

# Modeling and Performance Analysis of Wireless Sensor Network Systems Using Petri Nets

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**Abstract:** Wireless sensor network (WSN), composed by sensors, microprocessor and wireless communication interface, is an interesting field, and gains more and more attentions. The importance of sensor networks is highlighted by the number of recent funding initiatives, including the DARPA SENSIT program, military programs, and NSF Program Announcements. The wide application prospects make it developing rapidly in some fields such as health care, environment monitoring and military field. Petri nets are chosen for their ability to describe and study discrete event systems that characterized as being event systems that are characterized as being concurrent, asynchronous, parallel, nondeterministic, and stochastic. We construct ordinary and temporal Petri net models for wireless sensor network systems and use the model to solve the problem of location of nodes of wireless sensor network.

**Key words:** *Petri nets, wireless sensor network, modeling, location.*

## 1. Introduction

Today's telecommunication networks are facing the additional requirement to provide the ability to collect and compute information, i.e. to provide network intelligence, in addition to the classical transmission and switching functions. In this, wireless sensor network is one of the most important evolution directions that are attracting our attention. With the information from distributed wireless sensor networks, we can form smart environments<sup>[1]</sup>.

The challenges in the hierarchy of: detecting the relevant quantities, monitoring and collecting the data, assessing and evaluating the information, formulating meaningful user displays, and performing decision-making and alarm functions are enormous. Wireless sensor network is applied in many domains, such as military applications, traffic management, ecological and environmental monitoring, and so on.

Through the development of these years, Petri net has become the foundational theory of asynchronism communication. It is being integrated with the multimedia and object-oriented technology more and more tightly. And as a modeling technology of discrete affair system, it has many advantages obviously. Such as it offers uniform graphic to describe the characteristic of any system and it based on state of each affairs but not the change of them. Further more, high level Petri nets can model systems hierarchically.

This paper reports about motivation and experience of using PNs for description of WSN. We were driven by the application of WSN technology in coal mine rescue system. When a security accident happened to the coal mine, it is crucial to get the information about the locations of the miners timely and accurately. With an efficient wireless sensor network, we can get all kinds of information real time, such as the concentration of gas, the location of the miners. With such WSN, we can deal with the hidden troubles before them change to serious accidents. In this paper, we focus on the location of the miners.

## 2. Definition of Petri nets

Petri nets were originally developed to meet the need in specifying process synchronization, asynchronous events, concurrent operations, and conflicts or resource sharing for a variety of industrial automated systems at the discrete-event level.

A PN divides nodes into two kinds: places and transitions. Places are used to represent condition or status of a component in a system. They are pictured by circles. Transitions represent the events or operations. They are pictured by empty rectangles or solid bars. Two common events are "start" and "end." Instead of bidirectional links in some physical nets, a PN utilizes directed arcs to connect from places (called input places with respect to a transition) to transitions or from transitions to places (called output places). In other words, the information transfer from a place to a transition or from a transition to a place is one-way. Two way transfers between a place and transition is achieved by designing an arc from a transition to a place and another arc from the transition back to the place. Places, transitions, and directed arcs make a PN a directed graph, called the Petri net structure. The dynamics is introduced by allowing a place to hold either none or a positive number of tokens pictured by small solid dots. These dots could represent the number of resources or indicate whether a condition is true or not in a place. When all the input places hold enough number of tokens, an event modeled by a transition can happen, called transition firing. This firing changes the token distribution in the places, signifying the change of system states.

The introduction of tokens and their flow regulated through transitions allow one to visualize the material, control, and information flow clearly. Furthermore, one can perform a formal check of the properties related to the

underlying system's behavior, e.g., precedence relations among events, concurrent operations, appropriate synchronization, freedom from deadlocks, repetitive activities, and mutual exclusion of shared resources.

Structures and properties of Petri nets and firing rules are provided by T. Murata<sup>[2]</sup>. To be self-consistent, we include the following definitions:

- A Petri net is a 5-tuple,  $PN = (P, T, F, W, M_0)$  where:
- $P = \{p_1, p_2, \dots, p_m\}$  is a finite set of places,
- $T = \{t_1, t_2, \dots, t_n\}$  is a finite set of transitions,
- $F \subseteq (P \times T) \cup (T \times P)$  is a set of arcs (flow relation),
- $W: F \rightarrow \{1, 2, 3, \dots\}$  is a weight function,
- $M_0: P \rightarrow \{0, 1, 2, 3, \dots\}$  is the initial marking,  $P \cap T = \emptyset$  and  $P \cup T \neq \emptyset$

A Petri net structure  $N = (P, T, F, W)$  without any specific initial marking is denoted by  $N$ . A Petri net with the given initial marking is denoted by  $(N, M_0)$ . For a review of the application of Petri nets to communication systems, the reader is referred to P.M. Merlin<sup>[3]</sup>, following M.Reid<sup>[4]</sup> and W.M.Zuberek<sup>[4,5]</sup>; Michel Diaz<sup>[6]</sup>; Herman de Meer, Oliver-Rainer Dusterhoft, and Stefan Fischer<sup>[7]</sup>.

### 3. Modeling wireless sensor network system by Petri nets

The modeling of wireless sensor network system can be divided into two parts. The first step is to know the firing rules and model every component of the system to represent a logic working model of each component. The second is to use a brief example to show how to combine the components together and how the system works. In the second step, we use a topology graph with six points to show how to make the location.

In the first instance, we want to apply the model into the coal mine rescue system. With the development of our economies, more and more resources are produced and consumed. But unfortunately, mine accident took place now and then with many deaths. In order to reduce the rate of accident, many coal enterprises resort to monitor and control systems. WSNs, which based on the wireless transmission platform, offer these enterprises that don't qualify for cable transmission the monitor platform with high efficiency. But the cost of such WSNs becomes a big problem for many coal enterprises if we use too many devices. In order to persuade the firms to install such system to secure the safety, we manage to cut down the number of hardware with no reduction of system performance. For the map of the coal mine followed, we assume that person marked with one in the first laneway. We use (1, 1) to represent which, the first number represent the serial number of one person, the second number represent the serial number of one laneway. And the other pairs are (3, 2), (6, 3), (2, 4), (4, 5), (5, 6).

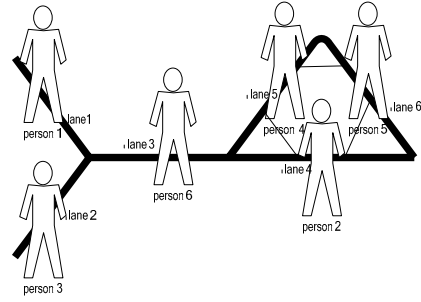


Fig.1: A brief map of a coal mine

We can assume that person 1 (lane 1) can communicate with person 6 (lane 3), but can't communicate with person 3 (lane 2). Communication details are as the follow table. And we assume the If we don't do anything that managed to reduce the number of devices, we may use as many as six devices as anchor nodes. But if we use some arithmetic, we can reduce the number to as few as two. In the interest of solve the problem automatically, we resort to Petri net.

Table 1: Communication details between lanes

lane	1	2	3	4	5	6
1	\	×	√	×	×	×
2	×	\	√	×	×	×
3	√	√	\	√	×	×
4	×	×	√	\	√	√
5	×	×	×	√	\	√
6	×	×	×	√	√	\

In which, the first row and column are the serial numbers of each lane. The  $\sqrt$  in the cross of row 3 and column 4 represent lane 2 can communicate with lane 3. The  $\times$  in the cross of row 3 and column 2 represent lane 2 can communicate with lane 1. Other communication statuses between each other are listed in the above table. For the sake of modeling the network, we have some more definitions as follows:

A marked Petri net  $PN = (P, T, F, M_0)$ , where:

$$\begin{aligned}
 P &= PL \cup Pc & PL &= \{p^1_1, p^1_2, \dots, p^1_n\} \\
 & & Pc &= \{p^c_1, p^c_2, \dots, p^c_n\}; \\
 T &= Ts \cup Tr & Ts &= \{t^s_1, t^s_2, \dots, t^s_n\} \\
 & & Tr &= \{t^r_1, t^r_2, \dots, t^r_n\};
 \end{aligned}$$

In this definition,  $p^l_i$  ( $1 \leq i \leq n$ ) is called a device location place,  $p^c_i \in \{0, 1\}$ ,  $p^c_i$  ( $1 \leq i \leq n$ ) is called a virtual channel place,  $p^c_i \in \{0, 1\}$ ,  $t^s_i$  ( $1 \leq i \leq n$ ) is a kind of transition represent sending message,  $t^r_i$  ( $1 \leq i \leq n$ ) is another kind of transition represent receiving message, F represents the arcs between places and transitions.

The firing rules are:

- 1) a transition  $t^s_i \in Ts$  is enabled if  $p^l_i = 1$ , that means there is a token in  $p^l_i$ .
- 2) a transition  $t^r_i \in Tr$  is enabled if  $p^c_i \cap p^l_{i+1} = 1$ , and if there are more than one inputs to  $p^l_{i+1}$ , then each transition fires in turn at the rate of 1 over the number of outputs to  $p^l_{i+1}$ .

Having known the firing rules, we now move to modeling the components of a simple WSN. In brief, we have the following elements as show in Fig.2.

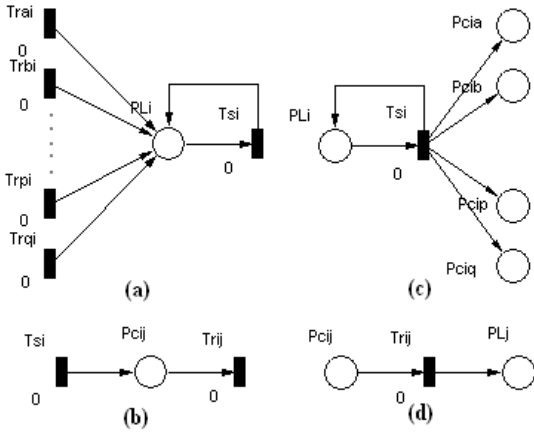


Fig.2 Structure of components of a simple WSN

Structure of PL (Fig.2(a)) means that each location place has two inputs at least. One input is the outcome of firing of itself and others are the outcome of other channel places. PL has such kind of input and output as show in Fig.2 (b) . It means that channel place has one input and one output. Structure of TS (Fig.2(c)) means that when sending message, TS receives one message from a location place and then sends it to one or more channel place. TR has such kind of input and output as show in Fig.2 (d) : It means that when receiving message, TR receives one token from its preceding channel place and its following location place and then sends one token to the following location place. We can conclude that the location place never consume any token.

Now, we apply the components to the chart of coal mine. In which, we use one PL to represent each laneway and a Pc to represent one virtual communication channel. Here, for an example, we assume there is only one miner in each laneway. And the model can also be applied to the circumstance that there are several miners in one laneway.

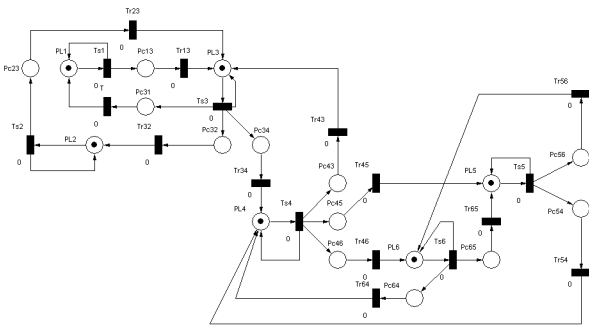


Fig.3 A Petri net of one brief coal mine

The marked Petri net  $PN = (P, T, F, M_0)$ , in the  $M_0 = (m_{p1}, m_{p2}, m_{p3}, \dots, m_{pn})$

$$\text{Where } m_{pi} = \begin{cases} 0 & \text{when } pi \in P^c \\ 0/1 & \text{when } pi \in P^l \end{cases}$$

#### 4. Applications in practice

In our model, we assume that

$$m_{pi} = \begin{cases} 0 & \text{when } pi \in P^c \\ 1 & \text{when } pi \in P^l \end{cases}$$

As that assumed in the third part, there is a person with one node in every laneway. We need to find which laneway the person is in.

First, each location place has one token and there is no token in channel place at the initial state.

Second, each node can communicate with the nodes around it. At each node, we construct one table to record its ID number and other IDs and their path that reaches it.

Third, any of Ts fires that mean a location place sends the message to channel place which is stored in the table (at the initial, send the node's ID).

Fourth, any of Tr fires that means the follower location place receive the message from channel place. Then the location place will check that whether the new message is longer than the former message stored in table. If the new message is shorter than the correspond message, the message will be replaced by the new message. Otherwise, the new message is abandoned. The program of the rule is:

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If X[h] ≠ ∅ do nothing
else X[h] = ∅
  if Y[h] = ∅
    Y[h] = X[h]
  else if len(X[h]+1) ≥ len(Y[h])
    else Y[h] = X[h]+Y[h]

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Fifth, we will deal with the ID messages which we got at the fourth step. For simplicity, we transfer the information of path to the hops. Then we get one table which contains the hops of each node to any other nodes. We can use graph theory and other mathematic theory to deal with it.

During the simulation, we confine some rules to the Petri net. Such as, at any time, if there are more than two outputs of Tsi, Tsi can only produce two tokens, one for PLi, and the other for Pci<sub>x</sub> (x is a variable). That's to say, no matter x have how many values (we can assume that x have n values), only one can get the token, and the probability is 1/n. And also, when the token produced by Tr<sub>x</sub> (x is a variable) received by PLi, it will immerge with the token existing in PLi.

From the simulation, we can map the graph exactly in a pretty short time. And the model can also used in other problems, such as location of the attained LED from the large LED plane, location of colorful bulbs out of a mansion.

#### 5. Conclusions-perspectives

This paper deals with the problem of the wireless sensor networks, through the location of particular assumption of each place have only one device.

From a prospective view point, the next extensions and objectives of the wireless sensor networks are:

1. To use the linear logic to validate the realized modeling.
2. To draw out a kind of arithmetic for the condition as the paper assumed.
3. To treat each device place has random numbers of devices (from 0 to a limited number).
4. To add redundancy mechanisms in order to be able to substitute a breakdown device site.
5. To treat the most common case in which the failure device is not known ahead.

## References

- [1] LEWIS F L, Wireless Sensor Networks [M]// Cook D J, Das S K, Wiley John. To appear in smart Environments: Technologies, Protocols, and Applications. New York, 2004
- [2] Murata T, Petri nets: properties, analysis and application [J]. Proc. IEEE, 1989. vol.44. 541–579.
- [3] Merlin P M. Specification and Validation of Protocols [J] IEEE trans. On communications. 1979. Vol. Com-27, 1671-1680
- [4] Reid M, Zuberek W M, Timed Petri Net Models of ATM LANs. [C]// Jonathan Billington, Michel Diaz, Grzeforz Rozenberg. Application of Petri nets to communication networks: advances in Petri nets. Springer,1999.150-175
- [5] Zuberek W M. Preemptive D-timed Petri Nets, Timeouts, Modeling and Analysis of communication protocols [C]// IEEE INFOCOM'87, San Francisco, California, 1987, 721-730
- [6] Diaz Michel. Modelling and analysis of communication and cooperation protocols using petri net based models[J]. Computer Networks, 1982, 6(6):419-441.
- [7] Herman de Meer, Oliver-Rainer Dusterhoft, Stefan Fischer. COSTPN for modeling and control of telecommunication Systems [C]// Jonathan Billington, Michel Diaz, Grzeforz Rozenberg. Application of Petri nets to communication networks: advances in Petri nets. Springer, 1999, 232-272