DOA Estimation by Virtual-ESPRIT Algorithm Using SLS Method

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1. Introduction

Virtual-ESPRIT (VESPA) [1] is a method for estimating the direction of arrival (DOA) using higher-order statistics. In this study, to further improve the estimation performance, we propose SLS-VESPA using the SLS (Structured (Structured Least Squares) method [2], and compare it with the conventional method TLS-VESPA [1].

2. DOA Estimation by SLS-VESPA

2.1 VESPA

We use a K-element uniform linear array with the element spacing of half wavelength and assume that L waves (plane waves) are incident on the array. Let $x_1(t), \cdots, x_K(t)$ be the received signals of each element, and then the receiving array vector $\boldsymbol{x}(t) \in \mathbb{C}^{K\times 1}$ is expressed as

$$\boldsymbol{x}(t) = [x_1(t), \cdots, x_K(t)]^T \tag{1}$$

VESPA uses the following cumulant matrices with the $1st$ and $2nd$ elements as guiding sensors.

$$\boldsymbol{C}_{11} = \mathrm{cum}\{x_1^*(t), x_1(t), \boldsymbol{x}(t), \boldsymbol{x}^H(t)\} \tag{2}$$

$$\boldsymbol{C}_{12} = \mathrm{cum}\{x_1^*(t), x_2(t), \boldsymbol{x}(t), \boldsymbol{x}^H(t)\} \tag{3}$$

Note that cum is a fourth-order statistic operation [1][2] computed by multiple snapshots of the receiving vector.

2.2. SLS-VESPA

Using the matrices $\boldsymbol{C}_{11}$ and $\boldsymbol{C}_{12}$, define the matrix $\boldsymbol{C}_r$

$$\boldsymbol{C}_r = \begin{bmatrix} \boldsymbol{C}_{11} \\ \boldsymbol{C}_{12} \end{bmatrix} \in \mathbb{C}^{2K\times L} \tag{4}$$

This $\boldsymbol{C}_r$ is singular value decomposed and represented only in the L-dimensional signal subspace as follows.

$$\boldsymbol{C}_r = \boldsymbol{U}_s \boldsymbol{\Sigma}_s \boldsymbol{V}_s^{\boldsymbol{H}} \tag{5}$$

where $\boldsymbol{U}_s \in \mathbb{C}^{2K\times L}$ is the left singular vector matrix, $\boldsymbol{V}_s \in \mathbb{C}^{K\times L}$ is the right singular vector matrix, and $\boldsymbol{\Sigma}_s \in \mathbb{C}^{L\times L}$ is a diagonal matrix of singular values.
Furthermore, We define $\boldsymbol{E}_x$ and $\boldsymbol{E}_y$ to be the matrices from rows 1 to K and from $K+1$ to $2K$ of $\boldsymbol{U}_s$, respectively, then

$$\boldsymbol{E}_x = \boldsymbol{AT} \in \mathbb{C}^{K\times L}, \boldsymbol{E}_y = \boldsymbol{A\Phi T} \in \mathbb{C}^{K\times L} \tag{6}$$

where $\boldsymbol{T}$ is an Lth-order regular matrix. If $\boldsymbol{E}_s$ is a $(K+1)\times L$ signal subspace left singular vector matrix containing one virtual element of VESPA, then

$$\boldsymbol{E}_x = \boldsymbol{J}_1\boldsymbol{E}_s, \quad \boldsymbol{E}_y = \boldsymbol{J}_2\boldsymbol{E}_s \tag{7}$$

where, $\boldsymbol{J}_1 = [\boldsymbol{I}_L\ \boldsymbol{0}_{K\times 1}], \boldsymbol{J}_2 = [\boldsymbol{0}_{K\times 1}\ \boldsymbol{I}_L]$.
Substituting equation (7) into equation (6), we obtain

$$\boldsymbol{J}_1\boldsymbol{E}_s\boldsymbol{\Psi} = \boldsymbol{J}_2\boldsymbol{E}_s, \boldsymbol{\Psi} = \boldsymbol{T}^{-1}\boldsymbol{\Phi T} \tag{8}$$

The SLS method [2] is applied to $\boldsymbol{E}_s$, and the error $\Delta\boldsymbol{E}_s$ and $\boldsymbol{\Psi}$ are calculated simultaneously. The DOA estimate for the L-wave is obtained from the fixed value of $\boldsymbol{\Psi}$ [2].

3. Performance Analysis by Simulation

Computer simulations of the DOA estimation were performed under the conditions shown in Table 1. The methods compared are the conventional method of TLS-VESPA, and the proposed method of SLS-VESPA, evaluated by RMSE (Root Mean Square Error). Fig. 1 shows the SNR characteristics of RMSE. From Fig. 1, we can see that the performance of SLS-VESPA improves significantly compared to TLS-VESPA.

4. Conclusion

It is shown that SLS-VESPA applying the SLS method [2] enhances the performance.

References

[1] E. Gönen, et al., IEEE Trans. on Signal Processing, Vol.45, No.9, pp.2265-2276, Sep. 1997. .

[2] M. Haardt, IEEE Trans. on Signal Processing, Vol.45, No.3, pp.792-799, Mar. 1997.

Table 1 Simulation conditions.

Number of elements	6	Signal modulation	BPSK
Number of waves	2	Number of snapshots	1000
DOAs	(0°, 30°)	Number of trials	500
Input powers	(1.0, 1.0)	SNR	−10~30 [dB]

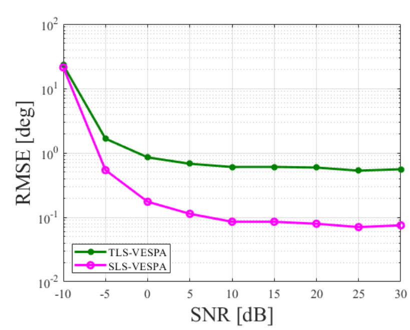

Fig. 1 RMSE of DOA estimates versus SNR.

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