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Advanced CI/OFDM System Robust to the Phase Noise

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Abstract — High PAPR (peak-to-average power ratio) and ICI (inter carrier interference) are serious problems in the OFDM (orthogonal frequency division multiplexing) system. Recently, CI/OFDM (carrier interferometry coded OFDM) system has been proposed for reducing the PAPR and it shows good PAPR reduction performance. However, the CI/OFDM system is very sensitive to the ICI (inter carrier interference) problem since the phase offset mismatch happens due to the frequency offset and phase noise. In this paper, we propose an ACI/OFDM (advanced-CI/OFDM) to simultaneously handle the PAPR and ICI problems for the overall BER performance improvement. This method basically increases the margin of phase offset in CI codes. Even though it shows a little bit higher PAPR than conventional CI/OFDM, but it improves the total BER performance. From the simulation results, we can show the performance comparison between the conventional OFDM, CI/OFDM and ACI/OFDM.

keywords : OFDM, CI/OFDM , ICI, PAPR, Phase noise

I. INTRODUCTION

OFDM system has the serious drawback of high PAPR and serious ICI problem unlike the single carrier system. Therefore, PAPR has to be reduced to improve communication performance of OFDM system and much researches has studied to solve this problem such as clipping, block coding, selective mapping (SLM) and partial transmit sequence (PTS) [1-6]. In spite of PAPR reduction, these methods trigger another important problem such that BER get worse and system complexity becomes tremendous. In the clipping method, the clipping noise makes the BER performance worse, since it is produced by cutting signal envelope above than threshold [3]. On the other hand, block coding, PTS, and SLM elevate the complexity of system and take very long signal processing time or they must transmit the phase-related side information [2-7]. In OFDM system, the ICI problem is caused by phase noise or carrier frequency offset and it seriously degrades system performance because it may break down the orthogonality between sub-carriers of OFDM communication system [8-11].

Recently, CI-OFDM system was proposed for the PAPR reduction using CI phase offset codes and it shows the BER improvement by frequency diversity effect in the narrow band interference channel [13-15]. This system spreads one information data into N sub-carriers and the orthogonal CI spread codes are multiplied. So, it can achieve the good BER performance because of the frequency diversity benefit in each bit. However, CI/OFDM system will be degraded when there is mismatch of phase offset due to the random phase noise.

In this paper, we like to analyze the effect of phase noise in CI/OFDM system and propose an ACI/OFDM to keep the low PAPR and solve the ICI problem. In ACI/OFDM system, the number of the CI phase codes is the same to the number of sub-carrier divided by number of the partitioned sub-blocks. Usually, the number of the partitioned sub-blocks is reasonably 4 to 8 considering the system realization and the complexity. So, the number of phase code can be greatly reduced and margin of phase offset increases by 4 to 8 times. This system becomes robust to the phase noise and ICI problem can be handled. Therefore, the proposed method gets the system performance gain with respect to the whole BER performance when we consider the phase noise and high PAPR problem.

Ⅱ. PAPR AND PHASE NOISE PROBLEMS

OFDM data is converted by serial to parallel N and is modulated by the IFFT. Then, OFDM signal can be given by

$$x(t)=\sum_{k=0}^{N-1} X_k e^{j2\pi f_k t}=\sum_{k=0}^{N-1} X_k e^{j2\pi kt/NT_s} \qquad (1)$$

where $j=\sqrt{-1}$, N is the total number of sub-carriers, T_s is symbol duration, $f_k = k/NT_s$. Complex base-band OFDM signal can be given by

$$x(n)=\sum_{k=0}^{N-1} X_k e^{j\frac{2\pi}{N}kn}, \qquad 0\le n\le N-1 \qquad (2)$$

In time domain, PAPR of OFDM signal is defined as follows

$$PAPR \equiv 10\log_{10}\frac{P_{peak}}{P_{av}} \quad [dB] \; . \qquad (3)$$

Where, P_{peak} and P_{av} can be given by

$$P_{peak} = \max|x(t)|^2 = |NA|^2 \; . \qquad (4)$$

$$P_{av} = \frac{1}{T}\int_0^T |x(t)|^2 dt = NA^2 \; . \qquad (5)$$

So, maximum PAPR of OFDM signal can be expressed as

$$PAPR = \frac{\max_{0<t<T_S} |s(t)|^2}{mean_{0<t<T_S} |s(t)|^2} \; . \tag{6}$$

$$P_{OFDM_{\max}} = \left(\sum_{i=1}^{N} A \right)^2 = (NA)^2 = \frac{1}{2}(NA_0)^2 = N^2 P_0 \; \cdot \tag{7}$$

$$PAPR_{\max} = \frac{N^2 P_0}{NP_0} = N \; \cdot \tag{8}$$

$$PAPR_{\max} = 10 \log_{10} N \quad [dB] \; . \tag{9}$$

OFDM signal is up-converted by transmitter oscillator and down-converted by receiver oscillator.

$$r(t) = \left\{ s(t) \cdot e^{j\phi_{TX}(t)} + n(t) \right\} \cdot e^{j\phi_{RX}(t)} \; . \tag{10}$$

where $n(t)$ is complex Gaussian noise, $\phi_{TX}(t)$ and $\phi_{RX}(t)$ is the phase noise process that happens at oscillators of transceiver. After removing cyclic prefix and FFT stage, the recovered output for the k th sub-carrier is as follows

$$Y_k = \sum_{m=0}^{N-1} r[m] \cdot e^{-j\frac{2\pi}{N}km} = \sum_{l=0}^{N-1} X_l \cdot PN_{l-k} + N_k \; . \tag{11}$$

N_k is the FFT version of AWGN influenced by phase noise and this has the variance value σ_n^2. And process $\phi[m]$ is $\phi_{TX}[m] + \phi_{RX}[m]$. Here, PN_k is as follows

$$PN_k = \frac{1}{N} \sum_{m=0}^{N-1} e^{j\phi[m]} \cdot e^{j\frac{2\pi}{N}km} \; . \tag{12}$$

In [9], we can approximate $e^{j\phi[m]}$ into $1 + j\phi[m]$, then can separate the signal and noise term. Degradation factor [11] is

$$DF = 10 \log \frac{(S/N)_{without\{\phi\}}}{(S/N)_{with\{\phi\}}} \; . \tag{13}$$

Signal to noise ratio plus phase noise can be given by

$$(S/N)_{with\{\phi\}} = \frac{P_s}{P_p + P_n} \; . \tag{14}$$

To find the power of each component, k th decision variable is expressed as follows

$$Y_k = X_k \cdot P_0 + \sum_{l=0, l \neq k}^{N-1} X_l \cdot P_{l-k} + N_k \; . \tag{15}$$

CPE component power can be given by

$$P_{CPE} = \mathrm{var}[P_0] = \mathrm{var}[\frac{1}{N} \sum_{m=0}^{N-1} e^{j\phi[m]}] = \frac{1}{N} \cdot (1 - e^{-4\sigma_\phi^2}) \; . \tag{16}$$

Therefore, variance of $e^{j\phi}$ can be

$$\mathrm{var}[e^{j\phi_{TX}}] = E[\left| e^{j2\phi_{TX}} \right|^2] - E^2[e^{j2\phi_{TX}}]. \tag{17}$$

where $E[\left| e^{j2\phi_{TX}} \right|^2] = E[\cos^2 2\phi_{TX} + \sin^2 2\phi_{TX}] = 1$, and

$$\begin{aligned} E[e^{j2\phi_{TX}}] &= \int_{-\infty}^{+\infty} \exp(j2\phi_{TX}) \cdot \frac{1}{\sqrt{2\pi\sigma_\phi}} \exp\left(-\frac{\phi_{Tx}^2}{2\sigma_\phi^2} \right) d\phi_{TX} \\ &= e^{-2\sigma_\phi^2} \end{aligned} \; . \tag{18}$$

Therefore, variance of $e^{j\phi}$ is $(1 - e^{-4j\sigma_\phi^2})$.

$$\begin{aligned} P_{ICI} &= \mathrm{var}\left[\sum_{l=0, l \neq k}^{N-1} X_l \cdot P_{l-k} \right] = P_s \cdot \left\{ \mathrm{var}\left[\sum_{l=0}^{N-1} P_l \right] - \mathrm{var}[P_0] \right\} \\ &= P_s \cdot \frac{N-1}{N} \cdot \left(1 - e^{-4\sigma_\phi^2} \right) \end{aligned} \; . \tag{19}$$

where the variance of $\sum_{l=0}^{N-1} P_l$ can be calculated as follows

$$\mathrm{var}\left[\sum_{l=0}^{N-1} P_l \right] = \frac{1}{N^2} \mathrm{var}\left[\sum_{l=0}^{N-1} \left(\sum_{m=0}^{N-1} e^{j\phi[m]} \cdot e^{j\frac{2\pi}{N}km} \right) \right] = \left(1 - e^{-4\sigma_\phi^2} \right). \tag{20}$$

$$\left| \sum_{l=0}^{N-1} \left(e^{j\frac{2\pi}{N}l} \right)^m \right|^2 = \begin{cases} N^2, & l = 0 \\ \dfrac{\sin(\pi l)}{\sin(\pi l / N)} \cdot e^{j\pi\left(\frac{N-1}{N}\right)l}, & otherwise \end{cases} \; . \tag{21}$$

So, signal to noise ratio plus phase noise can be given by

$$(S/N)_{with\{\phi\}} = \frac{P_s}{(P_{CPE} + P_{ICI}) + P_n} = \frac{P_s}{\frac{1}{N}(1 - e^{-4\sigma_\phi^2}) \cdot \{1 + (N-1) \cdot P_s\} + P_n} \; . \tag{22}$$

Ⅲ. CI/OFDM SYSTEM AND ANALYSIS

Phase offset of i th sub-carrier associated k th bit is $i\Delta\theta$. All of phase sequences of k th bit are ($e^{j0}, \ldots, e^{j\Delta\theta_k}, \ldots, e^{j(N-1)\Delta\theta_k}$) and this is the spreading sequence for the k th input data. Also, phase offset of CI/OFDM is given as $\Delta\theta_k$ is $(2\pi/N)k$.

$$X_k = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} a_k \cdot \exp(i\Delta\theta_k) \tag{23}$$

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \sum_{i=0}^{N-1} a_k \cdot e^{j\Delta\theta_k} \cdot e^{j\frac{2\pi}{N}kn} \; . \tag{24}$$

where a_k is the k th QPSK symbol. Assigning the phase offset $i\Delta\theta_k$ is to separate the N bit placed at same point in the receiver side [12-14]. Final transmit signal of N bit CI block is written as

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \sum_{i=0}^{N-1} a_k e^{j2\pi f_C t} e^{j2\pi f_i t} e^{i\Delta\theta_k} \; . \tag{25}$$

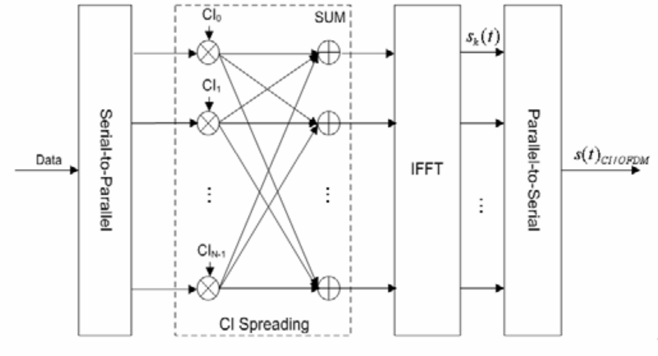

Fig 1. Block diagram of CI/OFDM transmitter.

Received signal can be given by

$$r(t)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}\sum_{i=0}^{N-1}(a_k e^{j2\pi f_C t}e^{j2\pi f_i t}e^{i\Delta\theta_k})+n(t)\ \cdot \qquad (26)$$

$$r(n)=x(n)\otimes h(n)+\upsilon(n) \qquad (27)$$

where $\upsilon(n)$ is the AWGN. For the simplicity of analysis, we suppose channel response, $h(n)=1$ and the perfect synchronization for the sake of simplicity.

Then, after removing cyclic prefix and FFT, the recovered output of the k th sub-carrier is as follows:

$$Y_k=\frac{1}{\sqrt{N}}\sum_{n\in K}r[n]\cdot e^{-j\frac{2\pi}{N}kn}=\sum_{i=0}^{N-1}a_k\cdot e^{i\Delta\theta_k}+N\ \cdot \qquad (28)$$

where s is the index of sub-carrier ($s,i,k\in K$) and K is defined as the set of sub-branch. X_f is the expression of $x(n)$ in frequency domain. Finally, the symbol a_k is

$$\hat{a}_k=\frac{1}{N}\sum_{l=0}^{N-1}a_l\cdot\sum_{s=0}^{N-1}e^{j\frac{2\pi}{N}(i-s)l}+N\quad =a_k+N\quad \cdot \qquad (29)$$

Therefore, peak power of CI/OFDM is lower than that of OFDM system. PAPR of CI/OFDM can be

$$PAPR_{CI-OFDM}=\frac{\frac{1}{2}(\max_{0<t<T_S}|s(t)|^2)}{NP_0}<<N\ \cdot \qquad (30)$$

CI/OFDM shows the degradation of system performance because of the mismatch of phase offset due to the random phase noise. When there is phase noise, the received signal of CI/OFDM is given by

$$r(n)=[x(n)\otimes h(n)+\upsilon(n)]\cdot e^{j\phi(n)}. \qquad (31)$$

After removing cyclic prefix and FFT, the recovered output for the k th sub-carrier is as follows

$$Y_k=\frac{1}{\sqrt{N}}\sum_{n\in K}r[n]\cdot e^{-j\frac{2\pi}{N}kn}=\sum_{f\in K}X_f\cdot Q_{f-k}+N_k\ \cdot \qquad (32)$$

where $n,f,k\in K$ and Q_{f-k} is the phase noise component. For the simplicity of analysis, we suppose channel response, $h(n)=1$. CI/OFDM demodulation for the symbol a_k is

$$\hat{a}_k=\sum_{i=0,i\in N}^{N-1}Y_k\cdot e^{-i\Delta\theta_k}=\sum_{i=0}^{N-1}Y_k\cdot e^{-i\frac{2\pi}{N}k} \qquad (33)$$

where the 1st component corresponds to the original data component with CPE, the 2nd component corresponds to the original data with ICI caused by original data components in the other sub-carriers, the 3rd component corresponds to the SCI caused by other data superimposed with original data in the same carriers, and the 4th component corresponds to the ICI caused by the other data in the other sub-carriers.

Ⅳ. PROPOSED ACI/OFDM SYSTEM

ACI/OFDM system is proposed for the bigger phase offset margin than the conventional CI/OFDM system so that the ACI/OFDM can be much robust to the phase noise.

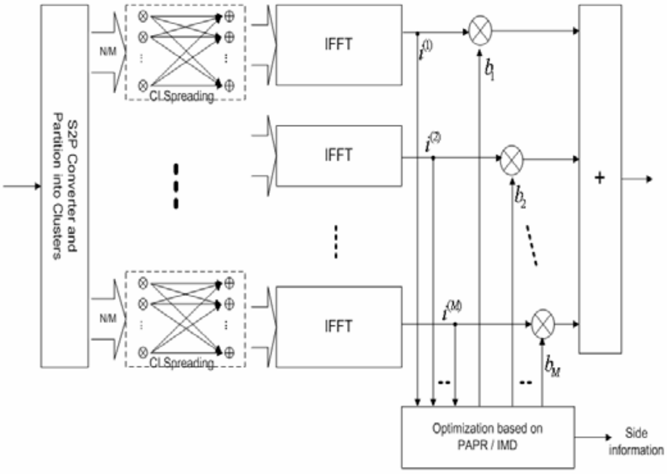

Fig 2. Transmitter block diagram of ACI/OFDM system.

In sub-block partitioning methods, there are 3 kinds of method such as adjacent, pseudo-random and interleaved methods. We will take the interleaved method to minimize the interference between adjacent channels. After S/P converter, input signal is divided to M sub-block. In this case, the phase offset interval is changed from $\Delta\theta_k=(2\pi/N)k$ to $\Delta\hat{\theta}_k=(2\pi M/N)k$, which can absolutely increase the phase difference. The received signal is multiplied with further separate CI phase code, $(2\pi/N)k$. Therefore, the transmitted signal for the k th bit of M th CI block can be given by

$$X_k=\frac{1}{\sqrt{N/M}}\sum_{i=0}^{N/M-1}a_k\cdot\exp(i\Delta\hat{\theta}_k) \qquad (34)$$

Each sub-block is multiplied with phase rotation factor, $b_{(a)}=e^{j\theta_a}$. Here, θ_a is $\{0,\pi\}$ or $\{0,\pi/2,\pi,3\pi/2\}$, which is easy to realize the system. Therefore, transmit symbol of ACI/OFDM is as follow

$$s=\min\left(\sum_{a=0}^{M-1}b_{(a)}IFFT\left[I^{(a)}\right]\right)=\min\left(\sum_{m=1}^{M}b_m i^{(m)}\right)\ \cdot \qquad (35)$$

This proposed ACI/OFDM system is less sensitive, more robust to the phase noise than conventional CI/OFDM because ICI component can be smaller and phase mismatch is hardly happened.

$$r(n)=[x(n)\otimes h(n)+\upsilon(n)]\cdot e^{j\phi(n)}. \qquad (36)$$

In the receiver, the recovered output for the k th sub-carrier is expressed by

$$Y_{m\_k}=\frac{1}{K}\sum_{n\in L}r[n^{(m)}]\cdot b^{(m)}\cdot e^{-j\frac{2\pi}{K}nk}=\sum_{f\in L}X_f\cdot Q_{f-k}+N_k \qquad (37)$$

where m is the index of sub-block, b is the transmitted phase rotation vector from transmitter and L is the sub-carrier number of one sub-block. $n,f,k\in L\ (0\le L\le N/M-1)$, ACI/OFDM demodulation for the transmitted symbol a_k^m is as follows

$$\hat{a}_k^{(m)} = \sum_{i=0,i\in N}^{K-1} Y_k \cdot e^{-i\Delta\theta_k} = \sum_{i=0}^{K-1} Y_k \cdot e^{-i\frac{2\pi}{N}k} = \frac{1}{N\cdot S'} \sum_{n\in S'} \sum_{f\in S'} \sum_{s=0}^{S'-1} \sum_{l=0}^{S'-1} a_l^{(m)} \cdot e^{j\left\{2\pi\left[\frac{(f-s)n}{K}+\frac{(lv-ki)}{S}\right]+\phi[n]\right\}} + N_k \tag{38}$$

Ⅴ. SIMULATION RESULTS AND DISCUSSION

For the simulation, we consider the simulation conditions of each system as follows: QPSK signal format, $T = 4\mu s, T_{CP} = 0, N = 64, M = 4$ and AWGN channel.

Figure 3 illustrates the comparison of BER performance without SSPA when there is phase noise. So, the proposed system shows best performance in the presence of phase noise. Figure 4 describes the BER performance when system considers both PAPR and phase noise.

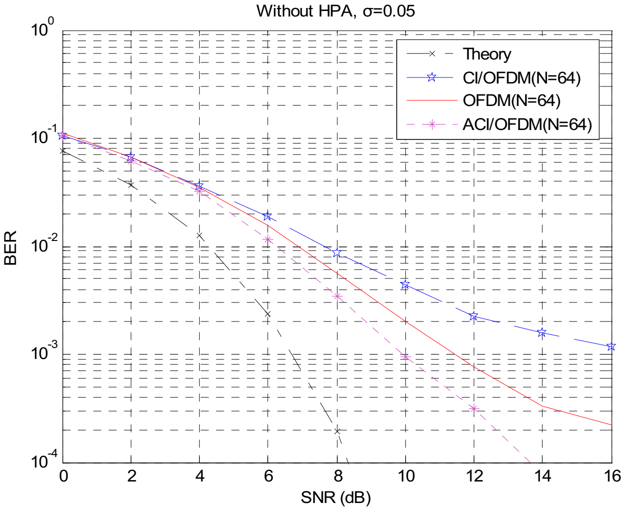

Fig 3. BER Performance without HPA.

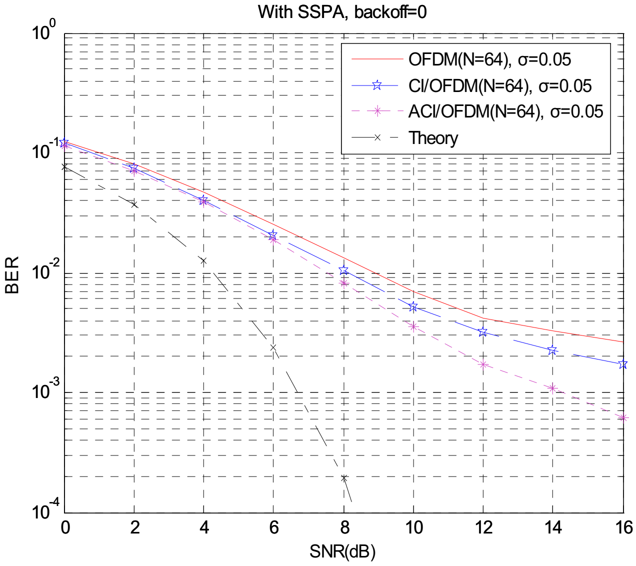

Fig 4. BER performances with SSPA(back-off=0).

Ⅵ. CONCLUSION

We propose an ACI/OFDM system to compensate the ICI problem and keep the low PAPR simultaneously. ACI/OFDM is designed to increase the phase margin. All data are divided into some partitioned sub-blocks by the sub-block mapping rule. When nonlinear HPA and phase noise is considered together, ACI/OFDM system shows about 1dB higher PAPR than CI/OFDM system because the spreading length is decreased by $N/2$, but this system is very robust to the phase noise. Therefore, the overall BER performance of ACI/OFDM system is better than other systems.

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