Comparison of the Message-Passing Algorithm SCCC and LDPC code for PMR Channel

Donghyuk Park[1], Jaejin Lee[2] and Yoan Shin[3]
School of Electronic Engineering Department, Soongsil University
511 Sangdo-Dong, Dongjak-Gu, Seoul, Korea
E-mail: [1]manakq@hotmail.com, [2]zlee@ssu.ac.kr, [3]yashin@e.ssu.ac.kr

Abstract: In storage system, the higher density perpendicular magnetic recording (PMR) channel requires more advanced signal setection method to satisfy the bit error rate performance. Recently the iterative decoding is in the limelight part of the coding theory for the storage system. Famous iterative codes are the LDPC code and the Turbo code. In this paper, we consider the serial concatenated convolutional code (SCCC) with the message-passing algorithm. LDPC code has the inner iterations by using Sum-Product Algorithm (SPA). The iterative decoding of LDPC code leads to increase the performance of the PMR channel. Meanwhile, the SCCC system has no inner iterations. Therefore we consider the channel iteration between channel detector and channel decoder in SCCC system. This iterative decoding scheme expects to improve the performance of the PMR channel. We increase the number of the channel iterations in order to improve the performance.

1. Introduction

Because of the multi-media data size increased, the storage capacity needs to be bigger and bigger. Therefore many researchers endeavor to increase the storage capacity. To increase the capacity of hard disk drive (HDD), many researchers studied the signal processing and the ECC. Especially, they have studied iterative code that is LDPC code and Turbo code [1][2][3]. The PMR hard disk drive has two types of the noises which are transition-jitter noise (TJN) and additive-white gaussian noise (AWGN). In the PMR channel, the channel model is the partial-response (PR) channel with ISI caused by memory channel.

In these days, the detection scheme is used by the noise-predictive maximum likelihood (NPML) in hard disk drive systems. However, it is hard to expect improving the performance by using the NPML only. Therefore many researchers study another detection scheme instead of the NPML. We evaluate two systems in this paper. One is the LDPC code with message-passing channel detector [4]. Another is serial concatenated convolutional code (SCCC) with message-passing algorithm channel detector and channel decoder. This uses the recursive systematic convolutional (RSC) code encoder and decoder that replace the LDPC encoder and decoder, respectively. Because, LDPC code decoding scheme has inner iterations but RSC decoder is not an iterative decoder.

The famous decoding algorithm of the convolutional code is the Viterbi algorithm. Viterbi algorithm is simple algorithm for implementation. The message-passing soft decision algorithm shows better performance than Viterbi algorithm for decoding of the convolutional code. The message-passing soft algorithm is similar to the maximum a posteriori (MAP) algorithm.

The SCCC system consists of channel detector and channel decoder. Channel detector is implemented by using message-passing soft decision algorithm that is forward-backward algorithm. Channel decoder is recursive convolutional code decoder that is message-passing soft decision algorithm like the channel detector.

The other system is LDPC system. It consists of a channel detector and a channel decoder, too. In LDPC system, the channel detector is implemented by using message-passing soft decision algorithm and the channel decoder is LDPC code decoder. LDPC code is well known by the message-passing algorithm therefore LDPC is able to implement parallel. Also, LDPC decoding has stop criterion, because the LDPC code is block code. Therefore LDPC decoding is faster than other serial decoding scheme. But LDPC code is required many inner iteration counts. Hence, LDPC decoding is faster than other iterative decoding mechanism, but it is slower than non-iterative decoding systems. SCCC system has no inner iterations.

Turbo-equalization scheme is the message-passing between the channel detector and the channel decoder. This recursive message-passing scheme is called channel iteration. We compare the performance of two systems as the different channel iteration counts.

In Section II, we present the modeling of the PMR channel. In Section III, we introduce the system of the SCCC with precoding and interleaver. Also, we discuss another system that consists of the message-passing detector and the LDPC code decoder. In Section IV, we explain the parameters of simulation. In Section V, we discuss the performance in this simulation. Finally, Conclusion is drawn in Section VI.

2. Modeling of the PMR Channel

The signal transition step response model of the PMR channel can be expressed as follows [5].

$$g(t) = A\tanh\left(\frac{2t}{0.597\pi T_{50}}\right) \tag{1}$$

where A is the maximum amplitude the signal and T_{50} is measured at 50% of the unipolar pulse amplitude. $K = T_{50}/T_b$ is the normalized recording density. T_b is the channel bit interval. In this paper, user bit density (UBD) is 1.7, hence the channel bit density is UBD/R where R is the code rate.

The noise model is the sum of AWGN and TJN [6]. We insert the TJN randomly. Each magnetic transition is shifted time that is gaussian distributed random variable.

The readback signal is given below.

$$r(t)=\sum_{k=-\infty}^{\infty} a_k[g(t-kT)-g(t-(k+1)T)]+n_w(t)+n_j(t) \quad (2)$$

where $n_w(t)$ and $n_j(t)$ are AWGN and TJN, respectively. We define channel signal-to-noise ratio (SNR) as

$$SNR=10\log_{10}\left(\frac{A^2}{\sigma_w^2+\sigma_j^2\|g'(t)\|^2}\right) \quad (3)$$

where σ_w^2 is the AWGN power, and $\sigma_j^2\|g'(t)\|^2$ is the TJN power. The simplified read channel model with AWGN and TJN is illustrated in Figure 1.

$$h(t)=g(t)-g(t-T) \quad (4)$$

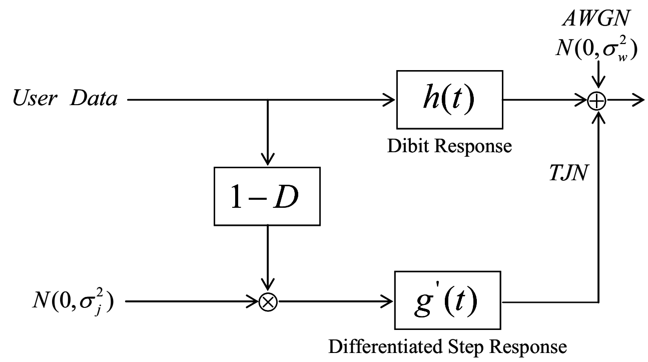

Figure 1. Simplified read channel model with AWGN and TJN.

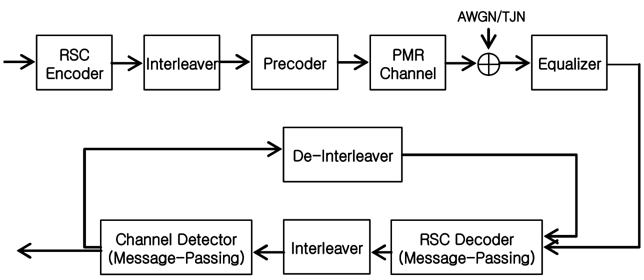

Figure 2. Block diagram of the SCCC system on PMR channel.

3. Description of Systems

We simulate two systems and discuss the performance. The first system is the SCCC system with the message-passing algorithm between the channel detector and channel decoder for the PMR channel. Another system is the LDPC system with message-passing algorithm of the channel detector, too. We consider the performance variation while different number of channel iterations. Both of the systems are message-passing channel detector used.

The channel detector is implemented by using the message-passing algorithm. This algorithm based the max-log message-passing [7].

This algorithm can be summarized as follows.

1. Initialize

$$L\_A_0(m)=L\_B_N(m)=\begin{cases}0 & ,m=0\\ -\infty & ,m\neq 0\end{cases} \quad (5)$$

$$L\_A_n(m)=L\_B_n(m)=-\ln(M) \quad ,\forall m \;; n\neq 0,N \quad (6)$$

2. For each node $n=0,...,N-1$, simultaneously generate the outputs $A_{n+1}(m), B_n(m), m=0,...,M-1$

$$L\_A_{n+1}(m)\approx \max\{L\_A_n(m')+\ln(\Pr[x_n])+\ln(\Pr[y_n\,|\,r_n])\} \quad (7)$$

$$L\_B_n(m')\approx \max\{L\_B_{n+1}(m)+\ln(\Pr[x_n])+\ln(\Pr[y_n\,|\,r_n])\} \quad (8)$$

3. For each node $n=0,...,N-1$, simultaneously generate $\ln(\Pr[x_n])$

$$\ln(\Pr[x_n])\approx \max\{L\_A_n(m')+\ln(\Pr[y_n\,|\,r_n])+L\_B_{n+1}(m)\} \quad (9)$$

In this algorithm, $L\_A(m)$ and $L\_B(m)$ are the probability of forward algorithm and backward algorithm, respectively, at node n, and state m in log-domain. N is the length of the message. M is the number of states. $\Pr[x_n]$ is the probability of the decoded message at node n. $\Pr[y_n\,|\,x_n]$ is the probability of the path between node $n-1$ and node n by trellis constraint. y_n is the decoded information by the channel detector at node n. r_n is the received data from the channel.

3. 1 Serial Concatenated Convolutional code (SCCC) System with the Message-Passing Algorithm for PMR Channel

The SCCC system for PMR channel is illustrated in Figure 2. The channel detector and channel decoder are implemented by the message-passing algorithm. The SCCC system has the random interleaver and de-interleaver for improving the performance. Equalizer (EQ) coefficients are updated by least-mean square (LMS) training. The EQ adapts the channel to the PR(12321) target with a $1/(1\oplus D^2\oplus D^3\oplus D^4\oplus D^5)$ precoder. The outer encoder and decoder are the recursive systematic convolutional (RSC) code encoder and decoder, respectively. The RSC code has the generator polynomial of $(31,23)_8$. This encoder is illustrated in Figure 3. Using the interleaver and de-interleaver, we can achieve the higher performance than that without interleaver and de-interleaver. Inputs of the channel detector are the channel information (γ) and Extrinsic information (EI) from the channel decoder. When there is no channel iteration, a priori (APR) information to the channel detector is zero. After one channel iteration, APR information to the channel detector is non-zero. Input of the channel decoder is the output information of the channel detector. Figure 4 indicates how to pass the soft information. y is the decoded information from the channel decoder (RSC decoder).

3. 2 LDPC System with the Message-Passing Algorithm for PMR Channel

LDPC code is an iterative coding scheme like a Turbo code. We implement the LDPC code by using the sum-product algorithm (SPA) on log domain. We use the log-likelihood ratio (LLR) value. LDPC decoding scheme uses the stop criterion by using property of the block codes. Figure 5 shows the block diagram of the LDPC system.

The performance is related the number of iterations in LDPC decoding. LDPC decoding produce the extrinsic information (EI), and EI passes back to the channel detector. This message passing is the channel iteration on the LDPC system. This soft-output soft-input (SISO) process is illustrated in Figure 6. y is the decoded information. Inputs of the channel detector are same as the channel detector of SCCC system. Input of the LDPC decoder is the output of the channel detector. The information is exchanged between bit-node and check-node of the LDPC decoder. This message-passing is the inner iteration of LDPC. The number of iterations is equal to the maximum limit stop. .

4. Description of this Simulation

Both of systems are implemented by using message passing algorithm on channel detector with PR(12321) target. Eq. (1) is the signal of the PMR channel. We modeled the PMR channel by using 61 channel tap memory. The PMR channel output signal sequence produced by convolution sum of the input signal sequence and each channel tap coefficient. The input sequence is the nonreturn-to-zero (NRZ) sequence $\{x_k\}$ of $+1$, -1. We use the 11 tap equalizer coefficient updated by training.

In this simulation, a codeword (4336 bits) is consisted of whole data bits (4096 bits = 512Bytes = 1 sector length of HDD) and parity bits (240 bits). The parity-check matrix of LDPC is 240 by 4336. We use the structured LDPC code for simulation. Parity length of the SCCC is 240 bits composed by puncturing. Trellis of the channel detector has an additive termination state. Thus, length of the trellis on the channel detector is $N+\alpha$, where N is the length of the codeword (4336 bits), α is the additive termination state. Because of the PR(12321) target, memory length (α) is 4. The information is sent to forward and backward directions on the trellis with the PR(12321) constraint.

The maximum number of the inner iterations is 10 in LDPC system. The maximum number of the channel iterations is 4 in LDPC system. The maximum number of the channel iterations is 4 in SCCC system.

We simulate the 20% AWGN and 80% TJN because of the TJN is the majority noise in the PMR channel. Table 1 shows the parameters of these systems in the simulation.

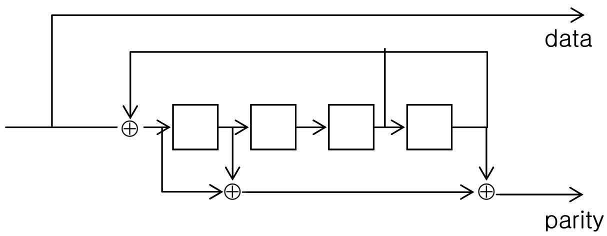

Figure 3. Recursive systematic convolutional (RSC) code encoder. $(31,23)_8$ RSC encoder.

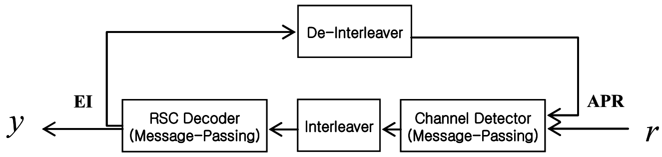

Figure 4. Soft-output Soft-input (SISO) model of the SCCC system with message-passing algorithm detector/decoder.

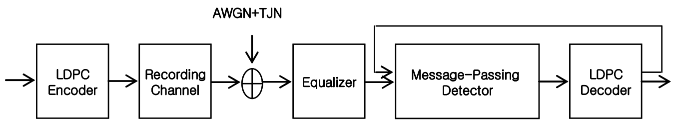

Figure 5. Block diagram of the LDPC system on PMR channel.

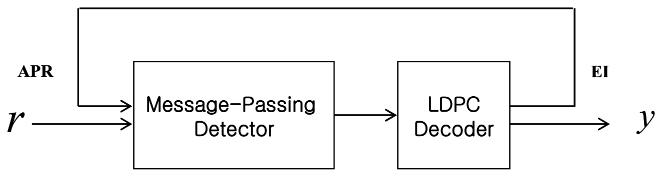

Figure 6. Soft-output Soft-input (SISO) model of the LDPC system with message-passing algorithm detector.

TABLE 1
SYSTEM PARAMETERS

	LDPC system	SCCC system
Codeword length	4336 bits	4336 bits
Parity length	240 bits	240 bits
Code rate	≈ 0.944	≈ 0.944
Noise energy allocation	20% AWGN 80% TJN	20% AWGN 80% TJN
User bit Density (UBD)	1.7	1.7
Maximum number of channel iterations	4	4
Maximum number of inner iterations (LDPC iterations)	10	-

5. Simulation Result

We simulate the SCCC system and the LDPC system for PMR channel and evaluate the performance of two systems with 20% AWGN and 80% TJN.

Figure 7 shows the bit error rate (BER) performance in accordance with the number of channel iterations applied. We can see that the BER is improved as the number of channel iterations is increased. BER increasing of the SCCC system is the bigger than increasing of the LDPC system as the number of channel iterations is increased. The reason is that the LDPC decoding scheme has better performance than the RSC decoding scheme. We observe the NPML is better than the SCCC system if there is no channel iteration. The reason is that the SCCC system is based on the PRML while the NPML is based on the noise-predictive filter. In the SCCC system, after four channel iterations, it performs 3.5dB better than the NPML at 10^{-6} BER. After four channel iterations, it performs 3.6dB better than the no channel iteration case at 10^{-6} BER. In the LDPC system, no channel iteration and after four channel iterations, it performs 3dB and 3.5dB better than the NPML at 10^{-6} BER, respectively. And after four channel iterations, it performs 0.5dB better than no channel iteration case at 10^{-6} BER.

In the result of this simulation, we can see that the iterative decoding improves the performance for the PMR channel. The iterative decoding contains both of iteration types that are inner iterations of the LDPC and the channel iterations. In the SCCC system, channel decoder is RSC decoder. It shows the performance improvement by using the channel iteration. In the comparison of two systems, the decoding of LDPC only has better performance than RSC decoding only. Because of the LDPC decoding is iterative decoding scheme. However we notice that their performances are similar after the four channel iterations at 10^{-6} BER. The iterative decoding has the better performance over 3dB than the NPML on the PMR channel.

6. Conclusion

After four channel iterations, the SCCC system is 3.5dB better than the NPML only at 10^{-6} BER. Also, in the LDPC system, no channel iteration and after four channel iterations, it performs 3dB and 3.5dB better than the NPML only at 10^{-6} BER, respectively. Also, we notice that, after the four channel iterations, the SCCC system has better performance over 3dB than the NPML only. Thus, we can see that two iterative decoding schemes with message-passing channel detector show better performance than non-iterative decoding scheme like NPML on the PMR channel. Two types of the iterative decoding schemes of SCCC and LDPC systems have the similar trend of performance when the number of iterations is large at 10^{-6} BER in PMR channel.

Without iterations, the LDPC system is 3dB better than the SCCC system because the LDPC code itself is superior to the convolutional code with message-passing detection.

References

[1] R. G. Gallager, *Low-density parity check codes,* Cambridge, MA MIT Press, 193.

[2] E. M. Kurtas, A. V. Kuznetsov, and I. Djurdjevic, "System perspectives for the application of structured LDPC codes to data storage devices," *IEEE Trans. Magnetics,* vol. 42, no. 2, pp. 200-207, Feb. 2006.

[3] W. E. Ryan, "Performance of high rate turbo codes on a PR4 equalized magnetic recording channel," in *Proceedings of IEEE International Conference on Communications,* vol. 2, pp. 947-951, June. 1998.

[4] X. Hu and B. V. K. V. Kumar, "Evaluation of low-density parity-check codes on perpendicular magnetic recording model," *IEEE Trans. Magnetics,* vol. 43, no. 2, pp. 727-732, Feb. 2007.

[5] M. Madden, M. Öberg, Z. Wu, and R. He, "Read channel for perpendicular magnetic recording," *IEEE Transaction on Magnetics*, vol. 40, no. 1, pp. 241-246, Jan. 2004.

[6] Y. Nishida, H. Sawaguchi, A. Kuroda, H. Takano, H. Aoi, Y. Nakamura, "Noise characteristics of double-layered perpendicular media," *Journal of Magnetism and Magnetic Materials*, vol. 235, no. 1, pp. 454-458, Oct. 2001.

[7] D. Park, J. Kim, J. Lee and J. Lee, "Message-passing iterative decoding between detector and RSC code decoder for PMR channel," *IEEE Transaction on Consumer Electronics*, submitted, 2008.

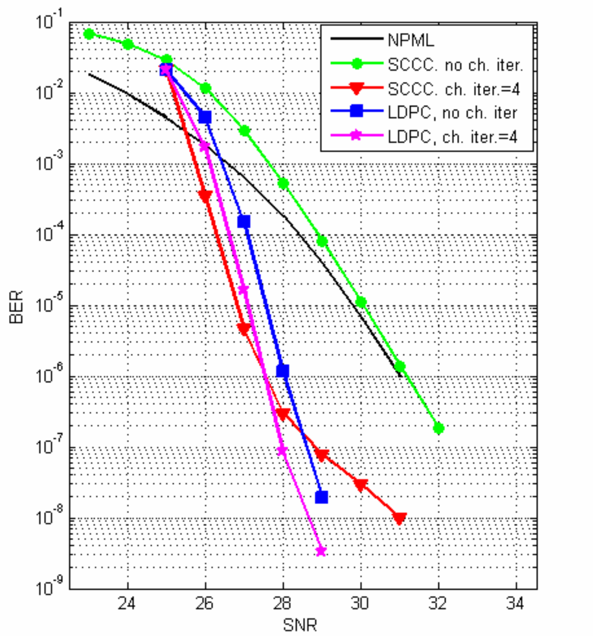

Figure 7. Simulation results for the SCCC system and LDPC system on PMR in accordance with the number of channel iterations.