Simultaneous Estimation of DOA and Angular Spread of Incident Radio Waves by DOA-Matrix Method with SLS and SAGE Algorithms

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***Abstract*—In estimating DOA of incident waves with high accuracy, we often have to take into consideration the angular spread of each wave due to reflection, diffraction, and scattering. As a method of estimating DOA and AS simultaneously, SAGE-DOA-Matrix method was proposed which uses DOA-Matrix method along with SAGE algorithm. In this paper, we further improve the estimation performance of the method by using SLS algorithm.**

***Keywords*—DOA, angular spread, DOA-Matrix method, SAGE algorithm, SLS algorithm**

I. Introduction

In order to clarify the radio environments, it is effective to estimate the DOA of individual incident waves at the receiving point. For the estimating method, high-resolution estimation algorithms, e.g. MUSIC and ESPRIT[1], with array antennas are attractive because of high estimation accuracy and high computational efficiency. However, we often have to take into consideration the angular spread of each wave due to reflection, diffraction, and scattering[2]. As a method of estimating DOA and AS simultaneously, we proposed SAGE-DOA-Matrix method which uses DOA-Matrix method along with SAGE algorithm[3]. One of the advantages is that it does not bear heavy computational load caused by two-dimensional peak search of spectrum on both DOA and AS. In this paper, we further improve the estimation performance of the method by using SLS (Structured Least Squares) algorithm[4]. Through computer simulation, we demonstrate the effectiveness of the improved method.

II. Array antenna and signal model

Fig. 1 shows the K-element linear array antenna with element spacing of d, which receives L clustered waves with angular spread. Then, the array input vector $\boldsymbol{x}(t)$ is expressed as follows.

$$\boldsymbol{x}(t) = \sum_{l=1}^{L} \left\{ \sum_{m=1}^{M_l} A_{lm}(t)\boldsymbol{a}_o(\theta_{lm}) \right\} + \boldsymbol{n}(t) \tag{1}$$

$$\boldsymbol{a}_o(\theta) = [a_{o1}(\theta), a_{o2}(\theta), \cdots, a_{oK}(\theta)]^T \tag{2}$$

$$a_{ok}(\theta) = e^{-j\frac{2\pi}{\lambda}(k-1)d\sin\theta} \quad (k = 1, \cdots, K) \tag{3}$$

where M_l is the number of element waves of the l-th clustered wave, $A_{lm}(t)$ and θ_{lm} are the complex amplitude and DOA of the m-th element wave in the l-th clustered wave, $\boldsymbol{a}_o(\theta)$ is the array response vector of each element wave, and $\boldsymbol{n}(t)$ is internal noise vector. Also, it is assumed that element waves in one clustered wave are fully correlated with each other.

Further assuming that the element waves are in phase and continuously distributed in the angular spread $\Delta\theta_l$, the array input vector can be expressed as follows.

$$\boldsymbol{x}(t) = \sum_{l=1}^{L} s_l(t)\boldsymbol{a}(\theta_l, \Delta\theta_l) + \boldsymbol{n}(t) \tag{4}$$

$$\boldsymbol{a}(\theta_l, \Delta\theta_l) = [a_1(\theta_l, \Delta\theta_l), \cdots, a_K(\theta_l, \Delta\theta_l)]^T \tag{5}$$

$$a_k(\theta, \Delta\theta) = a_{ok}(\theta)\mathrm{sinc}\left\{\frac{\pi}{\lambda}(k-1)d\Delta\theta\cos\theta\right\} \tag{6}$$

$$s_l(t) = \sum_{m=1}^{M_l} A_{lm}(t) \tag{7}$$

where $\boldsymbol{a}(\theta_l, \Delta\theta_l)$ is the integral-type mode vector[2].

III. AS Estimation by DOA-Matrix Method

We decompose K-element array into two overlapped subarrays with $(K-1)$ elements. One is from 1st to $(K-1)$-th element, and the other is from 2nd element to K-th element.

If we make the following approximation

$$\frac{\mathrm{sinc}\left\{\frac{\pi}{\lambda}(k-1)d\Delta\theta\cos\theta\right\}}{\mathrm{sinc}\left\{\frac{\pi}{\lambda}(k-2)d\Delta\theta\cos\theta\right\}} \simeq 1 \tag{8}$$

then the array input vectors of the two subarrays, $\boldsymbol{x}_1(t)$ and $\boldsymbol{x}_2(t)$, are expressed as follows.

$$\boldsymbol{x}_1(t) = \boldsymbol{A}_1\boldsymbol{s}(t) + \boldsymbol{n}_1(t) \tag{9}$$

$$\boldsymbol{x}_2(t) = \boldsymbol{A}_1\boldsymbol{\Phi}\boldsymbol{s}(t) + \boldsymbol{n}_2(t) \tag{10}$$

$$\boldsymbol{A}_1 = [\boldsymbol{a}_1(\theta_1, \Delta\theta_1), \cdots, \boldsymbol{a}_1(\theta_L, \Delta\theta_L)] \tag{11}$$

$$\boldsymbol{a}_1(\theta_l, \Delta\theta_l) = [a_1(\theta_l, \Delta\theta_l), \cdots, a_{K-1}(\theta_l, \Delta\theta_l)]^T \tag{12}$$

$$\boldsymbol{s}(t) = [s_1(t), s_2(t), \cdots, s_L(t)]^T \tag{13}$$

$$\boldsymbol{\Phi} = \mathrm{diag}(\phi_1, \cdots, \phi_L) \quad (\phi_l = e^{-j\frac{2\pi}{\lambda}d\sin\theta_l}) \tag{14}$$

where $\boldsymbol{n}_1(t)$ and $\boldsymbol{n}_2(t)$ are noise vectors of subarray 1 and subarray 2, respectively.

From the auto-correlation matrix $\boldsymbol{R}_{11} = E[\boldsymbol{x}_1(t)\boldsymbol{x}_1^H(t)]$ and the cross-correlation matrix $\boldsymbol{R}_{21} = E[\boldsymbol{x}_2(t)\boldsymbol{x}_1^H(t)]$, we make $\boldsymbol{R} = \boldsymbol{R}_{21}\boldsymbol{R}_{11}{}^{-1}$. Since $\boldsymbol{R}$ has the relation $\boldsymbol{R}\boldsymbol{A}_1 = \boldsymbol{A}_1\boldsymbol{\Phi}$, we can obtain the DOA estimates and AS estimates directly from the eigendecomposition of $\boldsymbol{R}$ [3].

This is DOA and AS estimation by DOA-Matrix method, and the one together with SAGE algorithm, which is improved for multiple clustered waves, is called SAGE-DOA-Matrix method[3].

IV. SLS Algorithm

SLS (Structured Least Squares) algorithm[4] is employed to remove the estimation errors from $\boldsymbol{a}_1(\theta_l, \Delta\theta_l)$ and ϕ_l ($l = 1, 2, \ldots, L$). These errors are due to an overlap of two subarrays.

SLS algorithm determines small modification $\Delta\boldsymbol{a}_1(\theta_l, \Delta\theta_l)$ and $\Delta\phi_l$ by minimizing the norm of residual vector defined by

$$\boldsymbol{\gamma}(\hat{\boldsymbol{a}}_1(\theta_l, \Delta\theta_l), \hat{\phi}_l) = \hat{\phi}_l \boldsymbol{J}_1^{(K-1)} \hat{\boldsymbol{a}}_1(\theta_l, \Delta\theta_l) - \boldsymbol{J}_2^{(K-1)} \hat{\boldsymbol{a}}_1(\theta_l, \Delta\theta_l) \quad (15)$$

$$\hat{\boldsymbol{a}}_1(\theta_l, \Delta\theta_l) = \boldsymbol{a}_1(\theta_l, \Delta\theta_l) + \Delta\boldsymbol{a}_1(\theta_l, \Delta\theta_l) \quad (16)$$

$$\hat{\phi}_l = \phi_l + \Delta\phi_l \quad (l = 1, \cdots, L) \quad (17)$$

$$\boldsymbol{J}_1^{(K-1)} = [\boldsymbol{I}_{K-2} \; \boldsymbol{0}_{(K-2)\times 1}] \quad (18)$$

$$\boldsymbol{J}_2^{(K-1)} = [\boldsymbol{0}_{(K-2)\times 1} \; \boldsymbol{I}_{K-2}] \quad (19)$$

and updates $\boldsymbol{a}_1(\theta_l, \Delta\theta_l)$ and ϕ_l by $\Delta\boldsymbol{a}_1(\theta_l, \Delta\theta_l)$ and $\Delta\phi_l$, respectively.

DOA-Matrix method using SLS algorithm is called SLS-DOA-Matrix method, and SAGE-DOA-Matrix method using SLS algorithm is called SAGE-SLS-DOA-Matrix method.

V. Computer Simulation

Computer simulation is carried out under the conditions described in Table I. Figs. 2 and 3 show the estimation accuracy as a function of input SNR. In both figures, three methods including MUSIC with integral-type mode vector[2] are compared. It is found from Fig.2 that SAGE-SLS-DOA-Matrix method has the lowest RMSE of DOA estimates for the SNR beyond 15dB, also revealing a tendency of monotonic decrease. On the other hand, Fig. 3 shows that SLS-DOA-Matrix method provides the most accurate estimates in AS estimation. Therefore, in the proposed method based on SLS-DOA-Matrix method, it is considered to be suitable to adopt the AS estimates obtained from SLS-DOA-Matrix method and the DOA estimates obtained from SAGE-SLS-DOA-Matrix method.

VI. Conclusion

Through computer simulation, the effectiveness of the proposed method has been demonstrated. Particularly, it is confirmed that incorporation of SLS algorithm into DOA-Matrix method contributes to improvement of AS estimation.

References

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TABLE I: Simulation conditions

Number of elements	15
Element Spacing	0.5λ
Number of clustered waves	2 (uncorrelated)
DOA of clustered waves	$(0°, 60°)$
AS of clustered waves	$(3°, 6°)$
Number of element waves of clustered waves	$(15, 30)$
Input SNR	$0 \sim 30$ [dB]
Number of snapshots	30
Number of trials	100

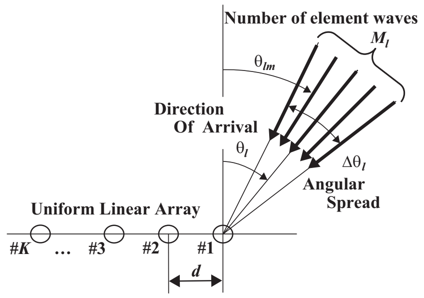

Fig. 1: Array antenna and incident waves with angular spread

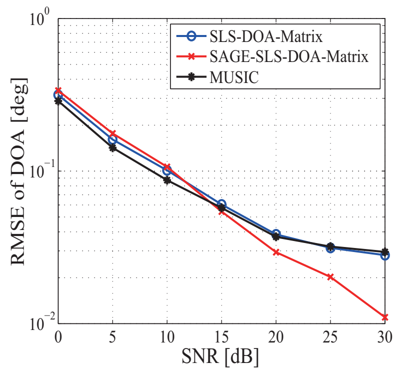

Fig. 2: RMSE of DOA estimates vs. SNR

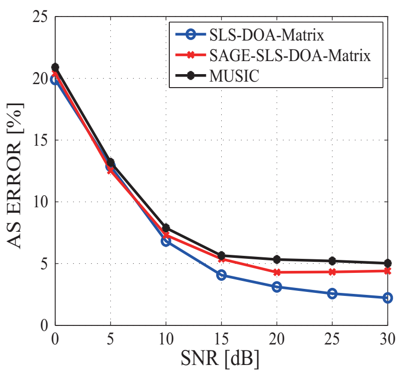

Fig. 3: Error of AS estimates vs. SNR