Computational Complexity Reduction of Orthogonal Precoding of N-continuous OFDM

Hikaru Kawasaki, Takahiro Matsui, Masaya Ohta, and Katsumi Yamashita
Department of Electrical and Information Systems,
Graduate School of Engineering, Osaka Prefecture University, Japan
Email: {ota,yamashita}@eis.osakafu-u.ac.jp

Abstract: N-continuous OFDM is a precoding method for sidelobe suppression of orthogonal frequency division multiplexing (OFDM) signals and is to seamlessly connect OFDM symbols up to the high order derivative for sidelobe suppression, which is suitable for suppressing out-of-band radiation. However, it degrade the error rate severely as increasing the continuous derivative order. Orthogonal precoding of N-continuous OFDM has both a sidelobe suppression performance and an ideal error rate; however, it requires a very large computational complexity for precoding and decoding. This paper proposes a matrix decomposition of the large-sized matrix in the orthogonal precoding of N-continuous OFDM to reduce the computational complexity. Numerical experiments show that the proposed method can drastically reduce the computational complexity.

Keywords— **OFDM, orthogonal precoding, singular-value decomposition, computational complexity reduction**

1. Introduction

The advantages of fast data transmission and robustness against multipath fading have led to orthogonal frequency division multiplexing (OFDM) being adopted in several telecommunications technologies. One of the drawbacks associated with the design of OFDM transmitters is that high out-of-band radiation is generated by the high sidelobes of the OFDM signal. A critical issue concerning OFDM-based cognitive radio systems is that unwanted in-band and out-of-band radiation interferes with the adjacent bands. Various methods of sidelobe suppression have been proposed [1]–[4].

N-continuous OFDM [1] is a precoding method to seamlessly connect OFDM symbols up to the high order derivative for sidelobe suppression, which is suitable for suppressing out-of-band radiation. However, the error rate performance is inevitably degraded due to an irreversible distortion introduced to the transmitted symbol by its precoding and it becomes larger as increasing the continuous derivative order.

Orthogonal precoding of N-continuous OFDM was initially given in [2]. In orthogonal precoding, it can achieve both the sidelobe suppression performance of N-continuous OFDM and the ideal error rate. On the other hand, the data rate loss occurs and the computational complexity for precoding and decoding is very huge due to the large-sized matrix. Then, Ref.[3] has presented an improved orthogonal precoding that the data rate loss can be limited to half compared with the means in [2]; however, the disadvantage of huge computational complexity is still left.

To reduce the computational complexity, this paper proposes a matrix decomposition of the large-sized matrix in the orthogonal precoding of N-continuous OFDM. Numerical experiments show that the proposed method can drastically reduce the computational complexity.

2. Orthogonal Precoding of N-continuous OFDM

In this paper, the OFDM signal is written as

$$s(t) = \sum_{i=0}^{\infty} s_i(t - iT), \tag{1}$$

where $T = T_s + T_g$, T_s is the OFDM symbol duration and T_g is the guard interval length. The i-th OFDM symbol $s_i(t)$ is written as

$$s_i(t) = \sum_{k \in \mathcal{K}} \bar{d}_{k,i} e^{j2\pi \frac{k}{T_s} t}, \tag{2}$$

where $\bar{d}_{k,i} \in \mathbb{C}$ is a precoded symbol transmitted in the k-th subcarrier of the OFDM symbol, $\mathcal{K} = \{k_0, \cdots, k_{K-1}\}$ is the set of the subcarrier indices and K is the number of subcarriers. To smoothly connect the consecutive OFDM symbols $s_i(t)$, $s_{i-1}(t)$ and their first $N(\ll K)$ derivatives continuous for sidelobe suppression, the scheme of N-continuous OFDM [1] presents the constraints such as

$$\frac{d^n}{dt^n} s_i(t) \bigg|_{t=-T_g} = \frac{d^n}{dt^n} s_{i-1}(t) \bigg|_{t=T_s}. \tag{3}$$

For the OFDM symbol (2), the constraints (3) can be cast in matrix form that such as

$$\mathbf{A\Phi}\bar{\mathbf{d}}_i = \mathbf{A}\bar{\mathbf{d}}_{i-1}, \tag{4}$$

where $\mathbf{A}$ is an $(N+1) \times K$ matrix with the elements $[\mathbf{A}]_{m,n} = (k_n)^{m-1}$, $\mathbf{\Phi} = \text{diag}\,(e^{j\phi k_0}, e^{j\phi k_1}, \ldots, e^{j\phi k_{K-1}})$ is a $K \times K$ diagonal matrix with $\phi = -2\pi T_g/T_s$, the $K \times 1$ vector $\bar{\mathbf{d}}_i = [\bar{d}_{i,k_0}, \bar{d}_{i,k_1}, \cdots, \bar{d}_{i,k_{K-1}}]^T \in \mathbb{C}^K$ is the result of precoding the $D \times 1$ vector $\mathbf{d}_i = [d_{i,0}, \cdots, d_{i,D-1}]^T$ containing $D\,(\leq K)$ information symbols in some finite symbol constellation. From $\mathbf{\Phi\Phi}^H = \mathbf{I}_K$, the constraint (4) can be rewritten as

$$\mathbf{B}\bar{\mathbf{d}}_i = \mathbf{B\Phi}^H \bar{\mathbf{d}}_{i-1}, \tag{5}$$

where $\mathbf{B} = \mathbf{A\Phi}$.

Ref.[3] has proposed the orthogonal precoding with $D = K - (N+1)$ that determines the solution of (5) as

$$\bar{\mathbf{d}}_i = \mathbf{V}_{[D]}\mathbf{d}_i + \mathbf{V}_{[N+1]}\mathbf{V}_{[N+1]}^H \mathbf{\Phi}^H \bar{\mathbf{d}}_{i-1}, \tag{6}$$

where

$$\mathbf{V}_{[N+1]} = \mathbf{V}\begin{bmatrix}\mathbf{I}_{N+1}\\ \mathbf{O}_{D\times(N+1)}\end{bmatrix} = [\mathbf{v}_0\ \mathbf{v}_1\ \dots\ \mathbf{v}_N], \tag{7}$$

$$\mathbf{V}_{[D]} = \mathbf{V}\begin{bmatrix}\mathbf{O}_{(N+1)\times D}\\ \mathbf{I}_{D}\end{bmatrix} = [\mathbf{v}_{N+1}\ \mathbf{v}_{N+2}\ \dots\ \mathbf{v}_{K-1}], \tag{8}$$

are the $K \times (N+1)$ and $K \times D$ matrices respectively, $\mathbf{V} = [\mathbf{V}_{[N+1]}\ \mathbf{V}_{[D]}] = [\mathbf{v}_0\ \dots\ \mathbf{v}_N\ \mathbf{v}_{N+1}\ \dots\ \mathbf{v}_{K-1}]$ is obtained from the singular-value decomposition (SVD) that factorizes $\mathbf{B}$ as

$$\mathbf{B} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H, \tag{9}$$

$\mathbf{U}$ is an $(N+1)\times(N+1)$ unitary matrix, and $\mathbf{\Sigma}$ is a diagonal $(N+1)\times K$ matrix containing the singular values of $\mathbf{B}$ in non-increasing order along its diagonal.

For the receiver, the decoding that inverts the transmitter precoding (6) are presented in [3] as

$$\mathbf{r}_i = \mathbf{V}_{[D]}^H \tilde{\mathbf{r}}_i, \tag{10}$$

where $\tilde{\mathbf{r}}_i$ is the i-th received OFDM symbol after the channel equalization, and (10) provides $\mathbf{r}_i = \mathbf{d}_i$ in the noiseless condition since $\mathbf{V}$ is unitary ($\mathbf{V}_{[D]}^H\mathbf{V}_{[D]} = \mathbf{I}_D$ and $\mathbf{V}_{[D]}^H\mathbf{V}_{[N+1]} = \mathbf{O}_{K\times(N+1)}$).

The precoding (6) and the decoding (10) require $K(K+N+2)$ and $K(K-N-1)$ multiplications, respectively. These are very huge and thus must be reduced.

3. Proposed System and Analysis

This paper proposes a matrix decomposition of the $K \times D$ matrix $\mathbf{V}_{[D]}$ to reduce the computational complexity in the orthogonal precoding [3]. As a general method to the computational complexity reduction in engineering, SVD has been used to obtain the decomposition of small-sized matrices assuming that the large-sized matrix is rank deficient. On the other hand, the SVD to $\mathbf{V}_{[D]}$ does not lead to reduction of the computational complexity because $\mathrm{rank}\{\mathbf{V}_{[D]}\} = \mathrm{rank}\{\mathbf{V}_{[D]}^H\mathbf{V}_{[D]}\} = \mathrm{rank}\{\mathbf{I}_D\}$ is full. Then $\mathbf{V}$ as the body of $\mathbf{V}_{[D]}$ in (8) is unitary whose rank is surely full, which means that the SVD to $\mathbf{V}$ is also not effective for the computational complexity reduction at all.

Therefore, we consider the SVD to $\mathbf{V} - \mathbf{I}_K$. The SVD of $\mathbf{V} - \mathbf{I}_K$ is written as

$$\mathbf{V} - \mathbf{I}_K = \mathbf{X}\mathbf{Y}\mathbf{Z}^H, \tag{11}$$

where $\mathbf{X} = [\mathbf{x}_0\ \mathbf{x}_1\ \dots\ \mathbf{x}_{K-1}]$ and $\mathbf{Z} = [\mathbf{z}_0\ \mathbf{z}_1\ \dots\ \mathbf{z}_{K-1}]$, are $K \times K$ unitary matrices, $\mathbf{Y}$ is a diagonal $K \times K$ matrix containing the singular values of $\mathbf{V} - \mathbf{I}_K$ in non-increasing order along its diagonal, expressed as

$$\mathbf{Y} = \mathrm{diag}\left(\sigma_0, \sigma_1, \cdots, \sigma_{K-1}\right), \tag{12}$$

and $\sigma_0 \geq \sigma_1 \geq \cdots \geq \sigma_{K-1}$ are the singular values of $\mathbf{V} - \mathbf{I}_K$.

From the Eckart–Young–Mirsky theorem, the matrix $\mathbf{V} - \mathbf{I}_K$ is approximated by replacing the singular values by zero except for the first L largest values, i.e.,

$$\mathbf{V} - \mathbf{I}_K \simeq \mathbf{X}\tilde{\mathbf{Y}}\mathbf{Z}^H, \tag{13}$$

where $\tilde{\mathbf{Y}}$ is a $K \times K$ diagonal matrix expressed as

$$\tilde{\mathbf{Y}} = \mathrm{diag}\left(\sigma_0, \sigma_1, \cdots, \sigma_{L-1}, 0, \cdots, 0\right). \tag{14}$$

Then we can obtain the decomposition of $\mathbf{V}$ such as

$$\mathbf{V} \simeq \mathbf{I}_K + \mathbf{X}\tilde{\mathbf{Y}}\mathbf{Z}^H = \mathbf{I}_K + \mathbf{Q}\mathbf{R}^H, \tag{15}$$

where $\mathbf{Q}$ is the $K \times L$ matrix that consists of the first L columns of the matrix $\mathbf{X}\tilde{\mathbf{Y}}$, expressed as

$$\mathbf{Q} = [\sigma_0\mathbf{x}_0\ \sigma_1\mathbf{x}_1\ \dots\ \sigma_{L-1}\mathbf{x}_{L-1}], \tag{16}$$

and $\mathbf{R}$ is the $K \times L$ matrix that consists of the first L columns of the matrix $\mathbf{Z}$, expressed as

$$\mathbf{R} = [\mathbf{z}_0\ \mathbf{z}_1\ \dots\ \mathbf{z}_{L-1}] = [\mathbf{z}'_0\ \mathbf{z}'_1\ \dots\ \mathbf{z}'_{K-1}]^H. \tag{17}$$

Substituting (15) into (6), the $K \times D$ matrix $\mathbf{V}_{[D]}$ also can be decomposed easily and we finally rewrite the precoding (10) and the decoding (15):

$$\bar{\mathbf{d}}_i \simeq \begin{bmatrix}\mathbf{O}_{(N+1)\times D}\\ \mathbf{I}_D\end{bmatrix}\mathbf{d}_i + \mathbf{Q}\mathbf{S}\mathbf{d}_i + \mathbf{V}_{[N+1]}\mathbf{V}_{[N+1]}^H\mathbf{\Phi}^H\bar{\mathbf{d}}_{i-1}, \tag{18}$$

$$\mathbf{r}_i \simeq \begin{bmatrix}\mathbf{O}_{D\times(N+1)}\ \mathbf{I}_D\end{bmatrix}\tilde{\mathbf{r}}_i + \mathbf{S}^H\mathbf{Q}^H\tilde{\mathbf{r}}_i, \tag{19}$$

where $\mathbf{S}$ is the $L \times D$ matrix composed of the last $D = K - (N+1)$ columns of $\mathbf{R}^H$, expressed as

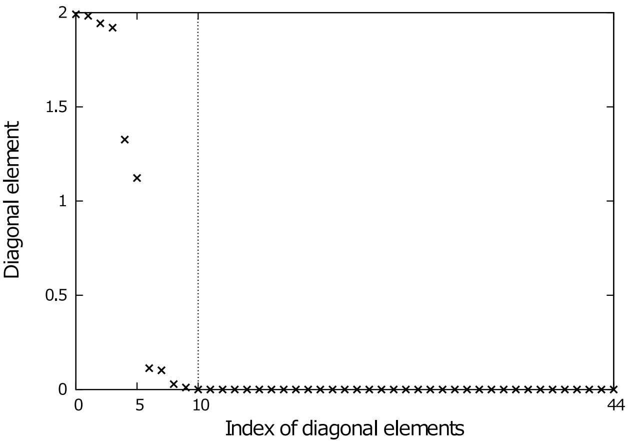
$$\mathbf{S} = [\mathbf{z}'_{N+1}\ \mathbf{z}'_{N+2}\ \dots\ \mathbf{z}'_{K-1}]. \tag{20}$$

We analyzed $\mathbf{Y}$ expressing the singular values $\mathbf{V} - \mathbf{I}_K$ under the experimental conditions in [2] and [3]. Figs. 1(a) and (b) show the all 45 and the first 100 diagonal elements of $\mathbf{Y}$, that is, the singular values $\sigma_0, \cdots, \sigma_{44}$ and $\sigma_0, \cdots, \sigma_{99}$, respectively. The results show that almost all diagonal elements can be considered as zeros, except for the first few values. The number of non-zero diagonal elements L can be found as $2(N+1)$ from these results.

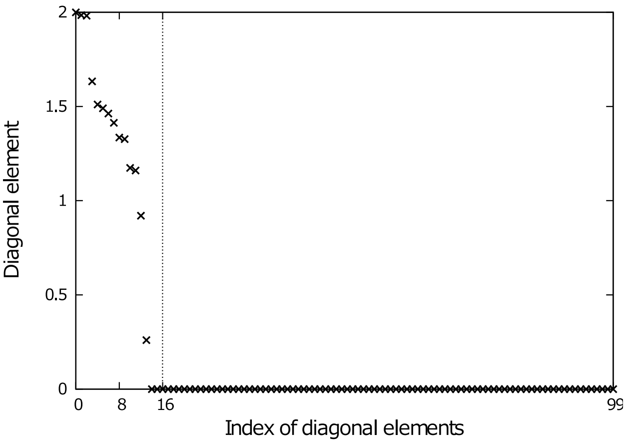
In the proposed system, the precoding (18) and the decoding (19) require $L(K+D) + 2(N+1)K = 2(N+1)(3K - N - 1)$ and $L(K+D) = 2(N+1)(2K - N - 1)$ multiplications if $L = 2(N+1)$, compared with $K(K+N+2)$ and $K(K-N-1)$ multiplications of the conventional orthogonal precoding of N-continuous OFDM, respectively.

4. Numerical Experiments

To evaluate the performance of the proposed method, we conducted numerical experiments with $L = 2(N+1)$ under the same conditions as Fig. 1.



(a) $N = 4$, $K = 45$ ($\mathcal{K} = \{-32, \cdots, 6\} \cup \{26, \cdots, 31\}$), $T_s = 1/15$ ms, $T_g = 1/8T_s$ and 16-QAM modulation.

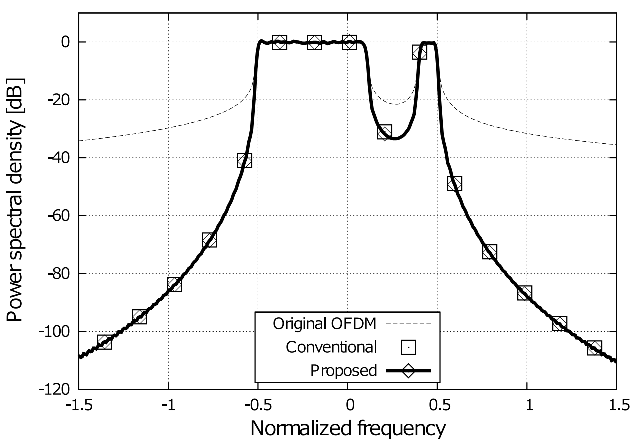


(b) $N = 7$, $K = 600$ ($\mathcal{K} = \{-500, \cdots, -201\} \cup \{201, \cdots, 500\}$), $T_s = 1/15$ ms, $T_g = 9T_s/128$ and QPSK modulation.

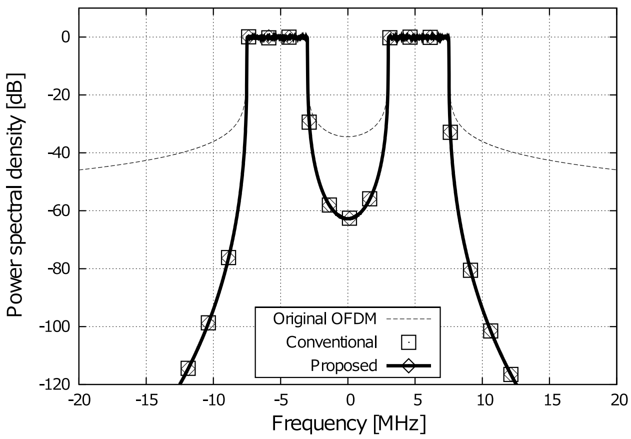
Figure 1. Singular values in $\mathbf{V} - \mathbf{I}_K$; the experimental conditions of Figs. 1(a) and 1(b) are based on those of Fig. 3 in [3] and Fig. 3(b) in [2], respectively.

We firstly verified that the proposed method does not degrade the performance of the conventional orthogonal precoding of N-continuous OFDM. Figure 2 shows the power spectral densities of the original OFDM, the conventional orthogonal precoding of N-continuous OFDM and the proposed method. Figure 3 shows the bit error rates in an additive white Gaussian noise (AWGN) channel. These show that the performance of the proposed method is identical to that of the conventional orthogonal precoding of N-continuous OFDM.

Next, we evaluated the computational complexity of the proposed method compared with the conventional orthogonal precoding of N-continuous OFDM. Table 1 shows the computational complexities in multiplications of precoding and decoding. The results show that the proposed method can reduce the computational complexity and the reduction becomes more drastic as increasing K, compared with the conventional orthogonal precoding. For example, Table 1(b) shows the proposed method requires only 7.9% in the precoding and 5.4% in the decoding, compared with the conventional orthogonal precoding.



(a) the experimental conditions of Fig. 1(a)



(b) the experimental conditions of Fig. 1(b)

Figure 2. Power spectral density of the original OFDM, conventional orthogonal precoding of N-continuous OFDM, and proposed method.

Table 1. Comparison of computational complexity in multiplications

(a) the experimental conditions of Fig. 1(a)

	Precoding	Decoding
Conventional [3]	$2,295$ (100%)	$1,800$ (100%)
Proposed	$1,345$ (57%)	850 (47%)

(Example: $K = 45$, $N = 4$, $L = 10$)

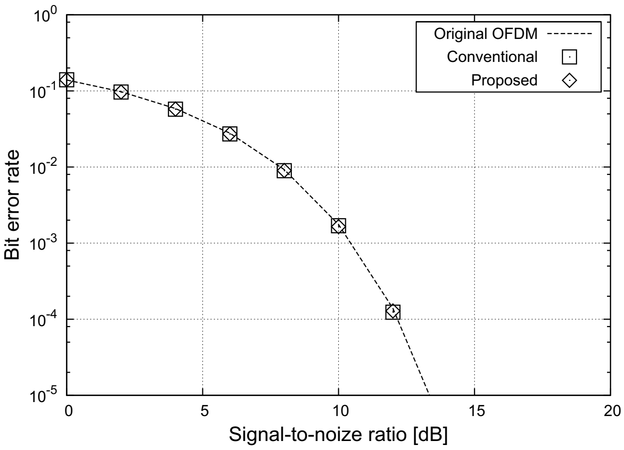
(b) the experimental conditions of Fig. 1(b)

	Precoding	Decoding
Conventional [3]	$365,400$ (100%)	$355,200$ (100%)
Proposed	$28,672$ (7.9%)	$19,072$ (5.4%)

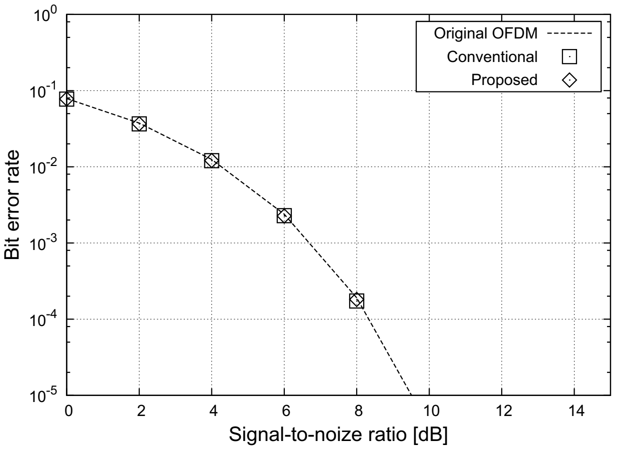
(Example: $K = 600$, $N = 7$, $L = 16$)

5. Conclusions

This paper has proposed a matrix decomposition of the large-sized matrix in the orthogonal precoding of N-continuous OFDM to reduce the computational complexity. Numerical experiments showed that the proposed method



(a) the experimental conditions of Fig. 1(a)



(b) the experimental conditions of Fig. 1(b)

Figure 3. Bit error rates in AWGN channel.

does not degrade the performances and can drastically reduce the computational complexity for the precoding, the decoding e.g., 7.9% and 5.4%, respectively, compared with the conventional orthogonal precoding of N-continuous OFDM.

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