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The Effects of Frequency Offset and Timing Jitter on the Performance of CSS System

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Abstract: Chirp spread spectrum (CSS) is a wideband modulation technique employing linear frequency sweep. We analyze the effects of frequency offset and timing jitter on the performance of CSS system. The exact bit error rate (BER) expressions of CSS system are derived in the presence of frequency offset and timing jitter. It is shown that analytical and experimental results match exactly and the frequency offset and the timing jitter degrade the CSS performance. In addition, we suggest tolerance criterion of the frequency offset and the timing jitter for reliable CSS based communication systems using simulation results.

1. Introduction

Numerous wireless communication techniques are having every human or electrical device be connected to networks anytime anywhere. Recently, the importance of wireless networks connecting the electrical devices such as personal computers, printers, storage devices and sensing human or another electrical devices in short range has been increasing. Wireless personal area network (WPAN) is designed to fulfill those roles, and the studies and applications on WPAN have been actively developed. Chirp spread spectrum (CSS) is one of selected techniques as the standards for WPAN.

CSS is a technique that spreads the energy of information bearing signals over a wide bandwidth applying linear frequency sweep. CSS is one of promising techniques for indoor wireless environments because of its features such as lower power consumption and robustness to multipath interference. IEEE 802.15.4a low rate alternative PHY task group (TG4a) for WPANs has selected CSS (operating in unlicensed 2.4GHz spectrum) as an optional PHY of the baseline specification [1]. Chirp based applications such as low power consuming sensor have been developed by Nanotron Technologies [2].

However, the effects of potential impairments such as frequency offset and timing jitter on the performance of CSS system have not been well studied. Though P. Zhang and H. Liu analyzed that the frequency offset causes the time shift in the matched filtered output of CSS system [3], the exact effect of frequency offset on the bit error rate (BER) performance is not derived. In this paper, thus, we provide analytical and experimental results on the effects of the frequency offset and timing jitter on the performance of CSS system. We derive the exact BER expression of CSS system in the presence of frequency offset and evaluated the effects of frequency offset and timing jitter by simulation. In addition, we can propose the tolerance ranges of frequency offset and timing jitter.

This paper is organized as follows. Section 2 describes a typical linear chirp waveform and CSS system structure. In Section 3, we analyze the effects of the frequency offset and timing jitter on the CSS system. The numerical results and the conclusion are presented in Section 4 and in Section 5, respectively.

2. System model

2. 1 Linear chirp signal

A typical linear chirp signal waveform $s(t)$ is expressed as,

$$s(t)=\cos(2\pi f_0 t+\pi\mu t^2), \qquad |t|<\frac{T_c}{2}, \tag{1}$$

where f_0 is the center frequency; T_c is the chirp duration; and μ is the chirp rate defined as the change rate of the instantaneous frequency. The chirp signal is called the up-chirp (down-chirp) signal when μ is positive (negative), which means the increase (decrease) of the frequency. The chirp bandwidth B is defined as the spreading range of the instantaneous frequency, and expressed as,

$$B = |\mu|\, T_c . \tag{2}$$

The impulse response of the matched filter is also a chirp signal, but with the opposite sign of μ. The impulse response matched to the up-chirp signal is the down-chirp signal. Then the matched filter output is given by [4],

$$g(t) = \sqrt{BT_c}\cos(2\pi f_0 t)\cdot\frac{\sin[\pi\mu t(T_c-|t|)]}{\pi\mu T_c t}, \quad |t|<T_c . \tag{3}$$

Since the matched filter output in (3) is a form of sinc function, the filter output has the maximum value at $t=0$ and the width of its main lobe is $2/B$. The matched filtered signal is compressed with a very short duration and its power is mainly concentrated in the main lobe. The pulse compression helps CSS system have high processing gain as much as BT_c, and makes it possible to transmit a chirp signal with low power consumption.

2. 2 Direct modulation (DM) using DPSK

The CSS system can be classified into two categories, according to the method of modulating a chirp signal: binary orthogonal keying (BOK) and direct modulation (DM) [5]. In the BOK method, the chirp types represent the information bit: up-chirp and down-chirp signals represent 1 and 0, respectively, for binary transmission. The DM method

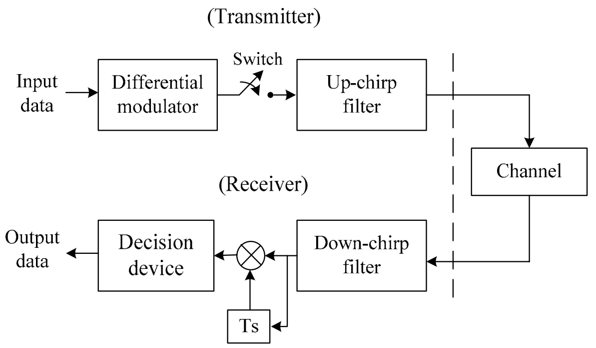


Figure 1. Block diagram of CSS system with DM-DPSK.

on the other hand, uses the chirp signal as a spreading code, so that the chirp modulation is independent from data modulation. This feature helps the DM scheme use various data modulation methods. Differential phase-shift keying (DPSK) is generally used because it can be detected non-coherently and is robust to envelope distortion.

In this paper, we employ the DM method with DPSK (DM-DPSK). Figure 1 describes the CSS system with DM-DPSK. This can be considered as DPSK scheme with a pair of the up-chirp and the down-chirp filters, and has less complexity than the CSS system with BOK in which more chirp filters are required as the data modulation order is increased.

3. Performance analysis

3. 1 Effect of frequency offset

The CSS systems have frequency offset caused by oscillator mismatch between a transmitter and a receiver and Doppler effect. The matched filter output $y(t)$ disturbed by a frequency offset f_d is given by [3]

$$y(t) = \sqrt{BT_c}\cos[2\pi(f_0+\frac{\mu\tau_d}{2})t] \cdot \frac{\sin[\pi\mu(\tau_d+t)(T_c-|t|)]}{\pi\mu T_c(\tau_d+t)}, \qquad (4)$$

where $\tau_d = \frac{f_d}{\mu}$. In (4), the frequency offset causes the time-shift in the filter output and also reduces the received symbol energy. We can quantify the energy loss due to the frequency offset by observing (3) and (4). The received symbol energy E_R^f is

$$E_R^f = E_{\max}\, sinc^2(\tau_d B), \qquad (5)$$

where $E_{\max}$ is the maximum energy achievable by matched filtering. The effect of frequency offset on CSS performance depends on $\tau_d B$. Since the larger value of $\tau_d B$ results in more serious performance degradation, it is with smaller value of B that the CSS system is robust to the effect of frequency offset. However, the CSS system is required to have the relatively large value of B since it is the spreading bandwidth which characterizes high processing gain of CSS system.

It is proven that the BER of CSS system with DM-DPSK is identical to that of a general DPSK scheme [5]. The exact BER expression for DQPSK without frequency offset is

$$P_b(e) = F(\zeta, \pi/4) + F(\zeta, 3\pi/4), \qquad (6)$$

where

$$F(\zeta, x) = \frac{\sin x}{2\pi}\int_0^{\pi/2} \exp\left[-\rho\frac{1-\cos x\cos\theta}{1-A_\gamma\cos x\cos\theta}\right] \cdot\left[\frac{1}{1-\cos x\cos\theta} - \frac{A_\gamma}{1-A_\gamma\cos x\cos\theta}\right]d\theta, \qquad (7)$$

and

$$\rho = \frac{\zeta\cdot K}{\zeta+K+1}, \qquad (8)$$

$$A_\gamma = \frac{\zeta}{\zeta+K+1}\cdot J_0(2\pi B_d/R), \qquad (9)$$

$\zeta = E/N_0$ is signal-to-noise power ratio (SNR) at the receiver; K is the channel parameter (which is infinity for the Gaussian channel and zero for the Rayleigh channel); B_d is the maximum Doppler frequency; R is the symbol rate; and $J_0(\cdot)$ is the zero-order Bessel function of first kind [6].

We can derive the BER expression of CSS system with DM-DQPSK in the presence of frequency offset by simply replacing E with E_R^f in (5). Note that the maximum SNR achievable at the receiver is

$$\zeta_0 = \frac{E_{\max}}{N_0} \qquad (10)$$

which the BER curves are drawn with respect to.

3. 2 Effect of timing jitter

Timing jitter may incur sampling errors, resulting in significant performance degradation in CSS systems [7]. In this paper, we assume the timing jitter to be a random variable with Gaussian distribution with zero mean and variance σ_j^2 [8]. The matched filter ouput with the timing jitter τ_j is given by

$$y(t) = \sqrt{BT_c}\cos[2\pi f_0(t+\tau_j)] \cdot \frac{\sin[\pi\mu(t+\tau_j)(T_c-|t+\tau_j|)]}{\pi\mu T_c(t+\tau_j)}. \qquad (11)$$

In (11), the incorrect sampling time caused by the timing jitter brings about the energy loss and the phase change of the sampled matched filter output. Calculating the average energy of (11) and comparing it with $E_{\max}$, we have the received symbol energy,

$$\begin{aligned} E_R^j &= E_{\max} \cdot \int_{-\infty}^{\infty} \cos^2(2\pi f_0 \tau_j) \\ &\quad \cdot \frac{\sin^2[\pi\mu\tau_j(T_c - |\tau_j|)]}{(\pi\mu T_c \tau_j)^2} \cdot f(\tau_j)\, d\tau_j \qquad (12) \\ &= E_{\max} \cdot L_j , \end{aligned}$$

where $f(\tau_j)$ is the probability density function of τ_j and L_j is the attenuation factor due to the timing jitter. The smaller value of L_j indicates more energy attenuation and CSS performance degradation.

We can substitute ζ in (8) and (9) with $L_j\zeta$ related to (12) and derive the exact BER expression of CSS system with DM-DQPSK in the presence of timing jitter as follows

$$\begin{aligned} P_b(e) = \int_{-\infty}^{\infty} &\left\{ F(L_j\zeta, \pi/4) + F(L_j\zeta, 3\pi/4) \right\} \\ &\cdot f(\tau_j)\, d\tau_j . \end{aligned} \qquad (13)$$

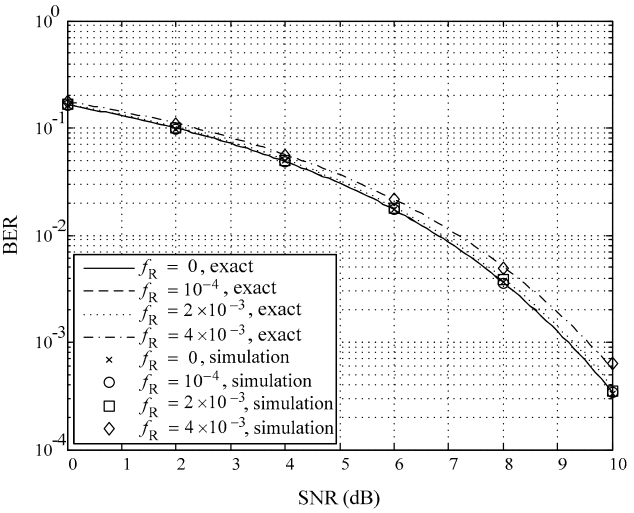
4. Numerical results

In this section, we present exact results for the BER performance of the CSS system with frequency offset and experimental results with timing jitter. Let us employ a relative frequency offset f_{R} and a relative variance σ_{R}^2 of timing jitter, which are defined as f_d / B and $\sigma_j^2 B$, respectively, since the performances are consistent with those quantities. The parameters used in the simulation are shown in Table 1.

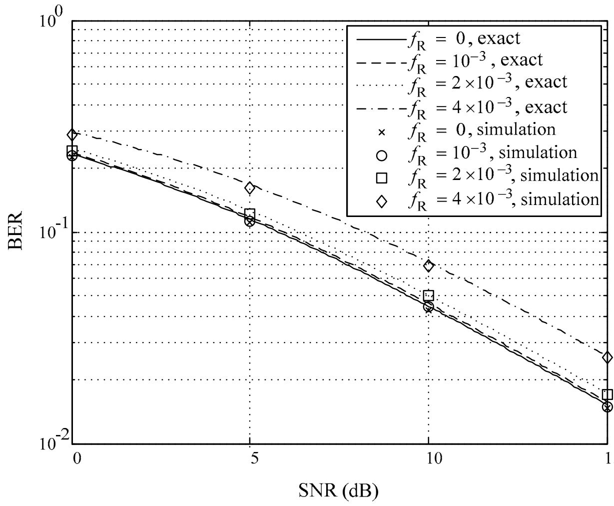
Figure 2 shows that the BER performance of the CSS system with DM-DQPSK in the presence of frequency offset. Analytical and experimental results match exactly and BER degradation incurs as frequency offset increases in both channels. On the other hand, the relative frequency offsets less than or equal to 10^{-4} in the AWGN channel and 10^{-3} in the slow Rayleigh fading channel do not affect the BER degradation. As a result, we can propose that the allowable ranges of the frequency offset are 0.01 % and 0.1 % of frequency offsets related to the chirp bandwidth in the AWGN and slow Rayleigh fading channels, respectively.

Table 1. Simulation parameters

Chirp duration (T_c)	0.5 μs
Chirp bandwidth (B)	200 MHz
Chirp rate (μ)	400 MHz/μs
Modulation method	DM-DQPSK
Channel environment	AWGN Slow Rayleigh fading



(a) AWGN channel

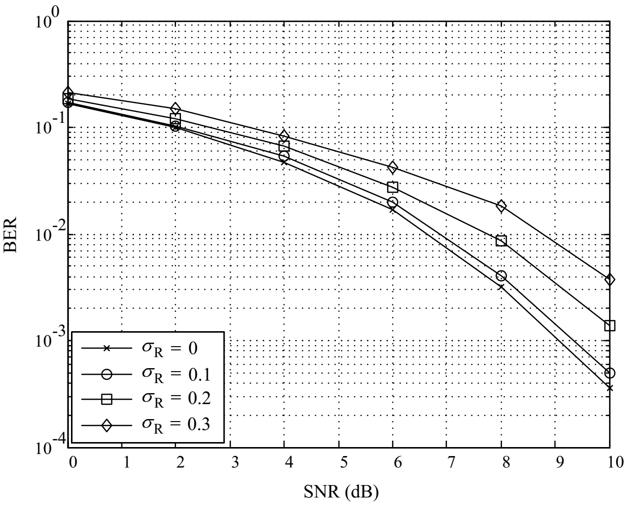


(b) Slow Rayleigh fading channel

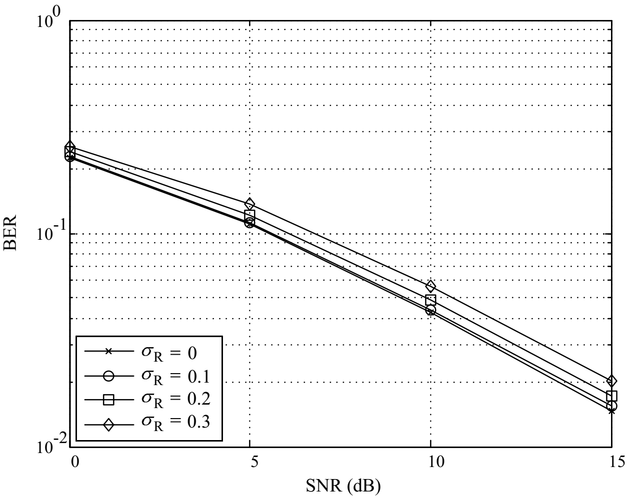
Figure 2. BER performance in the presence of frequency offset in the AWGN and slow Rayleigh fading channels.

Figure 3 shows that the effect of timing jitter on the BER performance of the CSS system with DM-DQPSK. The BER decreases as the relative variance of timing jitter increases. The BER curves with the relative standard deviation of timing jitter less than or equal to 0.1 in both channels are degraded by about 0.2 dB related to the curves without timing jitter. Therefore, the relative standard deviation as much as 0.1 can be the tolerance range of timing jitter which guarantees less than 0.2 dB BER degradation.

Figure 4 represents CSS performance when frequency offset and timing jitter simultaneously affect the system at a fixed SNR. The BER degradation is severer in the AWGN channel than in the slow Rayleigh fading channel with the increasing frequency offset. The more serious impact of frequency offset in the AWGN channel results in the difference of the tolerance range between the AWGN and slow Rayleigh fading channels in Figure 2. In Figure 4, we can also observe that the standard deviation of timing jitter less than or equal to 0.1 in both channels ensures about 10 % BER degradation when the frequency offsets are limited within the tolerance ranges.



(a) AWGN channel



(b) Slow Rayleigh fading channel

Figure 3. BER performance in the presence of timing jitter in the AWGN and slow Rayleigh fading channels.

5. Conclusion

In this paper, we have analyzed the effects of frequency offset and timing jitter on the performance of CSS system. We have derived the exact BER expression of CSS system in the presence of frequency offset and evaluated the effects of frequency offset and timing jitter by the simulation. As a consequence, we have observed that the frequency offset and timing jitter cause the CSS performance degradation. In addition, we have investigated the tolerance ranges of frequency offset and timing jitter in CSS systems.

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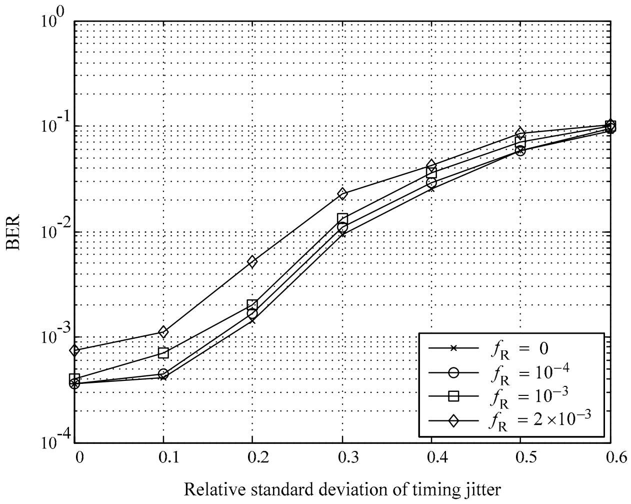
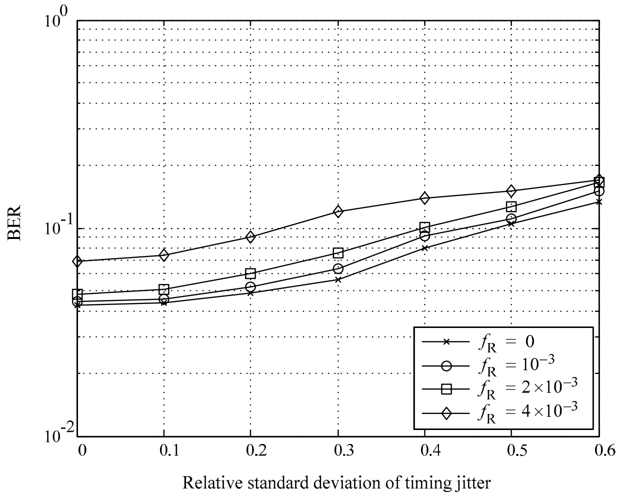


(a) AWGN channel



(b) Slow Rayleigh fading channel

Figure 4. BER performance in the presence of frequency offset and timing jitter at SNR=10 (dB).

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