Performance Improvement of TSVD-FOCUSS Algorithm in DOA Estimation Using Array Antenna

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1. Introduction

Compressed sensing with an array antenna has been investigated to estimate the DOA (directions of arrival) of the radio waves. In this study, we deal with one of the algorithms, TSVD-FOCUSS (Truncated Singular Value Decomposition Focal Undetermined System Solver) [1], and we try to improve estimation performance by switching the truncation rate (TR) in the algorithm during iterations.

2. TSVD-FOCUSS and Improvement Method

The array for DOA estimation is a uniform linear array composed of K isotropic elements. The number of waves incident on the array is L. Let N be the number of divisions (bins) of the estimated angle range ($-90° \sim 90°$), $\boldsymbol{A} \in \mathbb{C}^{K\times N}$ be the known mode matrix corresponding to the bins, and $\boldsymbol{s} \in \mathbb{C}^{N}$ be the unknown signal vector. Then, the received signal vector can be expressed as

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{n} \qquad (1)$$

where $\boldsymbol{n} \in \mathbb{C}^{K}$ is the internal noise vector. In TSVD-FOCUSS, $\boldsymbol{s}$ is estimated by solving the following problem:

$$\hat{\boldsymbol{s}} = \arg\min_{\boldsymbol{s}} \|\boldsymbol{s}\|_p^p \quad \text{s.t. } \boldsymbol{A}\boldsymbol{s} = \boldsymbol{x} \qquad (2)$$

where $\hat{\boldsymbol{s}}$ is the estimated value of $\boldsymbol{s}$, and $\|\cdot\|_p$ is l_p norm. When $p = 0$ and m represents the iteration number, $\hat{\boldsymbol{s}}$ is obtained by the following iterative algorithm [1]:

Step 1 : $\boldsymbol{W_m} = \mathrm{diag}(|\hat{\boldsymbol{s}}_m(1)|, \cdots |\hat{\boldsymbol{s}}_m(N)|)$

Step 2 : $\mathrm{svd}(\boldsymbol{A}\boldsymbol{W}_m) = \boldsymbol{U}\boldsymbol{S}\boldsymbol{V}^H$, $\boldsymbol{q}_m = \boldsymbol{V}_t \boldsymbol{S}_t^{-1} \boldsymbol{U}_t^H \boldsymbol{x}$

Step 3 : $\hat{\boldsymbol{s}}_m = \boldsymbol{W}_m \boldsymbol{q}_m$

where svd stands for singular value decomposition, and the matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ are left and right singular vectors of $\boldsymbol{A}\boldsymbol{W}_m$. $\boldsymbol{S}$ is the diagonal matrix of singular values σ_i ($\sigma_1 \geq \cdots \geq \sigma_K$). Also, $\boldsymbol{U}_t, \boldsymbol{S}_t$ and $\boldsymbol{V}_t$ are given by $\boldsymbol{U}, \boldsymbol{S}$ and $\boldsymbol{V}$ truncated to the first t components. Here, t is determined using the TR by

$$t = \arg\min_{t'} \left\{ t' \,\middle|\, \frac{\sum_{j=1}^{t'} \sigma_j}{\sum_{i=1}^{K} \sigma_i} > \mathrm{TR} \right\} \qquad (3)$$

To improve estimation accuracy, we compare $\|\boldsymbol{q}_m\|_2^2$ and a constant G at Step 2 of each iteration, and select TR_1 or TR_2 for TR as follows:

$$\text{if } \|\boldsymbol{q}_m\|_2^2 \geq G \text{ then } \mathrm{TR} = \mathrm{TR}_1 \text{ else } \mathrm{TR} = \mathrm{TR}_2$$

3. Performance Analysis by Computer Simulation

Computer simulation is performed under the conditions of Table 1. We use RMSE of DOA estimates to evaluate estimation accuracy. Fig.1 shows the SNR characteristics of RMSE for the fixed (TR = 0.95) and proposed switching methods. The result in Fig.1 shows that the estimation accuracy of switching method is higher than the fixed method especially in SNR = 5~10 dB.

4. Conclusion

We confirmed improvement of estimation accuracy by simple switching TR in TSVD-FOCUSS. A more detailed investigation about the values of TR and G is future work.

References

[1] I.F.Gorodnitsky and B.D.Rao, IEEE Trans. Signal Processing, vol. 45, No.3, pp 600-616, March 1997.

Table 1 Simulation Conditions.

Number of elements (K)	10
Elements spacing	0.5λ (a half wavelength)
Number of waves (L)	2
DOA	$0°, 30°$
Input SNR	$0 \sim 30$ dB
Angle range	$-90° \sim 90°$
Number of angle bins (N)	181 (1° intervals)
Number of trials	500
l_p norm of TSVD-FOCUSS	$p = 0$
TR ($\mathrm{TR}_1, \mathrm{TR}_2$)	0.95, 0.9
Constant G	10
Convergence conditions	$\frac{\| \hat{\boldsymbol{s}}_m - \hat{\boldsymbol{s}}_{m-1} \|_2}{\| \hat{\boldsymbol{s}}_m \|_2} < 10^{-3}$

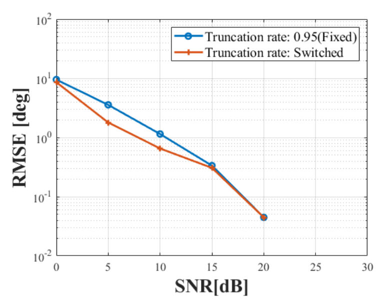

Fig.1 SNR characteristics of RMSE.

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