Multiple Edge Interaction Effect to Plane Wave Scattering by a Wide and Thick Slit

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***Abstract*—High frequency ray method has been applied to formulate E-polarized plane wave scattering by a wide and thick slit. Slit aperture region is considered as a waveguide which connects the upper and lower half spaces, and excitations of the waveguide modes are given by edge diffraction at the open end. Multiple edge diffraction effect as well as modal bouncing effect are also considered to improve the accuracy of the scattering field.**

I. INTRODUCTION

In recent years, with increases of the wireless usage, our attention has been made for outdoor-indoor connection thorough building walls and windows. As the frequency gets higher, electromagnetic waves decay fast by passing through concrete walls. Therefore, the windows on the building walls would be a primary gate for such transmitting waves, and the scattering by a wide and thick slit could be a canonical problem for analyzing the transmission property.

Diffraction by a slit is a classical problem for electromagnetic scattering analyses and many authors have been studied [1]–[3]. In the high frequency scattering by an object whose size is pretty large compared with the wavelength, the Geometrical Theory of Diffraction(GTD) [4] could be a powerful tool for the scattering analysis.

Analysis of the diffraction by a slit on an infinitely thin screen [1] is rather easy since one needs to consider only two edges for edge diffraction. On the other hand, the diffraction by a thick slit is difficult to solve and is analyzed mainly by the eigenfunction expansion method [5], or numerical approaches.

In this paper, we shall utilize a ray-mode conversion method to obtain the waveguide modal field inside the thick aperture. This approach has already been applied successfully for H polarized [6], and E polarized plane wave incidence cases [7], and we shall improve the accuracy of the scattering field by including the multiple edge diffraction effect for the latter case. In the following discussion, time harmonic factor $\mathrm{e}^{-i\omega t}$ is assumed and suppressed throughout the context.

II. FORMULATION

As illustrated in Fig. 1, an E-polarized plane wave:

$$u^i = E_y^i = e^{-ik(x\cos\theta_0 + z\cos\theta_0)} \tag{1}$$

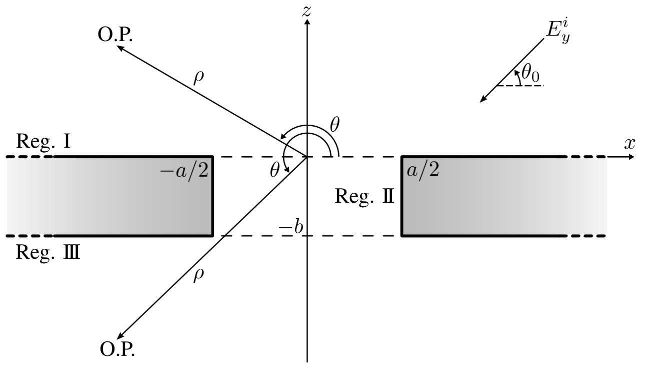


Fig. 1. Geometry of the problem

illuminates a slit on a thick perfect conducting screen. Width and thickness of the slit are a and b, respectively, and k is free space wavenumber.

First, let us classify the entire region into three; upper half space (Reg. I), aperture (Reg. Ⅱ), and lower half space (Reg. Ⅲ), respectively.

Based on the assumption that the slit aperture is pretty wide compared with the wavelength, the scattered field in each region may be formulated as a collection of reflected and diffracted rays. Specially in Reg. Ⅱ, the ray description is not suitable since an infinite number of multiply bouncing reflected rays exist due to the waveguide structure, and the convergence of the ray summation is very slow. One may apply a ray-mode conversion technique to obtain a complementary waveguide modal summation, which converges rapidly [8]. The total far field in Reg. I may be considered as a summation of the primary edge diffracted rays and successive modal reradiation fields from the aperture.

The primary edge diffracted wave u_0 may be written for the far field in Reg. I by GTD as

$$\begin{aligned} u_0 = {} & C(k\rho)D_{-1}\left(\theta, \theta_0; \frac{3}{2}\pi\right) e^{-ika(\cos\theta+\cos\theta_0)/2} \\ & + C(k\rho)D_{-1}\left(\theta + \frac{\pi}{2}, \theta_0 + \frac{\pi}{2}; \frac{3}{2}\pi\right) \\ & \cdot e^{ika(\cos\theta+\cos\theta_0)/2}. \end{aligned} \tag{2}$$

Here, $C(\chi)$=$(8\pi\chi)^{-1/2}e^{i(\chi+\pi/4)}$ is an asymptotic far field expression for 2D free space Green's function and $D_\tau(\phi, \phi_0; \phi_w)$ is the Keller's edge diffraction coefficient for a perfectly conducting wedge with the wedge angle $(2\pi - \phi_w)$ as [4]

$$D_\tau(\phi, \phi_0; \phi_w) = \frac{2\pi}{\phi_w} \sin\frac{\pi^2}{\phi_w} \left\{ \left(\cos\frac{\pi^2}{\phi_w} - \cos\frac{\phi - \phi_0}{\phi_w} \right)^{-1} + \tau \left(\cos\frac{\pi^2}{\phi_w} - \cos\frac{\phi + \phi_0}{\phi_w} \right)^{-1} \right\}. \tag{3}$$

The multiple edge diffraction occurs and this effect can not be ignored when the slit aperture becomes narrower. These multiple edge diffracted rays may be formulated also by GTD [9], and should be added to Eq. (2) as

$$\begin{aligned}
\bar{u}_0 = u_0 + C(k\rho) \Bigg\{ & D_{-1}\left(\pi, \theta_0; \frac{3}{2}\pi\right) D_{-1}\left(\frac{\pi}{2} + \theta, \frac{\pi}{2}; \frac{3}{2}\pi\right) \\
& \cdot e^{ika(\cos\theta - \cos\theta_0)/2} + D_{-1}\left(\frac{\pi}{2}, \frac{\pi}{2} + \theta_0; \frac{3}{2}\pi\right) \\
& \cdot D_{-1}\left(\theta, \pi; \frac{3}{2}\pi\right) e^{-ika(\cos\theta - \cos\theta_0)/2} \Bigg\} \\
& \cdot \sum_{s=1}^{\infty} \left(-\frac{1}{2}\right)^{2(s-1)} C\big((2s-1)ka\big) \\
+ C(k\rho) \Bigg\{ & D_{-1}\left(\pi, \theta_0; \frac{3}{2}\pi\right) D_{-1}\left(\theta, \pi; \frac{3}{2}\pi\right) \\
& \cdot e^{-ika(\cos\theta + \cos\theta_0)/2} + D_{-1}\left(\frac{\pi}{2}, \frac{\pi}{2} + \theta_0; \frac{3}{2}\pi\right) \\
& \cdot D_{-1}\left(\frac{\pi}{2} + \theta, \frac{\pi}{2}; \frac{3}{2}\pi\right) e^{ika(\cos\theta + \cos\theta_0)/2} \Bigg\} \\
& \cdot \sum_{t=1}^{\infty} \left(-\frac{1}{2}\right)^{2t-1} C(2tka).
\end{aligned} \tag{4}$$

A part of the above edge diffracted wave $\bar{u}_0$ also propagates into the slit aperture, and re-radiation occurs after several reflections and diffractions.

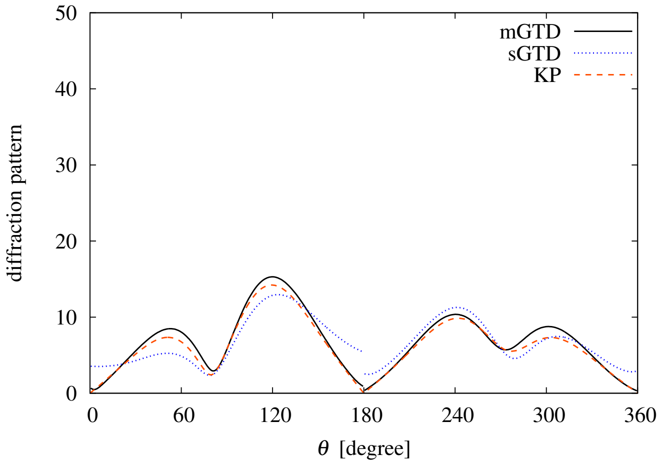
Thus, the total diffraction fields u_t^+ in Reg. I may be written in a compact matrix form as

$$\begin{aligned}
u_t^+ &= \bar{u}_0 + C(k\rho)[\boldsymbol{R}^+] \sum_{n=0}^{\infty} [\boldsymbol{B}]^{2n+1} [\boldsymbol{A}] \\
&= \bar{u}_0 + C(k\rho)[\boldsymbol{R}^+] \big[[\boldsymbol{I}] - [\boldsymbol{B}]^2\big]^{-1} [\boldsymbol{B}][\boldsymbol{A}].
\end{aligned} \tag{5}$$

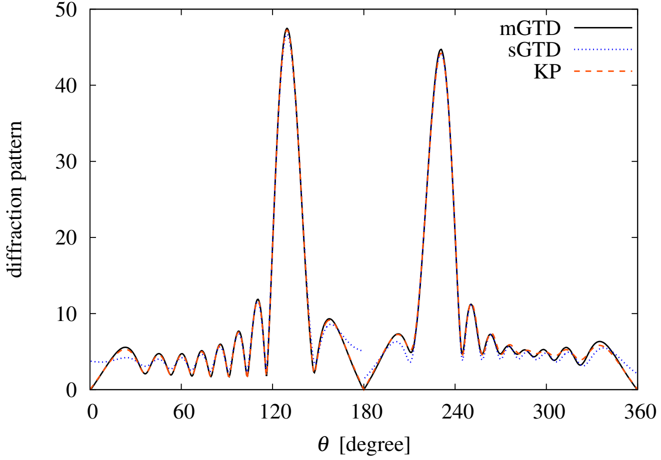
Similarly, for u_t^- in Reg. III,

$$\begin{aligned}
u_t^- &= C(k\rho)[\boldsymbol{R}^-] \sum_{n=0}^{\infty} [\boldsymbol{B}]^{2n} [\boldsymbol{A}] \\
&= C(k\rho)[\boldsymbol{R}^-] \big[[\boldsymbol{I}] - [\boldsymbol{B}]^2\big]^{-1} [\boldsymbol{A}].
\end{aligned} \tag{6}$$

In the above equations, $[\boldsymbol{R}^\pm]$ denotes modal diffraction row vector at the upper $(+)$ and the lower $(-)$ apertures, and $[\boldsymbol{A}]$ is a modal excitation column vector due to the primary edge diffraction. Also, matrix $[\boldsymbol{B}]$ is a modal coupling matrix at each open end, and $[\boldsymbol{I}]$ is an unit matrix, respectively [7]. Because of a space limit, we shall omit the detail description of the above matrix components here.



(a) $ka = 7$



(b) $ka = 30$

Fig. 2. Comparison of the far field pattern(width variation). $\theta_0 = 50°$, $kb = 2$. ——: Present GTD results with multiple edge diffracted waves; ·······: GTD results by the primary edge diffracted wave only [7]; - - -: KP method [3].

III. Numerical Results and Discussion

Figure 2 shows the far field diffraction patterns for different aperture widths. A common factor $C(k\rho)$ in Eqs. (4), (5), and (6) is omitted here. Newly derived results with the contribution of the multiple diffracted rays are plotted as mGTD, and previously derived results [7] are plotted as sGTD. As a reference, the figure also includes results obtained from the KP method [3], which is an analytical eigenfunction expansion method. One observes that our present result agrees well with the reference value. Especially, better agreement can be found, at the screen boundary directions: $\theta = 0°, 180°, 360°$, whereas the previous results without multiple edge diffraction don't satisfy the boundary condition. As one can expect, the result in Fig. 2 (b) for the wide slit aperture case $(ka = 30)$ matches better than the narrow case ($ka = 7$) in Fig. 2 (a). For the latter case, there are only 2 propagation modes, while 9 modes for the former case. Some differences in Fig. 2 (a) for the narrow aperture case may be improved by including the evanescent modal effect.

IV. CONCLUSION

In this paper, the E polarized plane wave scattering by a wide and thick slit has been analyzed by high frequency ray method. By including the multiple edge diffraction effect, our results match well with the reference result by KP method. So far, our formulation is made only by propagation waveguide modes, and the accuracy may become better by including the effect of the evanescent modes. This aspect is currently under study and will be reported in a separate paper.

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