Performance Analysis of Parallel FSO/MMW Systems with Adaptive Rate Under Weather Effects

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Abstract—This paper presents a concept of parallel Free-Space Optics/Millimeter-Wave (FSO/MMW) systems, in which adaptive rate is employed in both FSO and MMW links. We newly propose an analytical framework based on Markov chain model for system performance analysis. System performance metrics, including throughput and reliability rate, are analytically studied under the presence of various weather conditions. Numerical results quantitatively show how the adaptive-rate FSO/MMW significantly outperform conventional ones, and how weather conditions affect on the systems performance.

I. INTRODUCTION

In recent communication networks, the integration of FSO and RF links has been introduced for achieving high reliability, availability and throughput. This is logical due to the fact that these systems are not affected in the same way by transmission media [1]. The main degrading factors in FSO links are fog and atmospheric turbulence, whereas rain does not influence notably. RF links, on the contrary, are susceptible to heavy rain conditions, but not fog or atmospheric effects [2].

In some of the current hybrid systems, the RF link is only used as a backup when the FSO link is down, remaining silent when the FSO link is working normally (i.e., switch-over systems) [3]–[5]. In others, data is duplicated and transmitted simultaneously over both links (i.e., simultaneous transmission systems) [6], [7]. Both implementations lead to the wastage of bandwidth and under-utilization of the RF link. Furthermore, in the former implementation frequent switching between the FSO and RF links, called flapping, can lead to a collapse of the communication system, and in later one retransmission of data over the insecure RF link leads to an insecure communication system. Our study therefore focuses on another implementation, parallel transmission systems, in which each link takes responsibility of carrying one portion of the data (i.e., the RF link is used for transmitting actual information and does not act only as a backup for the FSO link) [8]-[11]. In addition, towards efficient data transmission for such systems, we are also interested in link adaptations, which have recently received significant research attention.

There have been several studies that consider link adaptations for parallel FSO/RF systems. They mainly imposed adaptive rate schemes, which were realized through severe constraints on the system design and performance analysis, including adaptive modulation/coding [8], hybrid-adaptive channel coding [9], [10], and adaptive symbol rate [11]. However, most previous designs assumed a relatively low data-rate RF link (e.g., a RF link working at 5.8 GHz was assumed in [9]). Additionally, most previous performance analyses mainly focused on effects of fading channels (e.g., atmospheric turbulence), and did not clearly investigated impacts of weather conditions.

In this paper, we consider the concept of adaptive rate for parallel FSO/MMW systems, which make use of a 60GHz MMW communications in the RF link. Using MMW data transmission allows the RF link to achieve data rates comparable to that of the FSO link. Obviously, this can result in an increase in system's throughput. With reference to more related works, fixed-rate (parallel) FSO/MMW systems have been studied in [12], [13], while adaptive-rate MMW and adaptive-rate FSO systems have been separately investigated in [14], [15] and [16]–[18], respectively. To the best of our knowledge, this is the first work that considers FSO/MMW systems, in which adaptive rate is employed in two links.

Additionally, we address the drawbacks of previous analysis frameworks for general parallel FSO/RF systems. In particular, we comprehensively consider effects of not only fading channels, but also different weather conditions (e.g., clear air, fog, and rain) in the system performance of considered systems, i.e., parallel FSO/MMW with adaptive rate. We newly develop an analytical framework, in which the operation of adaptive rate in the varying channel conditions is mapped into a two-dimensional (2D) Markov chain model. This framework allows us to conduct analysis on critical performance metrics, including throughput and reliability rate. Numerical results verify the advantages of considered system over conventional ones, including (i) fixed-rate FSO/MMW systems, (ii) adaptive-rate FSO systems, and (iii) adaptive-rate RF systems. We also discuss these systems performance under a variety of weather conditions ranging from clear weather to moderate fog or rain.

The remainder of the paper is organized as follows. The system descriptions, including the FSO/MMW system model, weather conditions effects, and the concept of adaptive rate, are described in Section II. The performance analysis, including the Markov chain model and performance metrics, is presented in Section III. The numerical results are given in Sections IV. Finally, Section V concludes the paper.

Notation: $\Gamma(.)$ is the ordinary Gamma function; $K_v(.)$ is the modified Bessel function of second kind and order v-th; $I_0(.)$ is the zero order modified Bessel function of the first kind; $Q_1(.,.)$ is the Marcum function; erfc(.) and erfcinv(.) are the complementary error function and its inverse function.



Fig. 1. Block diagram of a point-to-point parallel FSO/MMW system with adaptive rate.

II. System Descriptions

Figure 1 shows a point-to-point parallel FSO/MMW system, which includes two individual line-of-sight (LOS) FSO and MMW links. In this system, data is transmitted over both links, i.e., each link takes responsibility of carrying one portion of the data. In addition, the transmission rate in a link is adaptively adjusted to its channel condition. This is the main concept of adaptive rate for parallel FSO/MMW systems. In following, we first investigate the weather and fading channels effects in each individual link, and then present in detail the operation of adaptive rate.

A. FSO Link

The FSO channel can be modeled as

$$Y_f = A_f h_f X_f + N_f, \ A_f, h_f > 0, \tag{1}$$

where X_f denotes the transmitted signals, A_f denotes the channel attenuation, $N_f \sim \mathcal{N}(0, \sigma_{Nf}^2)$ is independent gaussian random variable representing the noise, and h_f denotes the atmospheric turbulence-induced fading gain. Assuming that perfect alignment between the transmitter and receiver, the attenuation factor is provided by

$$A_f = \frac{\pi D^2}{4(\theta L)^2} \exp(-L\alpha_f), \qquad (2)$$

where α_f denotes the weather dependent attenuation coefficient, θ is the transmit be am divergence, D is the aperture diameter of receiver, and L is the channel distance.

The atmospheric turbulence causes irradiance fluctuations, i.e., scintillation, on the received signals propagating along a horizontal path between the transmitter and receiver. Generally, a Gamma-Gamma (GG) distribution is used to model the two independent contributions of the small-scale and large-scale of turbulence, assuming a Gamma process governs each of them. The GG distribution is given by

$$f_{H_f}(h_f) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} h_f^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta} \left(2\sqrt{\alpha\beta h_f}\right), \quad (3)$$

where the pdf parameters α and β are given by

$$\alpha = \left\{ \exp\left[\frac{0.49\sigma_R^2}{\left(1+1.11\sigma_R^{12/5}\right)^{7/6}}\right] - 1 \right\}^{-1},$$

$$\beta = \left\{ \exp\left[\frac{0.51\sigma_R^2}{\left(1+0.69\sigma_R^{12/5}\right)^{5/6}}\right] - 1 \right\}^{-1}.$$
 (4)

where σ_R^2 is the Rytov variance, which is in fact the scintillation index of an unbounded plane wave in the weak turbulence regime, given by

$$\sigma_R^2 = 1.23 C_n^2 k^{7/6} L^{11/6}, \tag{5}$$

where $k = 2\pi/\lambda_f$ is the optical wave number with λ_f is the optical wavelength, and C_n^2 is the altitude-dependent index of the refractive structure parameter determining the turbulence strength. Typically, C_n^2 varies from 10^{-13} m^{-2/3} to 10^{-17} m^{-2/3} [16].

We define the instantaneous received signal-to-noise ratio (SNR) of FSO links as

$$\gamma_f = \overline{\gamma}_f h_f^2, \tag{6}$$

where $\overline{\gamma}_f$ is the average symbol SNR of FSO link, and it is given by $\overline{\gamma}_f = \mu^2 \eta^2 P_f^2 A_f^2 / \sigma_{Nf}^2$, where P_f is the average transmitted optical power, μ is the modulation index, and η is the responsivity of the receiver.

B. MMW link

The RF channel can be modeled as

$$Y_r = A_r h_r X_r + N_r, \ A_r, h_r > 0.$$
(7)

Assuming a carrier frequency of 60 GHz, the channel attenuation of MMW link is given by $A_r = 10^{A_r [\text{dB}]/10}$, where

$$A_r \left[\mathsf{dB} \right] = G_t + G_r - 20 \log_{10} \left(\frac{4\pi L}{\lambda_r} \right) - L(\alpha_{oxy} + \alpha_r), \quad (8)$$

where G_t and G_r denote, respectively, the transmit and receive antenna gains, λ_r is the wavelength of the RF system, and α_{oxy} and α_r are the attenuations caused by oxygen absorption and rain, respectively [12]. As a requirement for the FSO communications, a LOS path always exists. Hence, the fading gain h_r of RF channel is assumed to be modeled by a Rician distribution,

$$f_{H_r}(h_r) = \frac{h_r}{\sigma_r^2} \exp\left(-\frac{h_r^2 + s^2}{2\sigma_r^2}\right) I_0\left(\frac{h_r s}{\sigma_r^2}\right), \qquad (9)$$

where $2\sigma_r^2$ is the mean power of the sum of the non-specular component, and s^2 is the LOS component. By defining the Rician factor as: $K = s^2/2\sigma_r^2$, we can rewrite (9) as

$$f_{H_r}(h_r) = 2(K+1)h_r \exp\left(-K - (K+1)h_r^2\right) \\ \times I_0\left(2h_r\sqrt{K(K+1)}\right).$$
(10)

Similarly, we define the instantaneous received SNR of MMW link as

$$\gamma_r = \overline{\gamma}_r h_r^2 \tag{11}$$

where $\overline{\gamma}_r$ is the average symbol SNR MMW link, and it is given by $\overline{\gamma}_r = P_r A_r / \sigma_{Nr}^2$, where P_r represents the RF transmit power, and the RF noise variance is given by σ_{Nr}^2 [dBm]= $W_r N_0 + N_F$, where W_r , N_0 , and N_F denote the RF bandwidth, the noise power spectral density (in dBm/MHz), and the noise figure of the receiver, respectively [12].

In summary, it is important to emphasize that the values of channel attenuations A_f and A_r , i.e., (2) and (8), depend on the weather condition. To express weather-dependent parameters of FSO and MMW links, we adopt the specific values described in [7], [12], [13] in Tables I.

TABLE I. WEATHER-DEPENDENT PARAMETERS [7] [12] [13]

Weather Conditions	α_f (dB/km)	α_r (dB/km)	$C_n^2 \ ({\rm m}^{-2/3})$
Clear air	0.43	0	5×10^{-14}
Moderate fog	42.2	0	2×10^{-15}
Moderate rain (12.5 mm/h)	5.8	5.6	5×10^{-15}

C. Adaptive Rate

TABLE II. An Illustrative Example of Adaptive Rate with M-PSK for Parallel FSO/MMW System When J=I=5 and ${\rm BER}_0=10^{-3}$

SNR thresholds [dB]	Information rate [bits/symbol]	Modulation	Transmission mode
$(-\infty, 6.7895)$	0	No transmission	$TM_{f}^{(0)}, TM_{r}^{(0)}$
[6.7895, 9.7998)	1	BPSK	$TM_{f}^{(1)}, TM_{r}^{(1)}$
[9.7998, 14.7814)	2	QPSK	$TM_{f}^{(2)}, TM_{r}^{(2)}$
[14.7814, 20.3673)	3	8-PSK	$TM_{f}^{(3)}, TM_{r}^{(3)}$
[20.3673, 26.1286)	4	16-PSK	${ m TM}_{f}^{(4)},{ m TM}_{r}^{(4)}$
[26.1286, ∞)	5	32-PSK	${ m TM}_{f}^{(5)}, { m TM}_{r}^{(5)}$

The channel state information(s) (CSIs) are defined as the values of γ_f and γ_r , which are estimated at the receiver and fed back to the transmitter through a feedback channel¹. Based on CSIs, the transmission rate can be adjusted dynamically at the transmitter.

We assume that there are J + 1 and I + 1 available transmission modes (TMs) in FSO and MMW links:

$$\left\{ \mathbf{T}\mathbf{M}_{f}^{(0)},...,\mathbf{T}\mathbf{M}_{f}^{(J)}\right\} , \\ \left\{ \mathbf{T}\mathbf{M}_{r}^{(0)},...,\mathbf{T}\mathbf{M}_{r}^{(I)}\right\} ,$$

where $\operatorname{TM}_{f}^{j}$ and $\operatorname{TM}_{r}^{i}$ are corresponding to the transmission rates of $\operatorname{R}_{f}^{(j)}$ and $\operatorname{R}_{r}^{(i)}$, respectively. We also assume there are J+1 and I+1 set of non-overlapping SNR region boundaries:

$$\left\{ \begin{bmatrix} \gamma_f^{(0)}, \gamma_f^{(1)}), [\gamma_f^{(1)}, \gamma_f^{(2)}), ..., [\gamma_f^{(J)}, \gamma_f^{(J+1)}) \\ \\ \begin{bmatrix} \gamma_r^{(0)}, \gamma_r^{(1)}), [\gamma_r^{(1)}, \gamma_r^{(2)}), ..., [\gamma_r^{(I)}, \gamma_r^{(I+1)}) \\ \end{bmatrix} \right\}.$$

Here, it is noted that the upper and lower bounds are set as $\gamma_f^{(0)} = \gamma_r^{(0)} = 0$ and $\gamma_f^{(J+1)} = \gamma_r^{(I+1)} = \infty$.

A TM is associated with SNR region boundaries according to the following rule,

$$TM_f = TM_f^j \text{ (i.e., } R_f = R_f^j \text{) if } \gamma_f \in [\gamma_f^{(j)}, \gamma_f^{(j+1)}),$$

$$TM_r = TM_r^i \text{ (i.e., } R_r = R_r^i \text{) if } \gamma_r \in [\gamma_r^{(i)}, \gamma_r^{(i+1)}).$$

Here, it is noted that the lowest transmission rates are set to zero, i.e., $R_f^0 = R_r^0 = 0$; it means that no transmission takes place when the received SNR is relatively low.

In a practical system, the sets of transmission rates and SNR region boundaries have to be predesigned. Generally, adaptive modulation schemes are used to realize these designs. Practically, these flexibilities could be addressed via a software-based configuration or implementation of the operating function at the transmitter and receiver [19].

In the rest of this paper, for an illustrative example, we consider adaptive modulation with M-ary phase-shift keying (M-PSK) $(M = 2^m, m = 1, 2, ...)$ for both links. So, the transmission rate of a TM is represented as the number of bits per a transmitted symbol, the assignment of SNR region boundaries is done so that the SNR level at the boundary satisfies the BER target (BER₀) with the modulation scheme used in an AWGN channel. Table II gives an example of applying the adaptive M-PSK scheme for data transmission in parallel FSO/MMW systems, where J = I = 5 and BER₀ = 10^{-3} (see a design of SNR region boundaries in Appendix).

III. PERFORMANCE ANALYSIS

In this section a 2-D Markov chain model is proposed to qualify systems performance metrics in terms of throughput and availability.

A. 2-D Markov Chain Model

The operation of system can be formulated in discrete time, with one time unit equal to one transmission cycle. The system states are observed at the beginning of each cycle. Let $\zeta = (\mathrm{TM}_f(t), \mathrm{TM}_r(t))$ denote the system state at cycle t, where $\mathrm{TM}_f(t) \in \{0, ..., J\}$, and $\mathrm{TM}_r(t) \in \{0, ..., I\}$ represent the TM of FSO link, and TM of MMW link at this cycle, respectively. When we look at the set of time instants $t = 0, ..., \infty$ the transitions between states ζ are Markovian. So, an embedded Markov chain can be used to describe the underlying queuing process. The state space of this embedded finite-state Markov chain is $S = \{S_{\mu}\}_{\mu=1}^{N_s}$, where $N_s = (J + 1)(I + 1)$, and $S_{\mu} = (j, i)$ (Fig. 2). In the equilibrium, a state (j, i) has a probability of $\pi_{(j,i)}$. Let $\mathbf{\Pi} = [\pi_{(0,0)}, ..., \pi_{(0,I)}, ..., \pi_{(J,0)}, ..., \pi_{(J,I)}]$ be the matrix of steady-state probabilities. From Fig. 2, the steady-state probabilities can be directly found as

$$\pi_{(j,i)} = P(\gamma_f^{(j)} < \gamma_f \le \gamma_f^{(j+1)}) P(\gamma_r^{(i)} < \gamma_r \le \gamma_r^{(i+1)})$$
(12)
$$= \int_{\gamma_f^{(j)}}^{\gamma_f^{(j+1)}} f_{\gamma_f}(\gamma_f) \int_{\gamma_r^{(i)}}^{\gamma_r^{(i+1)}} f_{\gamma_r}(\gamma_r) d\gamma_r$$

$$= \left[F_{\gamma_f}(\gamma_f^{(j+1)} - F_{\gamma_f}(\gamma_f^{(j)})) \right] \left[F_{\gamma_r}(\gamma_r^{(i+1)}) - F_{\gamma_r}(\gamma_r^{(i)}) \right].$$

where $f_{\gamma_f}(.)$ and $F_{\gamma_f}(.)$ are pdf and cdf of γ_f , and $f_{\gamma_r}(.)$ and $F_{\gamma_r}(.)$ are pdf and cdf of γ_r . Using (3) and (6) for FSO link,

¹We assume that there is an erroneous feedback channel available between the receiver and transmitter. In practical, this channel can be realized by using one link of bi-directional FSO link (or MMW link).



Fig. 2. The state space of Markov chain for the adaptive rate $\ensuremath{\mathsf{FSO/MMW}}$ system.

(10) and (11) for MMW link, after transformation of random variables h_f and h_r , we obtain

$$f_{\gamma_f}(\gamma_f) = \frac{(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)\overline{\gamma}_f^{\frac{\alpha+\beta}{4}}} \gamma_f^{\frac{\alpha+\beta}{4}-1} \times K_{\alpha-\beta} \left(2\sqrt{\alpha\beta}\sqrt{\frac{\gamma_f}{\overline{\gamma}_f}}\right),$$
(13)

$$f_{\gamma_r}(\gamma_r) = \frac{K+1}{\overline{\gamma}_r} \exp\left(-K - (K+1)\frac{\gamma_r}{\overline{\gamma}_r}\right) \\ \times I_0\left(2\sqrt{K(K+1)\frac{\gamma_r}{\underline{-}}}\right), \tag{14}$$

$$F_{\gamma_f}(\gamma_f) = \frac{1}{\Gamma(\alpha)\Gamma(\beta)} G_{1,3}^{2,1} \left[\alpha \beta \sqrt{\frac{\gamma_f}{\overline{\gamma}_f}} \right]_{\alpha,\beta,0}^1 , \qquad (15)$$

$$F_{\gamma_r}(\gamma_r) = 1 - Q_1\left(\sqrt{2K}, \sqrt{\frac{2(K+1)\gamma_r}{\overline{\gamma}_r}}\right).$$
(16)

B. Throughput

For a state (j, i), each transmitted symbol in FSO link will carry j information bits for the mode adhering to a 2^j -PSK constellation, and each transmitted symbol in MMW link will carry i information bits for the mode adhering to a 2^i -PSK constellation. For the constant-power, adaptive discrete rate SIM assuming ideal Nyquist data pulses for each constellation, the average total throughput can be calculated as

$$T[\text{bits/s}] = \sum_{j=0}^{J} \sum_{i=0}^{I} \pi_{(j,i)} (jW_f + iW_r), \qquad (17)$$

where W_f is the bandwidth of the FSO link, and W_r is the bandwidth of the MMW link. In order to show the variation of normalized throughput, we have assumed $W_f = W_r =$

W. The normalized average throughput of hybrid FSO/MMW system then is defined as

$$T^{n}[\text{bits/s/Hz}] = \frac{T}{W} = \sum_{j=0}^{J} \sum_{i=0}^{I} \pi_{(j,i)}(j+i).$$
(18)

C. Availability Rate

In a parallel FSO/RF system, the availability rate is defined as the probability that the system is able to keep transmitting data under weather effects. Clearly, no transmission takes place when the system falls in state (0, 0). So, the availability rate can be simply calculated as

$$P^{a}[\%] = (1 - \pi_{(0,0)}) \times 100.$$
⁽¹⁹⁾

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, using the previously derived framework, we present selected numerical results illustrating the performance of different systems, including adaptive-rate FSO/MMW (FSO/MMW-AR), fixed-rate FSO/MMW (FSO/MMW-FR) [12], [13], adaptive-rate MMW (MMW-AR) [14], [15], and adaptive-rate FSO (FSO-AR) [16]–[18], for different weather conditions and channel distances². For the fixed-rate systems (i.e., FSO/MMW-FR), we assume that both links use 32-PSK as the common modulation; for other systems, the parameters of adaptive rate are assumed as in Table II. The system's parameters corresponding to two individual links, and the parameters corresponding to different weather conditions are given in Tables I and III, respectively.

 TABLE III.
 System Parameters [7] [12] [13]

FSO Link					
Parameter	Symbol	Value			
Laser wavelength	λ_f	1550 nm			
Transmitted power	P_f	10 mW			
Photodetector responsivity	η	0.5 A/W			
Noise variance	σ_{Nf}^2	10^{-14} A^2			
Beam divergence angle	ϕ	2 mrad			
Receiver diameter	D	20 cm			
MMW Link					
Parameter	Symbol	Value			
Carrier frequency	f_r	60 GHz			
Transmitted power	P_r	10 mW			
Bandwidth	W_r	250 MHz			
Transmit antenna gain	G_t	44 dBi			
Receive antenna gain	G_r	44 dBi			
Attenuation (due to oxygen)	α_{oxy}	15.1 dB/km			
Noise power spectral density	N_0	-114 dBm/MHz			
Receiver noise figure	N_F	5 dB			
Ricean Factor	K	6 dB			

A. System Comparisons

Let us first summarize the main conclusions that can be drawn from Figs. 3–5. In general, FSO/MMW-AR achieves the best performance in all cases thanks to combined advantages of both media diversity and adaptive rate. On the other hand, the system employed either media diversity (FSO/MMW-FR) or adaptive rate (FSO-AR and MMW-AR) could not always

²The proposed analytical framework is simplified for other systems as follows: (i) a 1-D Markov chain with J + 1 states can be considered for FSO-AR, (ii) similarly, a 1-D Markov chain with I + 1 states can be considered for MMW-AR, and (iii) a 2-D Markov chain with 4 states can be considered for FSO/MMW-FR [13].



Fig. 3. Under a clear weather condition.

maintain a good performance, especially in long channel distance and unfavorable weather conditions.



Fig. 4. Under a moderate fog condition.

B. Weather Effects

Next, we further compare the performance of considered systems under different weather conditions and channel distances. As expected, the system operates very well in the cases of clear air and short distance. For example, in less than 1000 m, the reliability nearly reaches 100 % for all systems in Fig. 3. Compared with the clear air, the systems performance gets worse in the case of either rainy or foggy conditions. As the attenuation coefficient in the FSO link is relatively high in the foggy condition, the data transmission in FSO-only system is totally blocked (i.e., $T^n = 0$, and $P^a = 0$) when the channel distance is higher than 500 m (Fig. 4). In rainy condition, the attenuation coefficient in both links are high ($\alpha_f = 5.8$ dB/km, and $\alpha_r = 5.6$ dB/km), neither media diversity nor adaptive rate could help to maintain data transmission, all systems are blocked in the channel distance higher than 1200 m (Fig. 5).



Fig. 5. Under a moderate rain condition (12.5 mm/h).

V. CONCLUSIONS

We have presented a concept of adaptive rate FSO/MMW systems to enhance overall system performances. A 2-D Markov chain model was proposed to theoretically analyze the system performance. Two critical system performance parameters, including throughput and availability, were derived under the presence of various weather conditions. The results shown that compared to systems employed either media diversity or adaptive rate, the adaptive rate FSO/MMW systems have significantly better performance, and that the impact of weather conditions and channel distance on the systems performance are severe.

APPENDIX SNR REGION BOUNDARIES DESIGN

The aim of adaptive rate is to provide the highest possible transmission rate while maintaining BER_0 . So, the design problem can be formally formulated as

$$\max M \mid M = 2^{m}$$

subject to BER(γ, m) \leq BER₀, (20)

where $\text{BER}(\gamma, m)$ is the conditional BER corresponding to M-PSK, which is given by

$$BER(\gamma, m) = \frac{A}{2} \operatorname{erfc}(\sqrt{\gamma}B)$$
(21)

where, A = 1, B = 1 when m = 1 (BPSK), A = 2/m, $B = \sin(\pi/2^m)$ when m > 2 [7]. From (20) and (21), we have

$$\gamma^{(m)} = \left[\frac{\operatorname{erfcinv}(\frac{2\operatorname{BER}_0}{A})}{B}\right]^2.$$
 (22)

Eventually, the SNR region boundaries can be expressed as

$$\gamma_f^{(j)} = \gamma^{(m)}, \ m = 1, 2, ..., J,$$
(23)

$$\gamma_r^{(i)} = \gamma^{(m)}, \ m = 1, 2, ..., I.$$
 (24)

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