Quantized Precoding Techniques for Spatially Correlated Channels without Correlation Feedback

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Abstract: This thesis deals with the quantized precoding techniques for spatially correlated MISO (multiple-input single-output) channels. The transformed codebook, which has been proposed recently for correlated channel, has problem of loosing diversity despite using additional feedback for channel correlation matrix. In this thesis, an efficient algorithm to estimate the correlation matrix without additional feedback and also prevent diversity loss is proposed. In the proposed algorithm, the estimation can be done using the codeword selection patterns. In order to estimate and get SNR gain simultaneously, the proposed algorithm uses both transformed codebook and specific forms of IID (independent identically distributed) codebook for estimation, which also prevents diversity loss.

1. Introduction

Space-time signal processing with multiple antenna transmitters can significantly reduce error rates by mitigating the effects of multipath fading caused by channel. Closed loop techniques obtain these benefits by adapting the transmitted signal to current forward link channel. One of the most well known closed loop system using perfect channel knowledge is transmit maximum ratio combining (transmit-MRC or MRT) which can provide full diversity and array gain of multiple antenna system.[1]

However, obtaining perfect knowledge of current channel at the transmitter is unrealistic in systems using frequency division duplexing (FDD), because forward and reverse link are separated in frequency. Thus, the receiver has to convey current channel information to the transmitter. As the feedback channel is limited, quantized precoder using a codebook of precoding vector is widely used.[2]-[5] Since the codebook is designed to substitute current channel and known at both transmitter and receiver, only the index of the codeword have to be fed back in quantized precoding systems.

Under the assumption of spatially uncorrelated channel (independent identically distributed channel, IID channel), the codebook design criteria in [3] and [4] minimize the maximum correlation between any pair of codewords. Interestingly, this IID codebook design is linked to the line packing problem in Grassmannian manifold, where the minimum chordal distance between any pair of straight lines passing through the origin should be maximized.[6]

For spatially correlated MISO channel, the codebook design criterion derived in [7] and [8] uses a simple modification of the existing IID codebook which is designed for uncorrelated channel. Under the assumption that both transmitter and receiver know the correlation, [7] and [8] transform the IID codebook with correlation matrix. This transformed codebook has average achieved SNR gain.

But we found a fatal problem of the transformed codebook, which loses diversity order in spite of requiring additional feedback for correlation matrix. This problem comes from large maximum distortion of the transformed codebook while the average achieved SNR is improved. In this paper, a quantized precoding system which has the ability of both estimating the correlation without feedback and preventing diversity loss is proposed. To prevent diversity loss, a new combined codebook is proposed, which also use IID codebook with transformed codebook to reduce maximum distortion. An efficient algorithm to estimate the correlation matrix without additional feedback is proposed.

2. Proposed QPC for Correlated Channel

The proposed quantized precoding system for correlated MISO channel with N_t transmit antennas and a single receive antenna is depicted in Figure 1. Assuming a narrowband channel, the discrete-time baseband signal model is

$$r = \sqrt{E_\mathrm{s}}\mathbf{h}^H\mathbf{w}_l s + n \tag{1}$$

where E_s is the average transmit symbol energy, s is the single dimensional transmit symbol, $\mathbf{h}^H$ is the $1 \times N_\mathrm{t}$ channel vector, r is the received symbol and n is the additive white Gaussian noise (AWGN) with zero mean and variance N_0. The quantized prcoder has a predetermined codebook

$$\mathcal{W} = \{\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_L\} \tag{2}$$

which contains channel information quantized into L codewords and each codeword has unit norm. The combined codebook proposed in this paper consists of the IID codebook (for estimating channel correlation) and the transformed codebook (by the estimated correlation). The correlation matrix of channel is estimated from the patterns of using IID codebook. And the transformed codebook is obtain by transforming the IID codebook with the estimated correlation matrix and normalized it as

$$\mathcal{W}_\mathrm{corr} = \left\{ \frac{\mathbf{R}^{\frac{1}{2}}\mathbf{w}_l}{\left\|\mathbf{R}^{\frac{1}{2}}\mathbf{w}_l\right\|} \middle| \mathbf{w}_l \in \mathcal{W}_\mathrm{IID} \right\} \tag{3}$$

The system selects a codeword which maximize the average received signal to noise ratio (SNR).

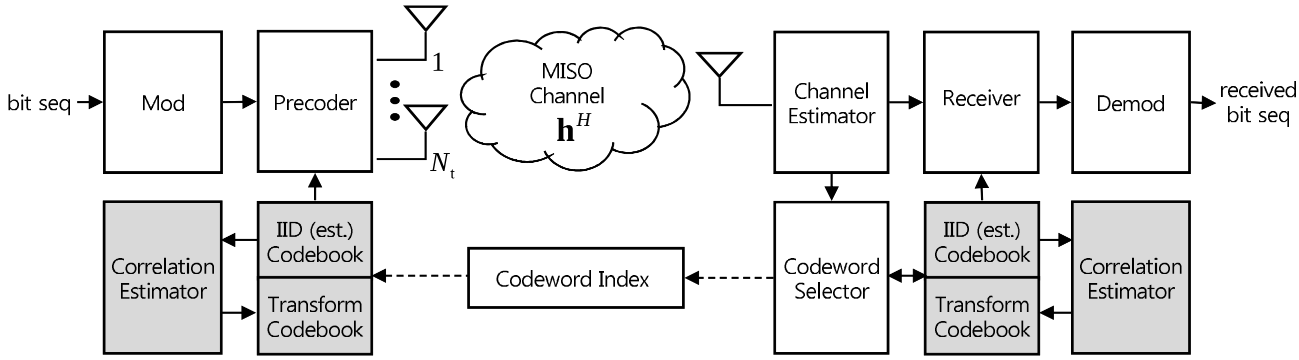


Figure 1. Proposed quantized precoding system for spatially correlated channels

2.1 Combined codebook

The conventional transformed codebook has SNR gain over the IID codebook, but loses diversity order as depicted in Figure 4. The transformed codebook is designed to maximize the normalized average achieved SNR, but it increases the maximum distortion (decreases the minimum achieved SNR) as depicted in Figure 2(b). When channel is highly correlated as Figure 2, most codewords in the transformed codebook are located very close to the statistically dominant direction, so the maximum distortion increased regardless of the codebook size. Although it has very low probability that the selected codeword has the maximum distortion (the minimum achieved SNR), but once it happens, error occurs with high probability despite of small noise (high SNR). Due to this property, the transformed codebook loses diversity. But the maximum distortion of the IID codebook is small and same whether the channel is correlated or not as depicted in Figure 2(a), because it is designed to minimize the maximum correlation between any pair of codewords.

To earn SNR gain from the transformed codebook and to prevent diversity loss from the IID codebook, the combined codebook which uses both IID codebook and transformed codebook simultaneously is proposed. As depicted in Figure 2, IID codewords (a') from the combined codebook (c) prevent maximum distortion of transformed codebook (b').

2.2 Estimation of correlation matrix

The transformed codebook is obtained by transforming the IID codebook by the correlation matrix as shown in (3). Since both transmitter and receiver should have same codebook, the process of estimating the correlation and feeding back the estimated correlation is required. To eliminate this additional feedback for transmitting the estimated correlation to the transmitter, the algorithm to estimate correlation matrix by using the patterns of selected codeword is proposed.

The correlation matrix can be written as

$$\mathbf{R} = \mathbb{E}\left[\|\mathbf{h}\|^2 \mathbf{w}_{\text{opt}} \mathbf{w}_{\text{opt}}^H\right] \tag{4}$$

using the optimum precoding vector $\mathbf{w}_{\text{opt}} = \mathbf{h}/\|\mathbf{h}\|$. From the codeword selection criterion, the selected codeword is the most similar one with optimal precoding vector $\mathbf{w}_{\text{opt}}$. Thus, correlation matrix can be estimated using the patterns of choosing codewords as

$$\hat{\mathbf{R}} = N_{\text{t}} \sum_{l=1}^{L} p_l \mathbf{w}_l \mathbf{w}_l^H \tag{5}$$

where p_l is the probability that the l-th codeword is used. However, each elements of the codeword must have equal absolute values in order to estimate the correlation matrix using (5). If they have unequal values, the diagonal entries of estimated correlation matrix are not guaranteed to be one despite the diagonal entries should be one.

One example satisfying above condition is a column vector chosen from the $N_{\text{t}} \times N_{\text{t}}$ DFT matrix. Arbitrary set of these vectors can be a codebook, which can have maximum N_{t} codewords. This codebook has good property that all codewords are orthonormal each other, which means the cross correlation between any pair of codewords is zero and has unit norm. To reduce estimation error due to limited number of codewords, tilting parameter σ which makes the codebook treated as bigger size of codebook is brought in as

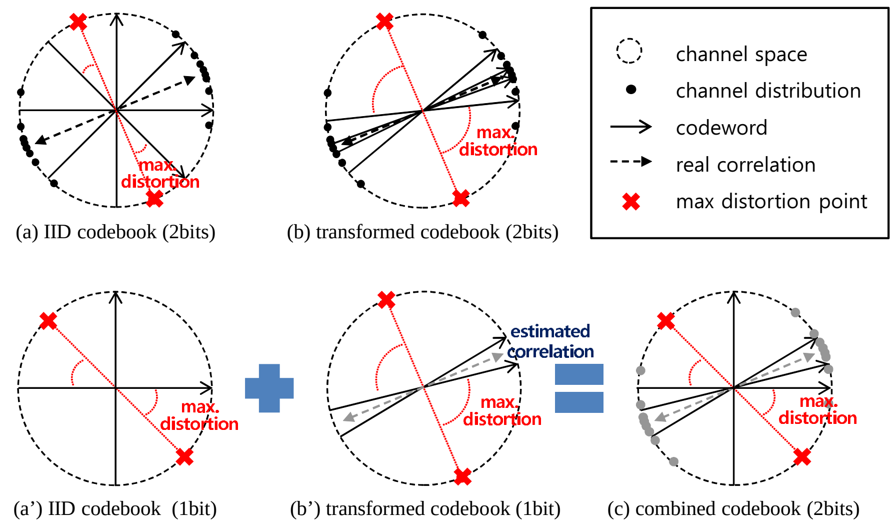
$$\mathbf{w}_l = \frac{1}{\sqrt{N_{\text{t}}}} \left[\begin{array}{ccc} e^{i\left(\frac{2\pi}{N_{\text{t}}}l+\sigma\right)\cdot 0} & \cdots & e^{i\left(\frac{2\pi}{N_{\text{t}}}l+\sigma\right)\cdot(N_{\text{t}}-1)} \end{array} \right]^T \tag{6}$$

which is still orthonormal each other for $l \leq N_{\text{t}}$. Without the tilting parameter, it is the same with choosing the first l columns of the DFT matrix. Using this codebook, the element of estimated correlation matrix is

$$\left\{\hat{\mathbf{R}}\right\}_{r,c} = \sum_{\sigma} \sum_{l} p_l(\sigma)\, e^{i\left(\frac{2\pi}{N_{\text{t}}}l+\sigma\right)(r-c)} \tag{7}$$

which means an estimated correlation between antenna r and c. Using this estimated correlation, we can construct the transformed codebook as (3).

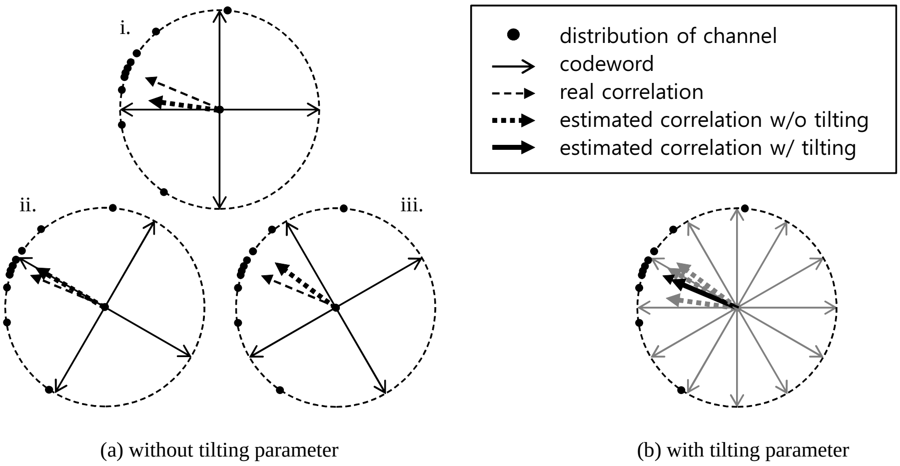
Figure 3 shows the procedure to estimate the correlation matrix with and without tilting parameter. Without tilting parameter, the correlation can not be estimated correctly, especially as shown in the case of iii in (a). But with tilting parameter, the estimated correlation in (b) is similar with averaging the correlation from i, ii, and iii in (a). This means that the tilting parameter can make estimation more accurate because the codebook using the tilting parameter is treated as a codebook that has more codewords.



(a) IID codebook (2bits) (b) transformed codebook (2bits)

(a') IID codebook (1bit) (b') transformed codebook (1bit) (c) combined codebook (2bits)

Figure 2. Illustration of codewords in IID, transformed, and combined codebook for highly correlated channel



(a) without tilting parameter (b) with tilting parameter

Figure 3. Estimation of correlation using the IID codebook

3. Simulation Results

In this section, numerical performance of the proposed quantized precoding system which has ability of estimating correlation without feedback and preventing diversity loss is evaluated to verify the benefits. For modeling the correlation matrix, a uniform linear array antenna with omnidirectional radiation is considered and the detail of generation is described in [9] and [10].

Figure 4 depicts the probability of bit error performance for proposed system which has 4 transmit antennas when azimuth spread is $1°$. All the codebooks were restricted to 3bits of feedback. The combined codebook consists of 2bits IID codebook and 2bits transformed codebook. For low SNR, the performance of the proposed system is almost the same with the conventional system using the transformed codebook which needs additional feedback for correlation matrix. For high SNR, the performance of proposed system has 1dB SNR gain with IID codebook and 2dB SNR gain with transformed codebook at 10^{-4} BER.

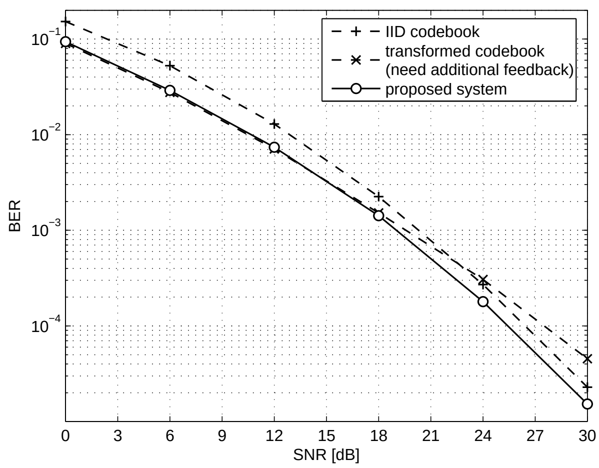


Figure 4. BER of 3bit quantized precoder ($N_t = 4$)

4. Conclusions

In this thesis, the quantized precoding system for spatially correlated channel is proposed. We have found some problems of the conventional transformed codebook that looses diversity in high SNR region and need additional feedback for correlation matrix. To prevent this diversity loss, combination method of transformed codebook and IID codebook is proposed. To eliminate the additional feedback, the method of estimating the correlation matrix by using the patterns of selected codeword is proposed. The estimation method requires the specific forms of IID codebook which is constructed for estimation. This IID codebook can be used for combining with transformed codebook, which prevents diversity loss.

From the view point of BER, the performance of the proposed quantized precoder is better than that of the conventional system using IID codebook. Although the proposed system has ability of estimating the correlation without feedback, it gives better performance than that of the conventional system using transformed codebook which requires additional feedback for correlation matrix.

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