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Iterative-Promoting Variable Step-Size LMS Algorithm based Adaptive Sparse Channel Estimation

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Abstract— **Least mean square (LMS) type adaptive algorithms have attracted much attention due to their low computational complexity. For estimating sparse channels, zero-attracting LMS (ZA-LMS), reweighted ZA-LMS (RZA-LMS) and reweighted L1-norm LMS (RL1-LMS) have been developed to exploit channel sparsity. However, these proposed algorithms may be difficult to make tradeoff between convergence speed and estimation performance with the invariable step-size. To solve this problem, we propose three sparse iterative-promoting variable step-size LMS (IP-VSS-LMS) channel estimation algorithms with sparse constraints, i.e. ZA, RZA, and RL1. The proposed algorithms are termed as ZA-IPVSS-LMS, RZA-IPVSS-LMS and RL1-IPVSS-LMS respectively. Simulation results are provided to confirm effectiveness of the proposed sparse channel estimation algorithms in different scenarios.**

***Keywords*— LMS; adaptive sparse channel estimation; sparse penalty; compressive sensing; variable step-size LMS (VSS-LMS).**

I. INTRODUCTION

The demand for high-speed wireless communications has been increasing rapidly. It is well known that broadband signal transmission is one of indispensable realization techniques in the next-generation wireless communication systems [1]–[3]. However, broadband signal transmission over the frequency-selective fading channel often incurs sparse multipath channel, where the dominant channel taps are very few while most of the channel taps are approximated as zeros or zeros [4]–[7]. To mitigate the frequency-selective fading as well as to take advantage of the inherent channel sparsity, accurate sparse channel estimation is required for coherent detection.

In last decade, many sparse adaptive channel estimation algorithms have been developed [8]–[11]. By using the invariable step-size (ISS), Gui. et. al developed zero-attracting ISS least mean square (ZA-ISS-LMS), reweighted ZA-ISS-LMS (RZA-ISS-LMS) [8] and Taheri et.al developed the reweighted ℓ_1-norm ISS-LMS (RL1-ISS-LMS) [11] to exploit the channel sparsity. However, these sparse LMS algorithms cannot make tradeoff between convergence speed and estimation performance by ISS. On the one hand, utilizing a smaller step-size can achieve a better estimation performance but scarifying the convergence speed. On the other hand, utilizing a larger step-size can improve convergence speed but deteriorating the estimation performance. Motivated by ISS-LMS-type algorithms, this paper proposes iterative-promoting variable step-size LMS (IPVSS-LMS) algorithms for estimating sparse channels. The proposed algorithms have a larger step-size in the initial stage, and then step-size is iteratively reduced. To achieve steady-state solution, the VSS is also bounded by a threshold. Hence, the proposed sparse IPVSS-LMS algorithms can balance the instantaneous estimation error and convergence speed. Since the threshold is adopted for VSS, the proposed sparse IPVSS-LMS algorithms do not scarify the additional convergence speed. Several representative simulations are conducted to confirm the proposed algorithms in different scenarios.

The remainder of the rest paper is organized as follows. A system model is described and sparse ISS-LMS algorithms are reviewed in Section II. In Section III, ZA-IPVSS-LMS, RZA-IPVSS-LMS and RL1-IPVSS-LMS are proposed. The simulation results are presented in Section IV. Finally, this paper is concluded in Section V.

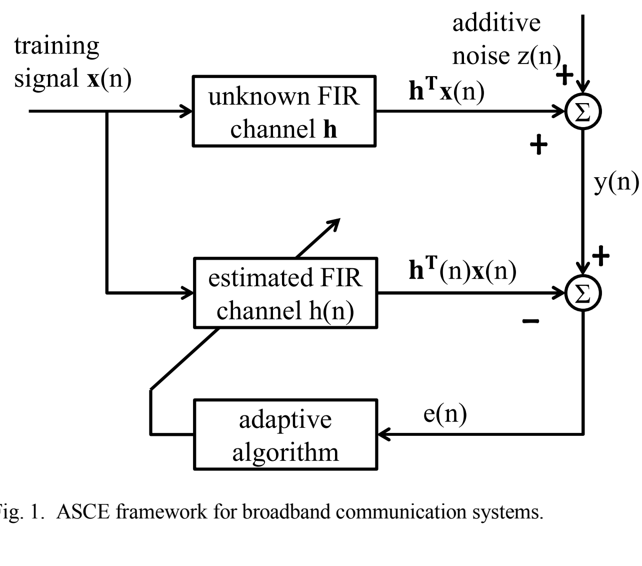

Fig. 1. ASCE framework for broadband communication systems.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Adaptive channel estimation scheme for estimating wireless channels is illustrated in Fig. 1. Assume that the training signal $\boldsymbol{x}(n)$ is transmitted over the sparse channel $\boldsymbol{h}=[h_0,h_1,...,h_{N-1}]^T$ which is supported only by K non-zero coefficients ($K \ll N$). The output signal $y(n)$ is observed as

$$y(n)=\boldsymbol{h}^T\boldsymbol{x}(n)+z(n), \tag{1}$$

where $\boldsymbol{x}(n)=[x(n),x(n-1),...,x(n-N+1)]^T$ denotes the N-length vector of training signal $\boldsymbol{x}(n)$; $z(n)$ is a random additive white Gaussian noise $z(n)\sim \mathcal{CN}(0,1)$ which is independent of $\boldsymbol{x}(n)$ .The objective of the adaptive filter is to estimate the unknown sparse channel $\tilde{\boldsymbol{h}}(n)$ by utilizing training signal $\boldsymbol{x}(n)$ and output signal $y(n)$. Then the n-th instantaneous estimation error $e(n)$ can be written as

$$e(n)=y(n)-\tilde{\boldsymbol{h}}(n)^T\boldsymbol{x}(n), \tag{2}$$

where $\tilde{\boldsymbol{h}}(n)$ is the estimated channel. According to Eq. (2), the cost function of the LMS algorithm can be derived as

$$L(n)=\frac{1}{2}e^2(n). \tag{3}$$

By differential derivative Eq. (3) with respect to $\tilde{\boldsymbol{h}}(n)$, the update equation of LMS algorithms is obtained as

$$\begin{aligned}\tilde{\boldsymbol{h}}(n+1)&=\tilde{\boldsymbol{h}}(n)+\mu\frac{\partial L(n)}{\partial\tilde{\boldsymbol{h}}(n)}\\&=\tilde{\boldsymbol{h}}(n)+\mu e(n)\boldsymbol{x}(n),\end{aligned} \tag{4}$$

where μ is a step size of gradient descend step-size. One can be observed that, ISS-LMS in Eq. (4) does not exploit the channel sparsity. To exploit the channel sparisity, sparse ISS-LMS is proposed as

$$\tilde{\boldsymbol{h}}(n+1)=\tilde{\boldsymbol{h}}(n)+\mu e(n)\boldsymbol{x}(n)-\lambda g(\tilde{\boldsymbol{h}}(n)), \tag{5}$$

where λ denotes regularization parameter, which can balance the estimation error term and channel sparsity constraint function $g(\tilde{\boldsymbol{h}}(n))$. We next review three sparse ISS-LMS algorithms. They are ZA-ISS-LMS, RZA- ISS-LMS, and RL1- ISS-LMS channel estimation algorithms.

A. ZA-ISS-LMS channel estimation algorithm

The cost function of ZA-LMS channel estimation algorithm [9] is given as

$$L_{ZA}(n)=\frac{1}{2}e^2(n)-\lambda_{ZA}\left\|\tilde{\boldsymbol{h}}(n)\right\|_1, \tag{6}$$

where λ_{ZA} is the weight associated with the penalty term and $\|\cdot\|_1$ denotes the ℓ_1-norm. Then, the updated equation of ZA-LMS channel estimation algorithm is derived as:

$$\begin{aligned}\tilde{\boldsymbol{h}}(n+1)&=\tilde{\boldsymbol{h}}(n)+\mu\frac{\partial L_{ZA}(n)}{\partial\tilde{\boldsymbol{h}}(n)}\\&=\tilde{\boldsymbol{h}}(n)+\mu e(n)\boldsymbol{x}(n)-\rho_{ZA}\operatorname{sgn}\left(\tilde{\boldsymbol{h}}(n)\right),\end{aligned} \tag{7}$$

where $\rho_{ZA}=\mu\lambda_{ZA}$ and $\operatorname{sgn}(\cdot)$ is defined as

$$\operatorname{sgn}(h)\triangleq\begin{cases}1, & h>0\\0, & h=0\\-1, & h<0\end{cases}. \tag{8}$$

Since the ZA-ISS-LMS channel estimation algorithm can exploit the channel sparisty, the steady-state performance gain of the channel estimate is obtained.

B. RZA-ISS-LMS channel estimation algorithm

The cost function of RZA-ISS-LMS channel estimation algorithm [9] is constructed as

$$L_{RZA}(n)=\frac{1}{2}e^2(n)-\lambda_{RZA}\sum_{i=1}^{N}\log\left(1+\varepsilon|h_i|\right), \tag{9}$$

where $\lambda_{RZA}>0$ is the regularization parameter and $\varepsilon>0$ is the reweight factor. Then i-th channel coefficient $\tilde{h}_i(n)$ is derived as

$$\begin{aligned}\tilde{h}_i(n+1)&=\tilde{h}_i(n)+\mu\frac{\partial L_{RZA}(n)}{\partial\tilde{h}_i(n)}\\&=\tilde{h}_i(n)+\mu e(n)x_i(n)-\rho_{RZA}\frac{\operatorname{sgn}\left(\tilde{h}_i(n)\right)}{1+\varepsilon\left|\tilde{h}_i(n)\right|},\end{aligned} \tag{10}$$

where $\rho_{RZA}=\mu\varepsilon\lambda_{RZA}$. The matrix-vector of the update equation can be also rewritten as

$$\tilde{\boldsymbol{h}}(n+1)=\tilde{\boldsymbol{h}}(n)+\mu e(n)\boldsymbol{x}(n)-\rho_{RZA}\frac{\operatorname{sgn}\left(\tilde{\boldsymbol{h}}(n)\right)}{1+\varepsilon\left|\tilde{\boldsymbol{h}}(n)\right|}. \tag{11}$$

Sparse penalty term of RZA-ISS-LMS channel estimation algorithm can attract channel coefficients as zero if those magnitudes are smaller than the threshold $1/\varepsilon_{RZA}$.

C. RL1-ISS-LMS channel estimation algorithm

The cost function of the RL1-ISS-LMS channel estimation algorithm [11] is constructed as

$$L_{RL1}(n)=\frac{1}{2}e^2(n)-\lambda_{RL1}\left\|\boldsymbol{f}(n)\tilde{\boldsymbol{h}}(n)\right\|_1, \tag{12}$$

where λ_{RL1} is the weight associated with the penalty term and reweighted vector $\boldsymbol{f}(n)$ is defined as

$$\boldsymbol{f}(n)=\begin{bmatrix}f_0(n)\\f_1(n)\\\vdots\\f_{N-1}(n)\end{bmatrix}=\begin{bmatrix}\dfrac{1}{\delta+|h_0(n-1)|}\\\dfrac{1}{\delta+|h_1(n-1)|}\\\vdots\\\dfrac{1}{\delta+|h_{N-1}(n-1)|}\end{bmatrix} \tag{13}$$

where δ denotes a positive number and thus $f_i(n) > 0$ for all $i = 0, 1, \cdots, N-1$. Then, the updated equation of RL1-ISS-LMS channel estimation algorithm is derived as

$$\begin{aligned}\tilde{\boldsymbol{h}}(n+1) &= \tilde{\boldsymbol{h}}(n) + \mu \frac{\partial L_{RL1}(n)}{\partial \tilde{\boldsymbol{h}}(n)} \\ &= \tilde{\boldsymbol{h}}(n) + \mu e(n)\boldsymbol{x}(n) - \rho_{RL1} \frac{\operatorname{sgn}\left(\tilde{\boldsymbol{h}}(n)\right)}{\delta + \left|\tilde{\boldsymbol{h}}(n-1)\right|},\end{aligned} \tag{14}$$

where $\rho_{RL1} = \mu\lambda_{RL1}$. To adaptive control the step-size, next we will propose sparse IPVSS-LMS channel estimation algorithms.

III. Proposed Sparse IPVSS-LMS Channel Estimation Algorithms

It is well known that the step-size is a critical parameter which determines the convergence speed, estimation performance and computational complexity [12]. Suitable step-size can govern the good steady-state estimation performance and fast convergence speed. The VSS $\mu(n)$ can be devised as

$$\mu(n) = \begin{cases} \mu/n, & if\ \mu(n) \geq \varphi \\ \varphi, & if\ \mu(n) < \varphi \end{cases} \tag{15}$$

where n is the number of iteration time and φ is a hard threshold to ensure the convergence when μ is enough small. By substituting Eq. (15) into Eq. (7), Eq. (11), and Eq. (13), respectively, three sparse IPVSS-LMS channel estimation algorithms, i.e., ZA-IPVSS-LMS, RZA-IPVSS-LMS, and RL1-IPVSS-LMS, are proposed as

$$\tilde{\boldsymbol{h}}(n+1) = \tilde{\boldsymbol{h}}(n) + \mu(n)\left\{e(n)\boldsymbol{x}(n) - \lambda_{ZA}\operatorname{sgn}\left(\tilde{\boldsymbol{h}}(n)\right)\right\}, \tag{16}$$

$$\tilde{\boldsymbol{h}}(n+1) = \tilde{\boldsymbol{h}}(n) + \mu(n)\left\{e(n)\boldsymbol{x}(n) - \frac{\varepsilon\lambda_{RZA}\operatorname{sgn}\left(\tilde{\boldsymbol{h}}(n)\right)}{1 + \varepsilon_{RZA}\left|\tilde{\boldsymbol{h}}(n)\right|}\right\}, \tag{17}$$

$$\tilde{\boldsymbol{h}}(n+1) = \tilde{\boldsymbol{h}}(n) + \mu(n)\left\{e(n)\boldsymbol{x}(n) - \frac{\lambda_{RL1}\operatorname{sgn}\left(\tilde{\boldsymbol{h}}(n)\right)}{\delta + \left|\tilde{\boldsymbol{h}}(n-1)\right|}\right\}. \tag{18}$$

In above three update equations, the step-size $\mu(n)$ is calculated by $1/n$ and threshold parameter φ, hence the computational complexity is very low. In addition, (15) implies that shows the adaptive vary curves in two steps by taking account of iterations as well as threshold parameter φ: Step 1): $\mu(n) = \mu/n$ when $\mu/n \geq \varphi$, a fast convergence speed is achieved; Step 2): $\mu(n) = \varphi$ when $\mu/n < \varphi$, the steady-state estimation performance can be achieved by the hard threshold parameter φ.

IV. Computer Simulations

In this section, the estimation performance of IPVSS-LMS for estimating channel is verified and compared with two ISS-LMS algorithms for estimating channel. One is used the step-size 0.005 and another is used the step-size 0.0005 which equals the hard threshold φ. The length of channel vector $\boldsymbol{h}$ is set as 128 with K ($K \in \{4, 8, 12\}$) non-zero coefficients. The values of dominant channel taps follow random Gaussian distribution and the positions of non-zero coefficients are random. The training signal is adopted by Pseudo-random (PN) binary sequence. The received signal-to-noise (SNR) is defined as $10\log(E_s/\sigma_n^2)$, where $E_s = 1$ is the unit transmission power. In this paper, we set SNR as 5, 10 and 15 to compare each other. The noise is set as additive white Gaussian.

The estimation performance is evaluated by average mean square error (MSE) which is defined by,

$$\mathrm{MSE}\left\{\tilde{\boldsymbol{h}}(n)\right\} \triangleq 10\log_{10} E\left\{\left\|\boldsymbol{h} - \tilde{\boldsymbol{h}}(n)\right\|_2^2\right\}. \tag{19}$$

We compare the performance of proposed channel estimators using 1000 independent Monte-Carlo runs for averaging. As shown in Figs. 2-7, in the case of different SNR regimes, VSS-LMS for estimating sparse channel which is proposed always has better estimation performance than ISS-LMS for estimating sparse channel with step-size μ_{upper}, achieves higher convergence speed than ISS-LMS for estimating channel with step-size μ_{lower}, and almost doesn't sacrifice the computational complexity.

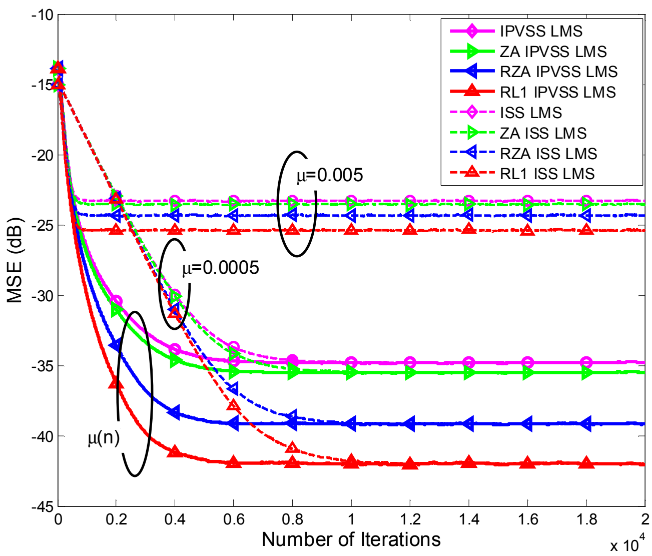

Fig. 2. MSE performance comparsion in the case of SNR=5dB and the channel sparsity K=4.

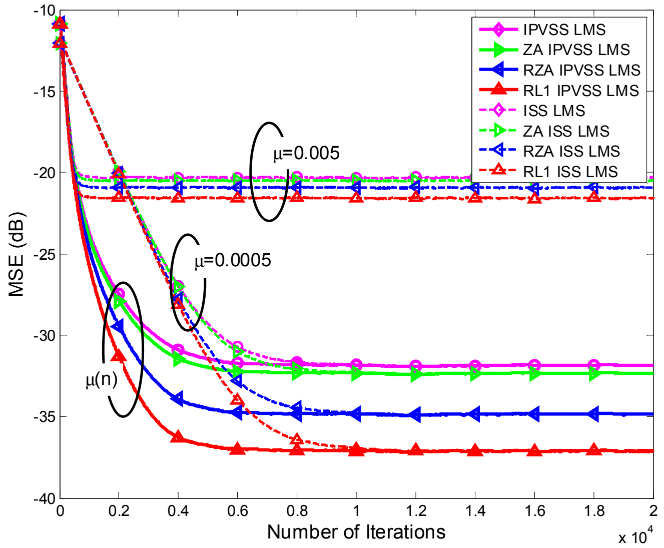

Fig. 3. MSE performance comparsion in the case of SNR=5dB and the channel sparsity *K*=8.

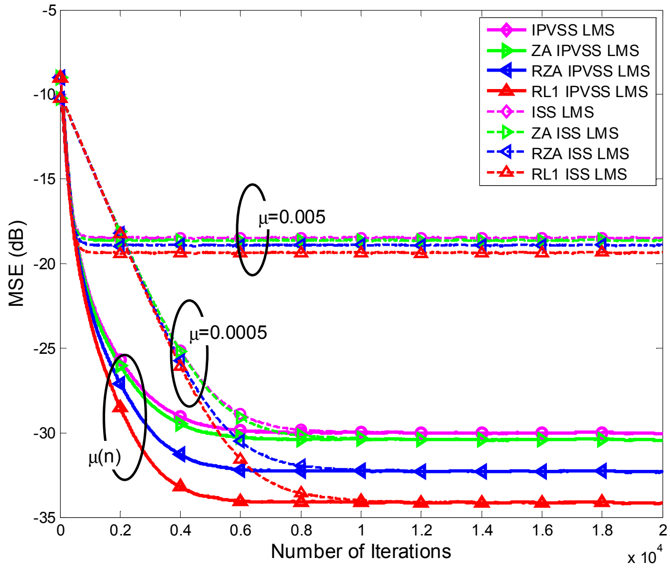

Fig. 4. MSE performance comparsion in the case of SNR=5dB and the channel sparsity *K*=12.

Let us take the Fig. 2 for example to illustrate the advantages of the proposed algorithms. In the case of 5dB, the number of non-zero coefficient is 4, they are compared with two groups of the performance curves of ISS-LMS for estimating sparse channel with different step- sizes (0.005 and 0.0005). In the first step, the proposed algorithms have a high speed convergence speed as same as ISS-LMS with step-size 0.005. When ISS-LMS with step-size 0.005 reach steady-state, the proposed algorithms continue to decline until $\mu(n)=0.0005$. In the second step, the steady-state performance curves of the proposed algorithms are same as ISS-LMS with step-size 0.0005. While it is obviously find that the proposed algorithms have a faster convergence speed than ISS-LMS with step-size 0.0005. In other words, the proposed algorithms have the convergence speed of ISS-LMS with step-size 0.0005 both the estimation performance of ISS-LMS with step-size 0.005.

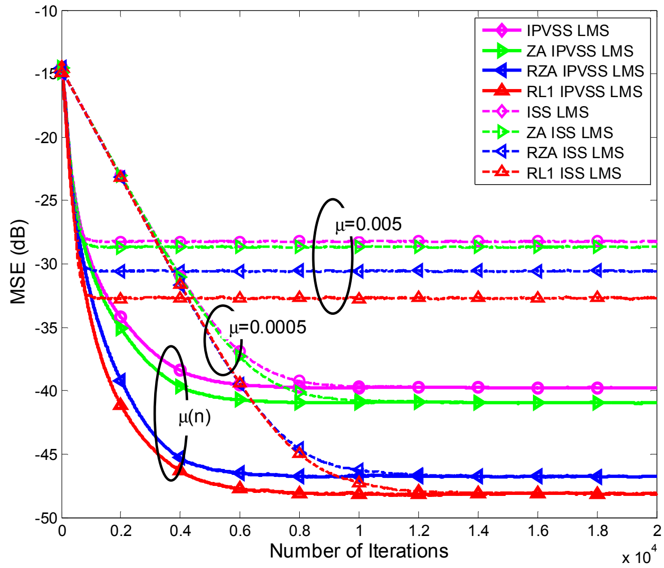

Fig. 5. MSE performance comparsion in the case of SNR=10dB and the channel sparsity K=4.

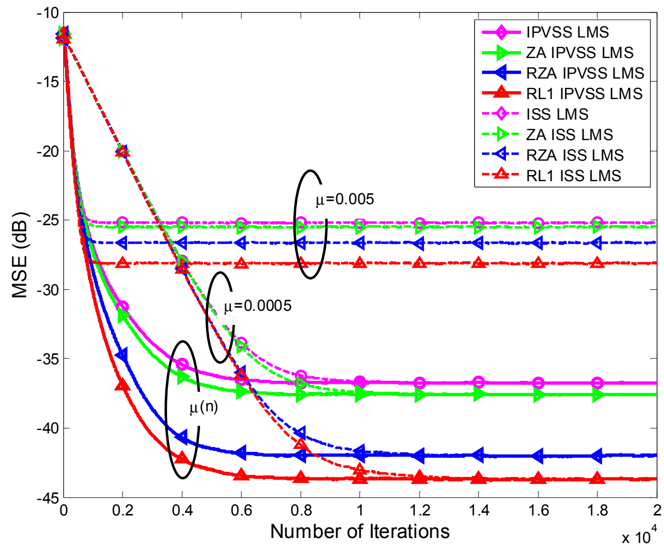

Fig. 6. MSE performance comparsion in the case of SNR=10dB and the channel sparsity *K*=8.

Fig. 8-10 demonstrate that the proposed channel estimation algorithms in the cased of 15dB, when different number of non-zero coefficient *K*. we can see that channel becomes smaller, namely the channel becomes sparser, the estimation performance of proposed algorithms can achieve better MSE performance than conventional sparse ISS-LMS algorithms with step-size 0.0005, while the convergence speed of the proposed IPVSS-LMS algorithms are faster than conventional sparse ISS-LMS algorithms. In Figs. 8-10, one can also find MSE performance of the proposed IPVSS-LMS algorithms are decided by the hard threshold φ, where smaller φ can bring better MSE performance but at the cost of convergence speed. Hence, selection of the threshold is also important technique to design the sparse IPVSS-LMS algorithms.

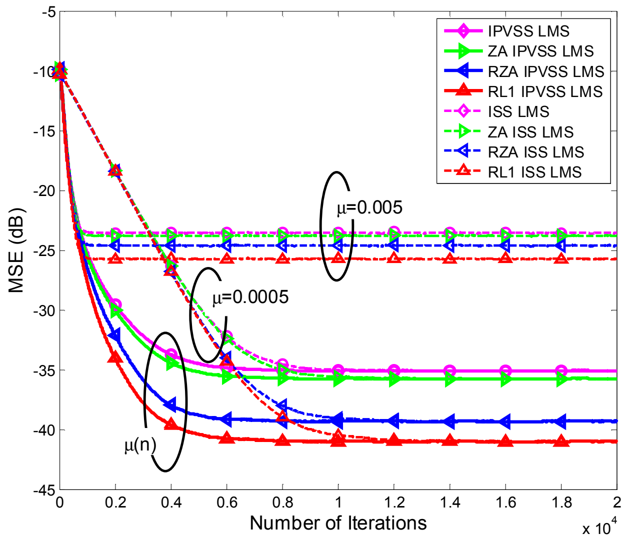

Fig. 7. MSE performance comparsion in the case of SNR=10dB and the channel sparsity K=12.

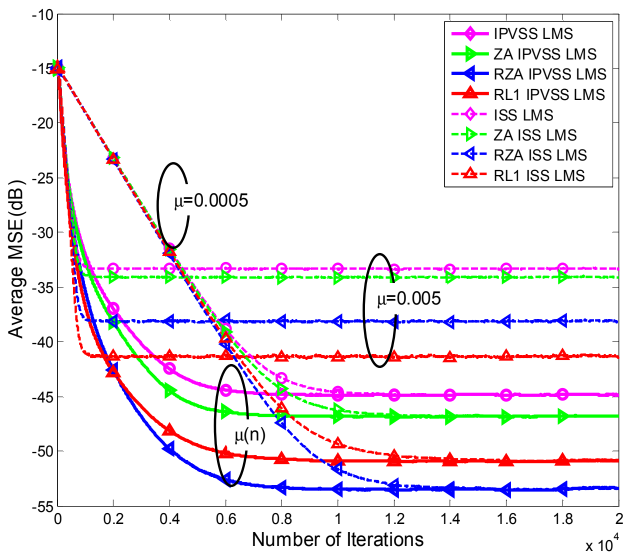

Fig. 8. MSE performance comparsion in the case of SNR=15dB and the channel sparsity K=4.

V. Conclusions

This paper has proposed three IPVSS-LMS algorithms to estimate sparse channels to accelerate the convergence speed. We first reviewed conventional sparse LMS algorithms, i.e., ZA-LMS, RZA-LMS and RL1-LMS and then presented the sparse IPVSS-LMS algorithms, i.e., ZA-IPVSS-LMS, RZA-IPVSS-LMS and RL1-IPVSS-LMS. The performance enhancement of the proposed channel estimation algorithms is achieved via designing VSS as well as exploiting channel sparsity. Compared to conventional sparse ISS-LMS algorithms, our proposed IPVSS-LMS algorithms can improve MSE performance while without scarifying convergence speed due to the fact that VSS is controlled only by iteration as well as threshold. Simulation results were provided to validate the proposed channel estimation algorithms.

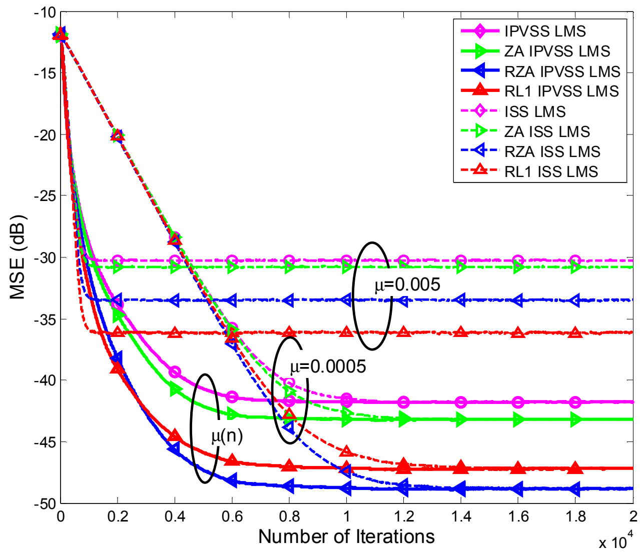

Fig. 9. MSE performance comparsions in the case of SNR=15dB and the channel sparsity K=8.

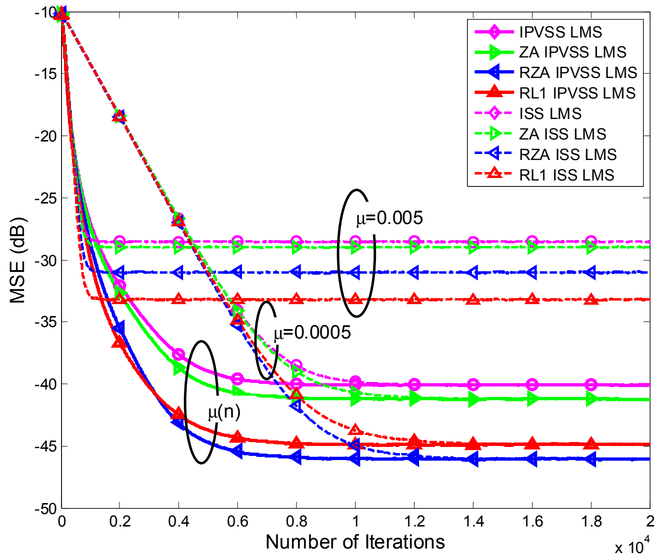

Fig. 10. MSE performance comparsions in the case of SNR=15dB and the channel sparsity K=12.

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