Phase Offset Error Detection Algorithm Without Parity Check Bit

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Abstract: This paper proposes an efficient error detection algorithm for phase offset enumeration of binary codes including PN (Pseudo Noise) sequences based on the number theoretical approach. The proposed error detection scheme does not need a parity check bit. The error detection failure probability of the proposed method is derived, and it is confirmed by simulation results. And very efficient circuit realization of the proposed algorithm is also discussed.

1. Introduction

Phase offsets of spreading sequences in the CDMA (Code Division Multiple Access) systems are used to achieve the acquisition and are used to distinguish each base station [1]. When the period of the sequence is not very long, the relative phase offset between the sequence and its shifted replica can be found by comparing them, but as the period of the sequence increases it becomes difficult to find the phase offset.

Willet proposed a method to calculate a phase offset of a maximal length sequence (*m*-sequence) [2]. For the phase offset of a binary sequence to be calculated by Willet's method, it is required that the sequence should be an *m*-sequence over GF(2)={0,1}. To overcome this restriction, more generalized method applicable to a binary sequence is proposed in [3]. Since this method is based on the number theoretical approach, it becomes very easy to find the phase offsets. Although [3] is more generalized and easy method compared to [2], it aslo did not show an error detection algorithm for phase offset enumeration. This paper presents an error detection algorithm for phase offset based on [3]. Thus this paper is an extension of [3]. The phase offset error detection method is derived based on the number theoretical approach and we show that the proposed scheme does not need any parity check bits. Furthermore the circuit realization of the proposed algorithm is also deiscussed. The error detection failure probability of the proposed algorithm is derived in closed form, and it is confirmed by simulation results over AWGN (Additive White Gaussian Noise) and Rayleigh fading channels.

2. Definitions and notations

Let $\mathbf{S}$ be the set of all *n*-tuple binary codes. We define a cyclic shift operator $T: \mathbf{S} \rightarrow \mathbf{S}$ by

$$T(\mathbf{C}) = (C_{n-1}, C_0, \cdots, C_{n-2}) \quad (1)$$

for every code $\mathbf{C} = (C_0, C_1, \cdots, C_{n-1}) \in \mathbf{S}$. For two integers i and j,

$$T^i(\mathbf{C}) = T^j(\mathbf{C}) = (C_{n-j}, C_{n-j+1}, \cdots, C_{n-j-1}) \quad (2)$$

where $i \equiv j \pmod n$. Here we define $T^0(\mathbf{C}) = \mathbf{C}$. We construct the following polynomial $C(x)$ corresponding to a code $T^j(\mathbf{C}) \in \mathbf{S}$.

$$C(x) = C_{n-j} + C_{n-j+1}x + \cdots + C_{n-j-1}x^{n-1} \quad (3)$$

We define an enumeration function $\mathrm{A}^l : \mathbf{S} \rightarrow \mathbf{Z}$ by

$$\mathrm{A}^l(T^j(\mathbf{C})) = \frac{d}{dx} x^l C(x) \bigg|_{x=1} \quad (4)$$

for all $T^j(\mathbf{C}) \in \mathbf{S}$, where $\mathbf{Z}$ is the set of all integer numbers and l is an integer [3]. Here we call the integer l weight of the accumulator function. The code $T^i(\mathbf{C})$ is said to be a reference code of the accumulator function with weight l if it satisfies $\mathrm{A}^l(T^i(\mathbf{C})) \equiv 0 \pmod n$. The function $\mathrm{A}^l(T^i(\mathbf{C}))$ is expressed as $\mathrm{A}_1^l(T^i(\mathbf{C}))$ if $C_i \in \{-1,1\}$ and $\mathrm{A}_0^l(T^i(\mathbf{C}))$ if $C_i \in \{0,1\}$, respectively.

3. Error detection algorithm

Theorem 1: Let k and $\hat{k}$ be the number of -1's or 0's in *n*-tuple binary codes $\mathbf{C}$ and $\hat{\mathbf{C}}$, respectively. And assume that the greatest common divisor of $2k$ and n, $(2k, n) = 1$ for $C_i \in \{-1,1\}$ and the greatest common divisor of k and n, $(k, n) = 1$ for $C_i \in \{0,1\}$, $i \in I = \{0,1,\cdots,n-1\}$. Then if $k \neq \hat{k}$,

$$a^*[\mathrm{A}^l(T(\hat{\mathbf{C}})) - \mathrm{A}^l(\hat{\mathbf{C}})] \neq 1 \pmod n \quad (5)$$

$$a^* \hat{C}(1) \neq 1 \pmod n . \quad (6)$$

where a^* is an arithmetic inverse of a modulo n such that

$$-2ka^* \equiv 1 \,(\text{mod } n) \text{ for } C_i \in \{-1,1\} \tag{7}$$

$$-ka^* \equiv 1 \,(\text{mod } n) \text{ for } C_i \in \{0,1\}\,. \tag{8}$$

And $\hat{C}(1)$ is defined by

$$\hat{C}(1) = \sum_{i=0}^{n-1} \hat{C}_i\,. \tag{9}$$

Proof: For the cases of $\hat{k} = 0$ and $\hat{k} = n$,

$$a^*[\mathrm{A}^l(T(\hat{\mathbf{C}})) - \mathrm{A}^l(\hat{\mathbf{C}})] \equiv 0 (\text{mod } n) \tag{10}$$

$$a^*\hat{C}(1) \equiv 0(\text{mod } n)\,. \tag{11}$$

Now we will show the theorem for $1 \le \hat{k} \le n-1$. Let $\hat{k} = k + k'$, where $k' \neq 0$ and $|k'| \le n-2$. From [3], we see that

$$\mathrm{A}^l(T(\hat{\mathbf{C}})) \equiv \mathrm{A}^{l+1}(\hat{\mathbf{C}}) \,(\text{mod } n) \tag{12}$$

$$\mathrm{A}^{l+1}(\hat{\mathbf{C}}) = (l+1)\hat{C}(1) + \mathrm{A}^0(\hat{\mathbf{C}}) \tag{13}$$

$$\mathrm{A}^l(\hat{\mathbf{C}}) = l\hat{C}(1) + \mathrm{A}^0(\hat{\mathbf{C}})\,. \tag{14}$$

Thus

$$a^*[\mathrm{A}^l(T(\hat{\mathbf{C}})) - \mathrm{A}^l(\hat{\mathbf{C}})] \equiv a^*\hat{C}(1) \,(\text{mod } n)\,. \tag{15}$$

where

$$\hat{C}(1) = \begin{cases} n - 2\hat{k}, & \text{for } C_i \in \{\text{-}1,1\} \\ n - \hat{k}, & \text{for } C_i \in \{0,1\} \end{cases}. \tag{16}$$

For $C_i \in \{\text{-}1,1\}$, (15) becomes

$$\begin{aligned} a^*[\mathrm{A}_1^l(T(\hat{\mathbf{C}})) - \mathrm{A}_1^l(\hat{\mathbf{C}})] &\equiv a^*(-2\hat{k}) \,(\text{mod } n) \\ &= a^*(-2(k+k')) \,(\text{mod } n) \\ &\equiv (1 + bk') \,(\text{mod } n) \end{aligned} \tag{17}$$

where $b \equiv -2a^* \,(\text{mod } n)$. From the assumption of $(2k, n) = 1$, we see that $(a^*, n) = 1$ and $(b, n) = 1$ [5]-[6]. Thus $bk' \neq 0 \,(\text{mod } n)$ and

$$\begin{aligned} a^*\hat{C}(1) &\equiv (1 + bk') \,(\text{mod } n) \\ &\neq 1 \,(\text{mod } n) \end{aligned}. \tag{18}$$

Similarly, we can also prove this theorem for $C_i \in \{0,1\}$. □

Corollary 1: Let k and $\hat{k}$ be the number of -1's or 0's in n-tuple binary codes $\mathbf{C}$ and $\hat{\mathbf{C}}$, respectively. And assume that the greatest common divisor of $2k$ and n, $(2k, n) = 1$ for $C_i \in \{-1,1\}$ and the greatest common divisor of k and n, $(k, n) = 1$ for $C_i \in \{0,1\}$, $i \in I = \{0, 1, \cdots, n-1\}$. Then $k = \hat{k}$ if and only if $a^*\hat{C}(1) \equiv 1 \,(\text{mod } n)$.

Proof: If $k = \hat{k}$, then

$$\hat{C}(1) = \begin{cases} n - 2k, & \text{for } C_i \in \{\text{-}1,1\} \\ n - k, & \text{for } C_i \in \{0,1\} \end{cases}$$

and thus $a^*\hat{C}(1) = 1 \,(\text{mod } n)$. From theorem 1, if $k \neq \hat{k}$, $a^*\hat{C}(1) \neq 1 \,(\text{mod } n)$. □

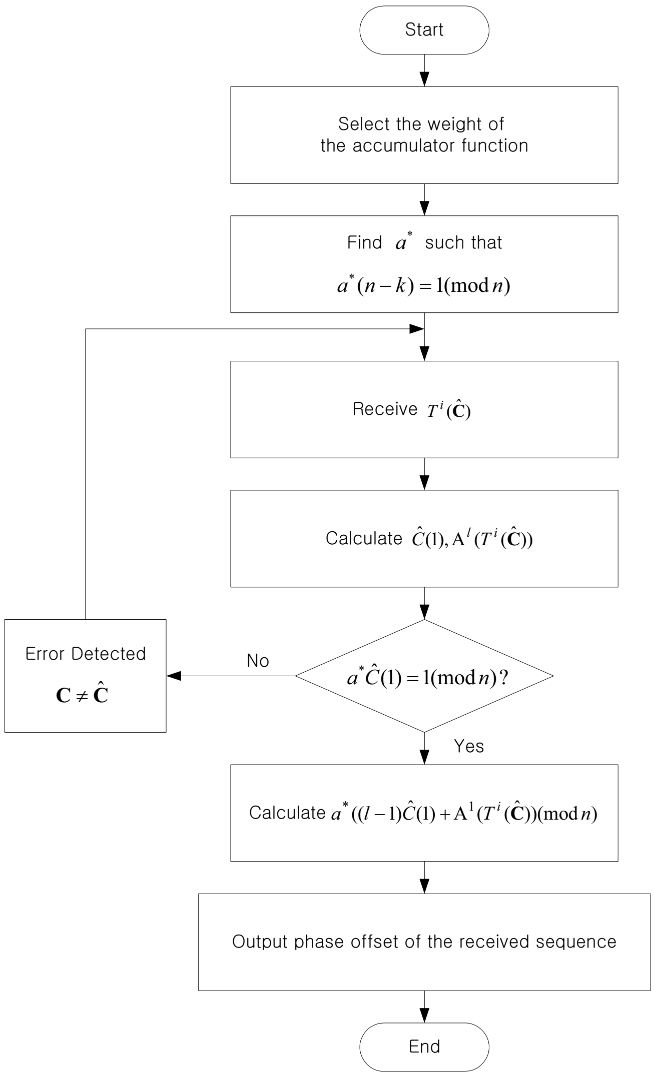

Fig. 1 Phase offset error detection algorithm.

Corollary 1 tells us that $a^*\hat{C}(1) \equiv 1 \pmod n$ is identical to $k = \hat{k}$. If $a^*\hat{C}(1) \equiv 1 \pmod n$, we assume that the received code has no error since $k = \hat{k}$. Otherwise phase offset error is declared becase this is the case of $k \neq \hat{k}$. If we add all elements in the received code $\hat{\mathbf{C}}$, we can obatin $\hat{C}(1)$. Fig. 1 shows the phase offset error detection algorithm based on the result of corollary 1.

4. Error detection failure probability

If the error pattern in the received code satisfies the following 3 conditions simultaneously,

1) The number of errors in the received code $\hat{\mathbf{C}}$ is even.
2) The number of errors in k -1's or 0's of $\hat{\mathbf{C}}$ is identical to the number of errors in $(n-k)$ 1's.
3) The number of errors in $\hat{\mathbf{C}}$ is less than or equal to $2\min(n-k,k)$.

the proposed phase offset error detection method fails to detect errors in the received code $\hat{\mathbf{C}} \neq \mathbf{C}$. Since this is the case of $\hat{k} = k$. Thus the derived error detection failure probability is

$$P_u = \sum_{i=1}^{\min(n-k,k)} \binom{n-k}{i}\binom{k}{i} p^{2i}(1-p)^{n-2i} \qquad (19)$$

where p is the average bit error probabiliy. The average bit error probability of coherent BPSK in AWGN (Additive White Gaussian Noise) channel is $p = \frac{1}{2}\left[1 - \mathrm{erf}\left(\sqrt{\frac{E_b}{N_0}}\right)\right]$, where $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2} du$ [6]. E_b is the received bit energy and N_0 is the single-sided noise power spectral density. $p = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}}\right)$ for Rayleigh fading channel, where $\bar{\gamma}_b$ is the average SNR (Signal-to-Noise Ratio) [6]. Figure 2 and 3 show the error detection failure probability for binary sequences with *n*=30, *k*=1,7,11,13, respectively over AWGN and Raleigh fading channels, respectively. The theoretical curves of figure 2 and 3 are very close to the simulation results. This confirms the derived error detection failure probability of (19) is exact. From figure 2 and 3, we see that P_u is propotional to k. This comes from the fact that the chances of 2) and 3) occurence decrease as the value of k becomes smaller. We find that P_u goes down at very low and high SNRs. This interesting phonomenon can be explained as follows. Very low SNR causes very high bit error rate and this reduces the chance of the condition 3) occurrence. Hence P_u goes down at very low SNR. Now consider the case of very high SNR. We know that bit error probability in the received code becomes very rare and thus P_u goes down at very high SNR.

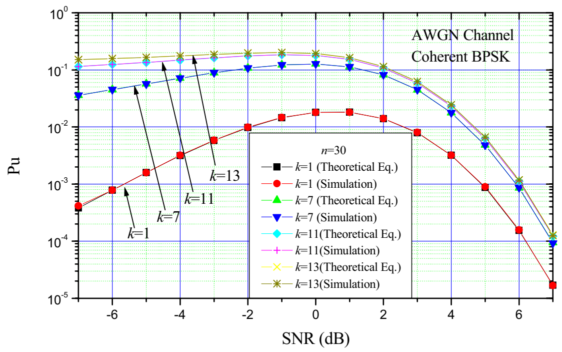

Fig. 2 Phase offset error detection failure probability over AWGN channel in the case of *n*=30, *k*=1,7,11,13.

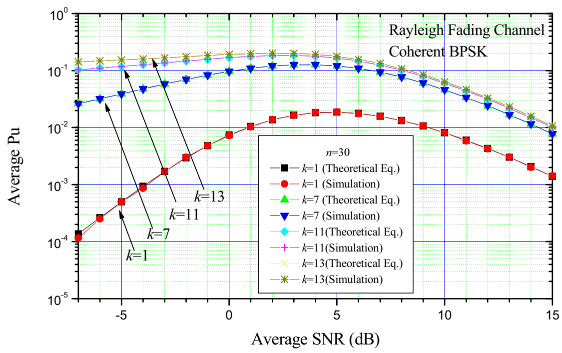

Fig. 3 Phase offset error detection failure probability over Rayleigh fading channel in the case of *n*=30, *k*=1,7,11,13.

Once P_u and p are fixed, P_u is simply determined by the value of k. The maximum value of $\min(n-k,k)$ is $n/2$ for even n and $(n-1)/2$ for odd n. Hence the maximum value of P_u is given by

$$\max P_u = \begin{cases} \sum_{i=1}^{n/2} \binom{n-k}{i}\binom{k}{i} p^{2i}(1-p)^{n-2i}, & n:\text{even} \\ \sum_{i=1}^{(n-1)/2} \binom{n-k}{i}\binom{k}{i} p^{2i}(1-p)^{n-2i}, & n:\text{odd} \end{cases} \qquad (20)$$

The length of PN sequences is odd [7] and

$$\min(n-k,k) = k = (n-1)/2 . \qquad (21)$$

Thus the phase offset error detection failure probability for PN sequences is $\max P_u$ for odd n of (20). Figure 4 and 5 show the error detection failure probability for PN sequences over AWGN and Rayleign fading channels, respectively. The theoretical curves are very close to the simulation results. This confirms the derived error detection failure probability of (20) is also correct.

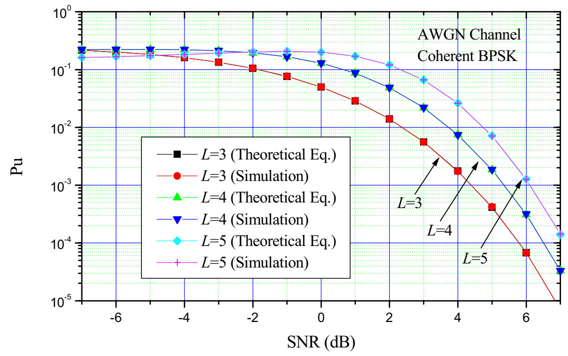

Fig. 4 Error detection failure probability for PN sequences with $n = 2^L - 1$, $L = 3,4,5$ over AWGN channel.

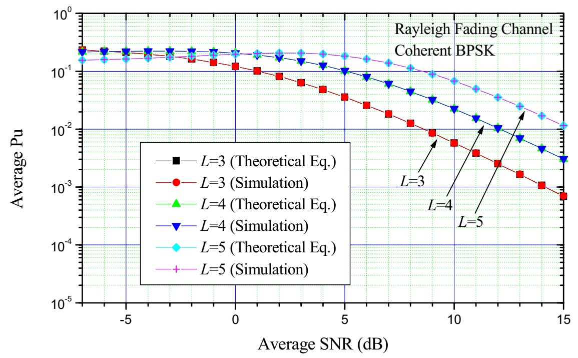

Fig. 5 Average error detection failure probability for PN sequences with $n = 2^L - 1$, $L = 3,4,5$ over Rayleigh fading channel.

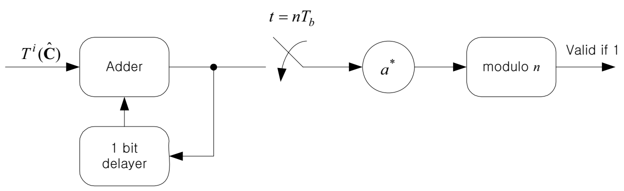

Fig. 6 Phase offset error detection circuit.

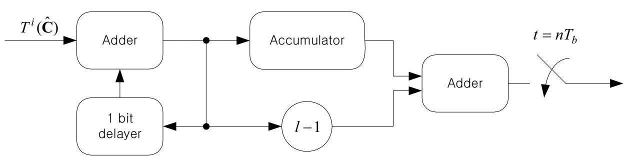

Fig. 7 Circuit for calculating the accumulator function with weight *l*.

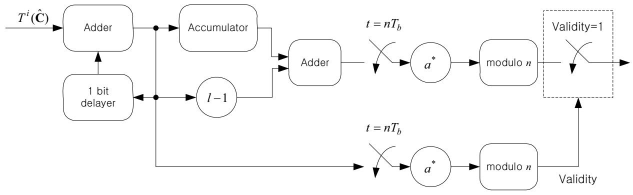

Fig. 8 Phase offset calculation curcuit with error detection.

5. Circuit realization

Phase offset error can be checked by just simply calculating $a^* \hat{C}(1)$. This is easily implemented by using figure 6. Figure 7 is the circuit for calculating the accumulaor function [3]. We can find some commonalities between figure 6 and 7. Therefore if we combine two circuits we can realize phase offset calculation circuit with error detection which is depicted in figure 8.

6. Conclusion

In this paper we have proposed the efficient phase offset error detection algorithm for binary sequences. By theorem 1 and corollary 1, if $(k, n) = 1$ for $C_i \in \{0, 1\}$ and $(2k, n) = 1$ for $C_i \in \{-1, 1\}$, the method is applicable to phase offset calculation and error detection without a parity check bit. The error detection failure probability of proposed algorithm has been derived. We confirmed the formula over AWGN and Rayleigh fading channels through intensive computer simulation. The simple circuit realization of the algorithm has been also described in detail.

References

[1] *Physical Layer Standard for cdma2000 Spread Spectrum Systems*, TIA/EIA/IS-2000-1a, March 2000.
[2] M. Willet, “The index of an m-sequence,” *SIAM J, Appl.*, vol 25, no.1, pp.24-27, July 1973.
[3] Y. Y. Han and Y. J. Song, “Phase Offset of Binary Code and Its Application to the CDMA Mobile Communications,” *IEICE Trans. Fundamentals*, vol. E81-A, no.6, pp.1145-1151, June 1998.
[4] I. Niven and H. S. Zukerman, *An Introduction to the Theory of Number*, John Wiley & Sons, 1980.
[5] K. Ireland and M. Rosen, *A Classical Introduction to Modern Number Theory*, Springer, 1990.
[6] J. G. Proakis, *Digital Communications*, McGraw-Hill, 4th edition, 2001.
[7] M. Simon, J. Omura, R. Scholtz, B. Levitt, *Spread Spectrum Communications Handbook*, McGraw-Hill, 1994.