Validation of Kirchhoff-Huygens Equation by using FDTD Method in the Radio Frequency

Kazuma KABASAWA[†], *Student Member*, Yusuke KUSAMA^{†a)}, *Member*

1. Introduction

Scattering and diffraction problem are important in the ratio frequency. In addition, there is a need for simulation methods that are light computational load. The objective of this study is to validate the Kirchhoff-Huygens equation and explore its directivity characteristics using the FDTD method. The equation simplifies the diffraction pattern calculations for aperture size, shape, and frequency changes. Previous studies focused on the optics with sharp directivity, though its validity unverified for radio frequencies [1]. Recently, a multiple screen diffraction model has been proposed as an alternative, aiming for faster computations in large domains compared to the computationally expensive mesh-division numerical method. This method provides one solution to this kind of problem.

2. Kirchhoff-Huygens formula

In Figure 1, the electric field at the observation point O when a plane wave is uniformly irradiated on the aperture plane S_A is given by the Kirchhoff-Huygens equation, where S_A is the aperture plane, r is the distance from the aperture plane to the observation point O, and θ is the angle between the direction of propagation axis z and the observation point O. When the observation surface is planar, the theoretical diffraction pattern is derived by using the Cartesian coordinate system (x, y, z). Consider a diffraction pattern in a two-dimensional plane with y = 0. Since the diffraction



Fig. 1 Plane wave ψ enters region *V* from the aperture plane S_A on the conductive screen S_C and is observed at observation point O.

^{a)} E-mail: kusama@toyo.jp

pattern can be evaluated in terms of the magnitude of the electric field $|E_0|$, equation (1) is obtained by excluding the phase term, where E_A is the uniform electric field amplitude at the aperture plane S_A and l_x is the aperture dimension along the *x*-axis. Once the wavelength λ of the incident wave and the aperture width l_x are set, the diffraction pattern can be calculated instantly.

$$|E_{0}(x,0,d)| = \frac{E_{A}S_{A}}{\lambda d} \frac{\sin\left(\frac{\pi l_{x}x}{\lambda d}\right)}{\frac{\pi l_{x}x}{\lambda d}}$$
(1)

3. Calculation results and summary

Figure 2 shows the directivity patterns for three different frequencies 1 GHz, 10 GHz, and 100 GHz. All of them are calculated with an aperture dimension is $l_x = 24$ mm constant. Upper sides are results of the FDTD method, and lower sides are results from the analytical calculation Eq. (1) excluded phase information. In all cases, it can be seen that the directivity becomes sharper as the frequency increases. Especially in the case of 100 GHz, the calculation using the FDTD takes three days by using Intel Core i7-11700, but the analytical calculation can be completed instantly. In the future, we intend to verify the relationship between frequency and aperture size, as well as to calculate the electric field reflected from the aperture surface.



Fig. 2 2D directivity patterns. Upper sides show FDTD calculation and lower sides show analytical calculation excluded phase information.

References

 Y. Kusama, "Application example of Green's second theorem in electromagnetics," Industrial Technology No.44, pp. 61-70, Feb.2022.7

[†] The author is with Toyo University Kawagoe-shi, 350-0815 Japan.