Complexity Reduction of Pico Cell Clustering for Interference Alignment in Heterogeneous Networks

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***Abstract*—Interference Alignment (IA) in heterogeneous networks (HetNets) is a promising technique that improves the spectral efficiency significantly. We showed in [1] that transmit antennas at pico BSs could be utilized more efficiently by clustering pico cells in IA in HetNet where the clustering formation was optimized so as to minimize the rate loss caused by inter-cluster interference. In [1] , the optimum clustering formation was selected by comparing all possible formations, that is, the value of the objective function for all possible formations was calculated. Therefore, we required the enormous complexity to construct pico cell clusters. In this paper, we propose a novel algorithm for clustering pico cells that reduces the complexity of the clustering process. In particular, we define rate-loss matrix that represents the rate loss caused by inter-pico interference, and translate the optimization problem to the construction of rate-loss matrix. Clearly, the proposed algorithm is sub-optimum in terms of achievable rate compared to all-search algorithm used in [1]. However, the simulation results show that the difference of achievable rate between the proposed algorithm and all-search algorithm is negligible and becomes smaller as the cluster size decreases. We evaluate the complexity of proposed algorithm quantitatively comparing to all-search algorithm, and show that our algorithm reduces the complexity of clustering process significantly while achieving almost same performance.**

***Keywords*—complexity reduction, interference alignment, heterogeneous networks**

I. INTRODUCTION

In wireless communications, interference is a critical problem because of the explosive spread of wireless devices. It is conventionally used to divide the communication resources (time, frequency, etc.) among users so that each user exclusively gets the resource and avoids interfering with each other. However, each user gets less amount of resources as the network accommodates more users. Therefore, the increase of users in network causes the degradation of the capacity each user achieves.

Interference alignment (IA) [3] is an attractive technique to maximize the degrees of freedom of the network. The authors in [3] showed that the degrees of freedom of $K/2$ can be achieved in K user interference channel by aligning the interference to a certain space. This result means that by using IA each user gets the half of the whole resources free from interference so that more users can be accommodated in the network, where the explosive increase of users is expected.

Heterogeneous network (HetNet) [4] is also a promising technique to meet the increase of users in the network. In HetNet, some small (pico) cells are deployed in the area of macro cell, which enables the further enhancement of the network capacity by offloading some macro user equipments (UEs) to pico cells around. Recently, it is revealed that applying IA in HetNet can achieve a significant increase of the network capacity [1], [5], [6].

In [2], the authors showed that by aligning inter-pico interference and the strongest interference from macro BS, each pico UE can eliminate those interferences simultaneously, which results in significant increase of the network capacity. However, each pico BS requires a large number of transmit antennas due to aligning all inter-pico interference and the residual interference that is not eliminated by IA may cause the degradation of the performance.

To solve these problems, we proposed 1) to construct pico cell clusters and align inter-pico interference only within each cluster and 2) to apply the combined weight and mitigate the residual interference by minimum mean square error (MMSE) weight [1]. Here, we defined a metric between pico cells based on the rate loss caused by inter-pico interference and constructed pico cell clusters by minimizing the sum of the metric between clusters, which indicates that we minimized the rate loss caused by inter-cluster interference. However, we calculated the sum of the metric for all possible clustering formations and by comparing derived metrics we determined the optimum formation. Therefore, we required the enormous complexity to construct pico cell clusters.

In this paper, we propose a novel algorithm for clustering pico cells to reduce the complexity of clustering process. In particular, we define rate-loss matrix that represents the rate loss caused by inter-pico interference, and translate the optimization problem to the construction of rate-loss matrix. Clearly, the proposed algorithm is sub-optimum in terms of achievable rate compared to all-search algorithm that is the algorithm to search all possible clusters to determine the optimum cluster and used in [1]. However, the simulation results show that the difference of achievable rate between the proposed algorithm and all-search algorithm is negligible and becomes smaller as the cluster size decreases. We evaluate the complexity of proposed algorithm quantitatively comparing to all-search algorithm, and show that our algorithm reduces the complexity of clustering process significantly while achieving almost same performance.

This paper is organized as follows. Some preliminaries are presented in Section II where system model, design of the weights, and rate-loss matrix are described, respectively. The proposed algorithm is described and analyzed in Section III and the simulation results are shown in Section IV. Finally, the conclusion of this paper is stated in Section V.

II. Preliminary

A. System model

We assume the downlink of HetNet where the access mode is open access. In the network, there is one macro cell in which K_p pico cells exist. In macro cell, one macro BS and K_m macro UEs exist. In each pico cell, one pico BS and one corresponding pico UE exist. Macro BS, pico BSs, and UEs equip with N_{MT} transmit antennas, N_{PT} transmit antennas, and N_R receive antennas, respectively. Macro BS (pico BS) transmits d data streams to each macro UE (corresponding pico UE).

The system model considered is shown in Fig. 1.

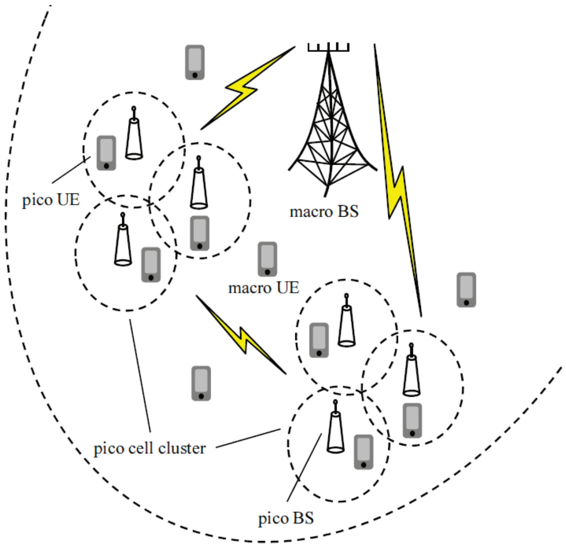

Fig. 1. system model

We construct clusters that consist of some pico cells. Here, we denote a clustering formation by C that is defined as follows.

$$C \stackrel{\text{def}}{=} \{C^1, C^2, \ldots, C^N\} \in \Omega \tag{1}$$

where C^n, Ω, and N represent the nth cluster, the set of all possible clustering formations, and the number of clusters in the network, respectively.

C^n is further denoted as follows.

$$C^n \stackrel{\text{def}}{=} \{b_1^n, b_2^n, \ldots, b_{|C^n|}^n\} \subset \{1, 2, \ldots, K_p\} \ (n = 1, 2, \ldots, N) \tag{2}$$

where b_m^n and $|C^n|$ represent mth pico cell included in the cluster C^n and the number of pico cells in cluster C^n, respectively.

For simplicity, we assume that each cluster consists of L pico cells.

$$|C^n| = L, \forall n \tag{3}$$

Note here that the proposed algorithm imposes no limitation on the value of $|C^n|$.

B. Design of the weights

The weights at each BS and UE are designed as shown in Table I.

The precoding weight at macro BS is designed by using block diagonalization (BD) scheme so as to direct the null space to macro UEs except for the desired macro UE and to eliminate inter-stream interference in the desired macro UE. The postcoding weight at macro UE is designed so as to eliminate inter-stream interference. The precoding weight at pico BS is designed so as to align the intra-cluster interference to the strongest interference from macro BS. The postcoding weight at pico UE is designed so as to null out the aligned interference by ZF weight and to mitigate the residual interference by MMSE weight.

C. Rate-loss matrix

The rate loss at pico cell a when interfered by pico cell b is represented as follows.

$$\begin{aligned}\Delta_{a,b} &= R_a - R_{a,b} \\ &= \log_2(1 + \rho_{aa}P_a) - \log_2\left(1 + \frac{\rho_{aa}P_a}{1 + \rho_{ab}P_b}\right)\end{aligned} \tag{4}$$

where R_a and $R_{a,b}$ represent the rate achieved at pico cell a without interference and the rate achieved at pico cell a with the interference from pico cell b, respectively. ρ_{ab} represents the path loss between pico BS b and pico UE a, and P_a represents the transmit power at pico BS a.

Rate-loss matrix is defined as the following matrix where the element in ith row and jth column represents the rate loss at pico cell i when interfered by pico cell j, that is, $\Delta_{i,j}$.

$$\begin{aligned}\boldsymbol{\Delta} &\stackrel{\text{def}}{=} \begin{bmatrix} \Delta_{1,1} & \Delta_{1,2} & \cdots & \Delta_{1,K_p} \\ \Delta_{2,1} & \Delta_{2,2} & \cdots & \Delta_{2,Kp} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta_{K_p,1} & \Delta_{K_p,2} & \cdots & \Delta_{K_p,K_p} \end{bmatrix} \\ &= \begin{bmatrix} 0 & \Delta_{1,2} & \cdots & \Delta_{1,K_p} \\ \Delta_{2,1} & 0 & \cdots & \Delta_{2,K_p} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta_{K_p,1} & \Delta_{K_p,2} & \cdots & 0 \end{bmatrix}\end{aligned} \tag{5}$$

Note that the rate loss between the same pico cell, that is, $\Delta_{a,a}$ is equal to 0.

We also define a metric between pico cell a and pico cell b as follows.

$$w(a,b) \stackrel{\text{def}}{=} \Delta_{a,b} + \Delta_{b,a} \tag{6}$$

Eq. (6) represents the sum of the rate loss when pico cell a and pico cell b interfere with each other, and is used as a key metric in the clustering optimization problem. Note that $\Delta_{a,b}$ and $\Delta_{b,a}$ are symmetric with respect to the diagonal in rate-loss matrix.

TABLE I. DESIGN OF WEIGHTS

type of weight		how to design
precoding weight	macro BS	direct the null space to macro UEs except for the desired UE and eliminate inter-stream interference as in [1]
	pico BS	align the intra-cluster interference to the strongest interference from macro BS as in [2]
postcoding weight	macro UE	eliminate inter-stream interference as in [1]
	pico UE	ZF weight nulls out the aligned interference and MMSE weight mitigate the residual interference as in [2]

III. CLUSTERING OPTIMIZATION

A. Translation of the problem

Clustering formation is optimized based on rate-loss matrix defined in eq. (5). Here, we construct clusters such that minimize the sum of the metrics (defined in eq. (6)) between clusters, which indicates minimizing the rate loss caused by inter-cluster interference. By using the characteristic that the metric is the sum of elements which are symmetric with respect to the diagonal of rate-loss matrix, we translate the optimization problem to the construction of rate-loss matrix.

Clustering optimization problem is translated by the following three steps.

- STEP 1 : the minimization of the sum of metrics between clusters is translated to the maximization of the sum of elements in rate-loss matrix
- STEP 2 : the maximization overall rate-loss matrix is divided to some local maximizations in each cluster.
- STEP 3 : each local maximization is translated to step maximization along the diagonal of rate-loss matrix.

Each step is described in detail below.

1) STEP 1: After applying a clustering formation C to rate-loss matrix, the following matrix is obtained.

$$\begin{aligned}\boldsymbol{\Delta}(C) &= \boldsymbol{\Delta}_D(C) + \boldsymbol{\Delta}_{\bar{D}}(C) \\ &= \begin{bmatrix} \boldsymbol{\Delta}_D(C^1) & \times & \dots & \times \\ \times & \boldsymbol{\Delta}_D(C^2) & \dots & \times \\ \vdots & \vdots & \ddots & \vdots \\ \times & \times & \dots & \boldsymbol{\Delta}_D(C^N) \end{bmatrix}\end{aligned} \tag{7}$$

where $\boldsymbol{\Delta}_D(C)$ is the block diagonal matrix that consists of block diagonal elements of $\boldsymbol{\Delta}(C)$ corresponding to each cluster, and is expressed as follows.

$$\boldsymbol{\Delta}_D(C) \overset{\text{def}}{=} \begin{bmatrix} \boldsymbol{\Delta}_D(C^1) & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Delta}_D(C^2) & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \boldsymbol{\Delta}_D(C^N) \end{bmatrix} \tag{8}$$

Note here that $\boldsymbol{\Delta}_D(C^n)$ is a square matrix and its size is $|C^n|$.

$\boldsymbol{\Delta}_{\bar{D}}(C)$ is the complement matrix of $\boldsymbol{\Delta}_D(C)$, and is given as follows.

$$\boldsymbol{\Delta}_{\bar{D}}(C) \overset{\text{def}}{=} \boldsymbol{\Delta}(C) - \boldsymbol{\Delta}_D(C) = \begin{bmatrix} \mathbf{0} & \times & \dots & \times \\ \times & \mathbf{0} & \dots & \times \\ \vdots & \vdots & \ddots & \vdots \\ \times & \times & \dots & \mathbf{0} \end{bmatrix} \tag{9}$$

Each element of $\boldsymbol{\Delta}_D(C)$ represents the rate loss caused by intra-cluster interference, and we call here the sum of these elements as internal sum-rate loss. On the other hand, each element of $\boldsymbol{\Delta}_{\bar{D}}(C)$ represents the rate loss caused by inter-cluster interference, and we call here the sum of these elements as external sum-rate loss.

We also define the operator su($\mathbf{A}$) which calculates the sum of the elements of matrix $\mathbf{A}$. By using su($\cdot$), we obtain the following equation.

$$\mathrm{su}\left(\boldsymbol{\Delta}(C)\right) = \mathrm{su}\left(\boldsymbol{\Delta}_D(C)\right) + \mathrm{su}\left(\boldsymbol{\Delta}_{\bar{D}}(C)\right) = \mathrm{su}\left(\boldsymbol{\Delta}\right) \tag{10}$$

where su$(\boldsymbol{\Delta}_D(C))$ represents internal sum rate-loss, and su$\big(\boldsymbol{\Delta}\bar{D}(C)\big)$ represents external sum rate-loss, respectively. In particular, since su$(\boldsymbol{\Delta})$ is constant no matter what the clustering formation is, minimizing external sum rate -loss is equal to maximizing internal sum rate-loss.

The clustering formation is optimized by minimizing the sum of the rate loss caused by inter-cluster interference, and is expressed as the minimization of external sum rate-loss. As mentioned above, this is equal to the maximization of internal sum rate-loss. Therefore, we translate the clustering optimization problem to the maximization of internal sum rate-loss.

2) STEP 2: The maximization of internal sum rate-loss is transformed as follows.

$$\begin{aligned} & \max_{C \in \Omega} \mathrm{su}\left(\boldsymbol{\Delta}_D(C)\right) \\ =\ & \max_{\{C^1,\dots,C^N\} \in \Omega} \sum_{n=1}^{N} \mathrm{su}\left(\boldsymbol{\Delta}_D(C^n)\right) \\ \approx\ & \sum_{n=1}^{N} \max_{\substack{C^n \subset \{1,\dots,K_p\} \\ C^n \cap C^k = \emptyset, k<n}} \mathrm{su}\left(\boldsymbol{\Delta}_D(C^n)\right) \\ =\ & \sum_{n=1}^{N-1} \max_{\substack{C^n \subset \{1,\dots,K_p\} \\ C^n \cap C^k = \emptyset, k<n}} \mathrm{su}\left(\boldsymbol{\Delta}_D(C^n)\right) \end{aligned} \tag{11}$$

Eq. (11) means that the maximization process of su$(\boldsymbol{\Delta}_D(C))$ can be divided to the sum of the local maximization of su$(\boldsymbol{\Delta}_D(C^n))$ in each cluster. Note that the maximization in eq. (11) represents that $|C^n|$ pico cells are selected so as to maximize su$(\boldsymbol{\Delta}_D(C^n))$ from pico cells that have not selected yet.

The last equality is followed by the fact that the last cluster is uniquely determined by selecting from C^1 to C^{N-1}.

TABLE II. COMPLEXITY COMPARISON (LEFT : ALL SEARCH, RIGHT : PROPOSAL)

N \ K_p	12		24		36	
2	33264	2232	3.9×10^8	17568	2.9×10^{12}	58968
4	554400	2376	1.4×10^{13}	18576	2.9×10^{20}	62208
6	249480	2280	510048	17760	2.1×10^8	59400

3) STEP 3: The local maximization in eq. (11) is transformed as follows.

$$\begin{aligned}
&\max_{\substack{C^n \subset \{1,\ldots,K_p\} \\ C^n \cap C^k = \emptyset, k<n}} \mathrm{su}\left(\boldsymbol{\Delta}_D(C^n)\right) \\
&= \max_{\substack{\{b_1,\ldots,b_{|C^n|}\} \subset \{1,\ldots,K_p\} \\ \{b_1,\ldots,b_{|C^n|}\} \cap C^k = \emptyset, k<n}} \mathrm{su}\left(\boldsymbol{\Delta}_D\left(\{b_1,\ldots,b_{|C^n|}\}\right)\right) \\
&= \max_{\substack{\{b_1,\ldots,b_{|C^n|}\} \subset \{1,\ldots,K_p\} \\ \{b_1,\ldots,b_{|C^n|}\} \cap C^k = \emptyset, k<n}} \sum_{m=1}^{|C^n|} \sum_{a=T^{n-1}+1}^{T^{n-1}+m-1} w\left(a, b_m\right) \\
&\approx \sum_{m=2}^{|C^n|} \max_{\substack{b_m \in \{1,\ldots,K_p\} \\ b_m \notin C^k, k<n \\ b_m \neq b_l, l<m}} \sum_{a=T^{n-1}+1}^{T^{n-1}+m-1} w\left(a, b_m\right) \qquad (12)
\end{aligned}$$

where T^n represents the sum of the number of pico cells included in from C^1 to C^n, and is expressed as $\sum_{k=1}^{n} |C^k|$.

The maximization in eq. (12) means that the pico cell b_m that maximizes the objective function $\sum_{a=T^{n-1}+1}^{T^{n-1}+m-1} w\left(a, b_m\right)$ is selected in the set of pico cells which are not selected yet.

Since the first pico cell in each cluster can not be selected by eq. (12), we select the first pico cell as follows.

$$b_1 = \arg\max_{\substack{b_1 \in \{1,\ldots,K_p\} \\ b_1 \notin C^k, k<n}} \sum_{a=T^{n-1}+1}^{K_p} w\left(a, b_1\right) \qquad (13)$$

Algorithm 1 Clustering optimization algorithm

INPUT: $T^0 = 0, \boldsymbol{\Delta}, \Theta = (1, 2, \ldots, K_p)$
for $n = 1, 2, \ldots, N-1$ **do**
  $T^n = T^{n-1} + |C^n|$
  for $m = 1, 2, \ldots, |C^n|$ **do**
    if m = 1 **then**
      $b_m^* = \operatorname{argmax}_{b_m \in \left\{T^{n-1}+1, T^{n-1}+2, \ldots, K_p\right\}} \sum_{a=T^{n-1}+1}^{K_p} w(a, b_m)$
    else
      $b_m^* = \operatorname{argmax}_{b_m \in \left\{T^{n-1}+m, T^{n-1}+m+1, \ldots, K_p\right\}} \sum_{a=T^{n-1}+1}^{T^{n-1}+m-1} w(a, b_m)$
    end if
    exchange the b_m^* th row (column) and the $(T^{n-1}+m)$ th row (column) of Δ
    exchange the b_m^* th element to the $(T^{n-1}+m)$ th element of Θ
  end for
end for

B. Proposed Algorithm

The proposed algorithm is summarized as in Algorithm 1 where Θ is the vector that represents the clustering formation finally derived.

C. Complexity analysis

The objective functions of both all-search algorithm and proposed algorithm are only comprised of the addition of the rate loss. Therefore, we use the total number of additions in the optimization problem as the complexity. The complexities of all-search algorithm and proposed algorithm are evaluated below where S denotes the number of calculations of the objective function for simplicity.

1) All-search: The optimization problem in all-search algorithm is expressed as follows.

$$\begin{aligned}
C^* &= \operatorname*{argmax}_{C \in \Omega} \ \mathrm{su}\left(\Delta_D\left(C\right)\right) \\
&= \operatorname*{argmax}_{C \in \Omega} \left(\sum_{n=1}^{N} \sum_{a=T^{n-1}+1}^{T^n} \sum_{b=T^{n-1}+1}^{T^n} \Delta_{a,b} \right) \qquad (14)
\end{aligned}$$

In the objective function of eq. (14), the number of additions is $\sum_{n=1}^{N} \left(T^n - \left(T^{n-1}+1\right)+1\right)^2 = \sum_{n=1}^{N} |C^n|^2 = NL^2$. Therefore, the total number of additions in the optimization problem is given as SNL^2 where S is equal to the number of possible clusters and given as follows.

$$S = \frac{\prod_{i=0}^{N-2} \binom{L(N-i)}{L}}{N!} \qquad (15)$$

2) Proposal: The optimization problem in the proposed algorithm is expressed as follows.

$$b_m^n = \operatorname*{argmax}_{b \in \{T^{n-1}+m, \ldots, K_p\}} \begin{cases} \sum_{a=T^{n-1}+1}^{K_p} w(a, b_m), & \text{for } m = 1 \\ \sum_{a=T^{n-1}+1}^{T^{n-1}+m-1} w(a, b_m), & \text{for } m \neq 1 \end{cases} \qquad (16)$$

where $n \in \{1, 2, \ldots, N-1\}$ and $m \in \{1, 2, \ldots, |C^n|\}$ for given n, respectively.

In the objective function of eq. (16), the total (maximum) number of additions is $2m \leq 2K_p = 2NL$ and factor 2 is due to $w(a, b) = \Delta_{a,b} + \Delta_{b,a}$. Therefore, the total number of additions in the optimization problem is given as $2SNL$ where S is represented as follows.

$$S = \sum_{n=1}^{N-1} \sum_{m=1}^{|C^n|} \left(K_p - \left(T^{(n-1)} + m\right) + 1\right) \qquad (17)$$

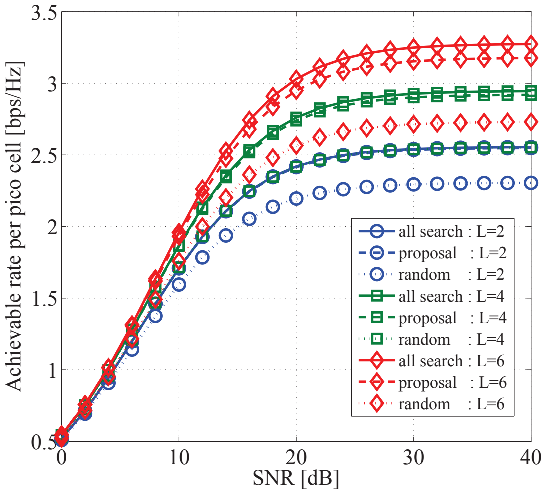

Fig. 2. Average rate per pico cell at each cluster size

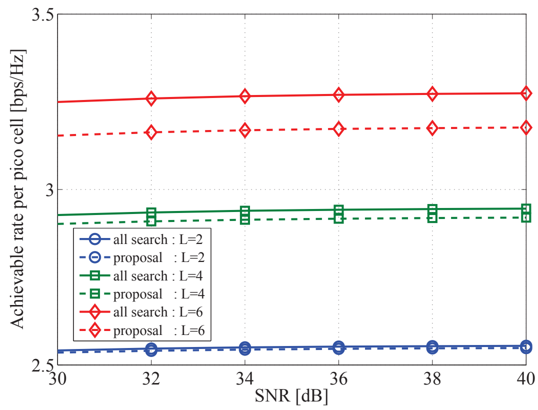

Fig. 3. Comparison between all search and proposal

3) Comparison: The result of complexity comparison is shown in TABLE II where the number of pico cells (K_p) is 12, 24, 36, respectively, and the number of clusters (N) is 2, 4, 6, respectively for each K_p. Note that in TABLE II, the left side is the complexity of all-search algorithm and the right side is that of the proposed algorithm.

As shown in TABLE II, the proposed algorithm reduces the complexity significantly compared to all-search algorithm.

IV. Performance Evaluation

The simulation parameters are shown in TABLE III. The configuration for the simulation is same as in [1]. Thus, we omit the detail explanation for the configuration.

Fig. 2 shows the average rate per pico cell for three cases where case 1 is when the clustering formation derived by all search is used (all search), case 2 is when the clustering formation derived by the proposed algorithm is used (proposal), and case 3 is when the random clustering formation is used (random), respectively. In Fig. 2, the horizontal line represents SNR in dB and the vertical line represents the average rate per pico cell in bps/Hz. As shown in the figure, the rate is higher in the order of all search, proposal, and random. Therefore, the proposed algorithm is sub-optimum in terms of achievable rate. However, the difference between all search and proposal is negligible as shown in the figure.

To facilitate the comparison of all search and proposal, the enlarged figure from 30 dB to 40 dB in Fig. 2 is shown in Fig. 3. As shown in Fig. 3, the difference between all search and proposal becomes smaller as the cluster size decreases. In particular, the performance is almost same when the cluster size is 2. Therefore, the difference between all search and proposal can be negligible as the cluster size becomes smaller, that is, the effect of clustering becomes smaller.

V. Conclusion

In this paper, we proposed a novel algorithm for pico cell clustering for interference alignment in heterogeneous networks to reduce the complexity of clustering process. In particular, we translate the optimization problem to the construction of an appropriately defined matrix (rate-loss matrix). The simulation results showed that even though the proposed algorithm is sub-optimum in terms of achievable rate, the difference between the proposed algorithm and all-search algorithm is negligible. We also evaluated the complexity of the proposed algorithm comparing to all-search algorithm, and showed that a significant reduction of the complexity is achieved while achieving almost same performance.

TABLE III. Simulation Parameters

macro BS		1		
macro UE		12		
pico BS		12		
pico UE		1 per pico cell		
radius of macro cell		500 m		
radius of pico cell		50 m		
cluster size		2	4	6
transmit stream		2		
Rx antenna		4		
Tx antenna	macro BS	44		
	pico BS	4	12	20
channel coefficient		i.i.d. Gaussian		
path loss model		$-38.46 - \log_{10}$ (distance (km))		

References

[1] R. Seno, T. Ohtsuki, W. Jiang, and Y. Takatori, "Interference Alignment in Heterogeneous Networks using Pico Cell Clustering," *IEEE Vehicular Technology Conference (VTC)*, Sept. 2015, submitted

[2] Q. Niu, Z. Zeng, T. Zhang, Q. Gao, and S. Sun, "Joint Interference Alignment and Power Allocation in Heterogeneous Networks," *IEEE Personal, Indoor and Mobile Radio Communications (PIMRC)*, pp. 811--815, Sept. 2014

[3] V. Cadambe and S. Jafar, "Interference alignment and the degrees of freedom of the K user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425-3441, Aug. 2008.

[4] V. Chandrasekhar and J.G. Andrews, "Femtocell networks: a survey," *IEEE Commun. Mag.*, vol. 46, no. 9, pp. 59-67, Sept. 2008.

[5] W. Shin, W. Noh, K. Jang, and H. Choi, "Hierarchical Interference Alignment for Downlink Heterogeneous Networks," *IEEE Trans. on Wireless Communications*, vol.11, No. 12, pp. 4549-4558, Dec. 2012

[6] H. Lee and Y. Ko, "Linear Transceiver Design Based on Interference Alignment for MIMO Heterogeneous Networks," *IEEE Personal, Indoor and Mobile Radio Communications (PIMRC)*, pp. 1645-1650, Sept. 2012