we compared the simple equation to MoM and measurements. In terms of fabrication errors or measurement errors, we consider the simple expression derived in this study to be accurate.

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#### REFERENCES

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