

# Outage Statistics for Beckmann Fading Channels in Non-Isotropic Scattering Environments

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**Abstract**—In this paper, the outage statistics are studied for non-isotropic Beckmann fading channel model. Non-isotropic scattering generally results in an asymmetrical Doppler power spectral density (PSD). In this context, an expression for the outage probability (OP) (or equivalently the cumulative distribution function (CDF)) of the fading envelope is first derived. Then, the probability density function (PDF) of the rate of change of the fading envelope is investigated. Thereafter, an expression for the average rate of outages (ARO) (or equivalently the level-crossing rate (LCR)) is provided. Finally, by making use of the analytical results of the ARO and OP, an expression for the average duration of outages (ADO) (or equivalently the average duration of fades (ADF)) is attained. The obtained results, which are given in the form of finite-range integrals that can efficiently be computed numerically, are verified to include a variety of already known expressions for models that are special cases of the Beckmann model. Numerical and simulation results are provided to verify the validity of the derivations and to analyze the effect of the fading parameters on the obtained metrics.

**Keywords**—Beckmann channel model, non-isotropic scattering, outage probability, average rate of outages, average duration of outages.

## I. INTRODUCTION

The multipath propagation and fading caused by the Doppler effect are the main contributors to the impairments and distortions affecting the quality of wireless communication systems. For an adequate design and performance analysis of these systems, the modeling of the channel as well as the knowledge of its statistics are required. It is well known that the most common fading distributions are based on the Gaussian model. The main reasons for this are the ease of the physical interpretation of the propagation scenario, the mathematical tractability such as in the multivariate statistical analysis, and also the simplicity of the simulations [1]. The unifying statistical model of the Gaussian-based models is the Beckmann model [2]. In this multipath fading channel model, the channel gain is described by a complex Gaussian process, the real and imaginary parts of which have different mean values and variances. The Beckmann model includes Rayleigh, Rice, Hoyt, and one-sided Gaussian distributions as special cases [2]. It should be mentioned that the Beckmann model is also referred to as the generalized Rice [3] and generalized Gaussian [4] models.

The statistical properties of the Beckmann channel model

and its importance for the performance analysis of wireless communications has been studied in several papers. For instance, the moment generating function of the Beckmann fading envelope has been determined in [5]. The first-order statistics of the Beckmann model, combined with a lognormal process that accounts for the shadowing effects, have been investigated and applied to the modeling of satellite channels in [3]. Under isotropic scattering conditions, the second-order statistics of Beckmann channel model have been derived in [6], where the model was verified by measurement data of mobile satellite channels. The statistics of the level-crossing properties of the channel phase and random frequency modulation (FM) noise have been investigated in [7]. An analysis of collaborative spectrum sensing in cognitive radio has been carried out in [4] by employing the Beckmann model. Allowing the Gaussian processes, from which the Beckmann model is constructed, to be correlated, the first-order statistics of the envelope and phase of the model have been derived in [8] and used to analyze the error performance of binary frequency-shift keying modulation schemes. Particularly, in [6] and [7], where the original results of the level-crossing analysis have been presented, isotropic scattering has been assumed. However, as is reported in [9] and references therein, this assumption is valid only in limited circumstances, because in many propagation environments, radio wave scattering is non-isotropic. Therefore, it is of interest to gain an insight into the impact of non-isotropic scattering on the statistical properties of the Beckmann channel model. These challenges form the basis for the motivation of our paper.

This paper contributes to the analysis of Beckmann fading channel model by investigating the outage statistics that capture non-isotropic scattering effects. We start by providing an expression for the OP. Here, it should be emphasized that this performance metric is independent of the scattering scenarios, as the OP is obtained from the first-order statistics. We then address the derivation of the ARO. By making use of the OP and ARO expressions, the ADO is attained. All the derived quantities are verified by reducing them to those corresponding to particular cases of the Beckmann model. Simulations, for the case of a mobile-to-mobile (M2M) Beckmann fading channel model, are also performed to check the correctness of the derivations.

The paper is organized as follows. In Section II, we briefly present the Beckmann fading channel model and derive an

expression for the OP. Considering non-isotropic scattering scenarios, the joint PDF of the fading process and its time derivative is derived in Section III. Relying on this joint PDF, we present in Section IV the analytical expressions for the ARO and ADO. In Section V, numerical and simulation results are presented for an M2M Beckmann fading channel model. We conclude the paper in Section VI.

## II. BECKMANN FADING CHANNEL MODEL AND OP

In the Beckmann fading channel model, the complex channel gain  $\mu(t)$  is given by

$$\mu(t) = A \exp(j\theta_0) + \mu_1(t) + j\mu_2(t) \quad (1)$$

where  $A$  is the amplitude and  $\theta_0$  is a constant phase shift of the line-of-sight (LOS) component. In (1),  $\mu_1(t)$  and  $\mu_2(t)$  are zero-mean Gaussian processes with unequal variances  $\sigma_1^2$  and  $\sigma_2^2$ , respectively. The envelope  $R(t)$  and the phase  $\vartheta(t)$  of the Beckmann fading channel model are deduced from (1) as

$$R(t) = \sqrt{(A \cos(\theta_0) + \mu_1(t))^2 + (A \sin(\theta_0) + \mu_2(t))^2} \quad (2)$$

and

$$\vartheta(t) = \arctan\left(\frac{A \sin(\theta_0) + \mu_2(t)}{A \cos(\theta_0) + \mu_1(t)}\right). \quad (3)$$

The PDF  $p_R(z)$  of the process  $R(t)$  is given by [2], [3]

$$p_R(z) = \frac{z}{2\pi\sigma_1\sigma_2} e^{-A^2\gamma(\theta_0)} \int_0^{2\pi} e^{(-z^2\gamma(\theta) + z\rho(\theta))} d\theta \quad (4)$$

where

$$\gamma(\theta) = \frac{\cos^2(\theta)}{2\sigma_1^2} + \frac{\sin^2(\theta)}{2\sigma_2^2} \quad (5)$$

and

$$\rho(\theta) = A \left( \frac{\cos(\theta) \cos(\theta_0)}{\sigma_1^2} + \frac{\sin(\theta) \sin(\theta_0)}{\sigma_2^2} \right). \quad (6)$$

As can be noted, the model has the drawback that the PDF of the envelope does not have a closed-form solution, and hence its evaluation combined with other performance metrics requires numerical integration. Two alternative equivalent forms of  $p_R(z)$ , in terms of an infinite sum and standard functions, can be found in [4].

Now, considering the above fading channel model, we first investigate the OP. In addition to its usefulness as a performance measure, the OP plays a key role in the evaluation of the ADO. This metric will be denoted by  $P_{\text{out}}(r)$ , which represents the probability that  $R(t) \leq r$ , where  $r$  is a predetermined threshold, i.e.,  $P_{\text{out}}(r) = \Pr(R \leq r)$ , in which  $\Pr(\cdot)$  stands for the probability operator. Thus, the OP is given by the CDF  $F_R(r)$  of  $R(t)$  according to

$$P_{\text{out}}(r) = F_R(r) = \int_0^r p_R(z) dz. \quad (7)$$

Then, by substituting (4) in (7) and using [10, Eq. (2.33.1)], the CDF  $F_R(r)$  of the Beckmann process can be obtained as

$$F_R(r) = \frac{1}{2\pi\sigma_1\sigma_2} e^{-A^2\gamma(\theta_0)} \int_0^{2\pi} \frac{1}{2\gamma(\theta)} \left( 1 - e^{(-\gamma(\theta)r^2 + \rho(\theta)r)} \right) + \frac{\rho(\theta)\sqrt{\pi}}{4\gamma^{\frac{3}{2}}(\theta)} e^{\frac{\rho^2(\theta)}{4\gamma(\theta)}} \left( \operatorname{erf}\left[\frac{\rho(\theta)}{2\sqrt{\gamma(\theta)}}\right] + \operatorname{erf}\left[\frac{2\gamma(\theta)r - \rho(\theta)}{2\sqrt{\gamma(\theta)}}\right] \right) d\theta \quad (8)$$

where  $\operatorname{erf}(\cdot)$  represents the error function [10, Eq. (8.250.1)]. The CDF in (8) can be reduced to known results of fading models that are special cases of the Beckmann model. For instance, by setting  $\sigma_1^2 = \sigma_2^2$ , it can be shown that (8) reduces to the CDF of Rice fading channels [11, Eq. (33)]. If, in addition,  $A$  is set to zero, we obtain the CDF corresponding to Rayleigh channels given by [12, Eq. (1.1.17)]. On the other hand, by setting  $A = 0$  in (8) and using the expression of the generalized Marcum Q-function given in [13, Eq. (3)], then (8) reduces to the CDF of the Hoyt fading channel model [14, Eq. (3)].

Unlike the OP, the rate and average duration of outages depend strongly on the scattering environment. In the sequel, our purpose is to determine these two statistical quantities under the general condition of non-isotropic scattering. To this end, we first need to derive the joint PDF  $p_{R\dot{R}}(z, \dot{z})$  of the process  $R(t)$  and its time derivative  $\dot{R}(t)$ .

## III. JOINT PDF $p_{R\dot{R}}(z, \dot{z})$

The amplitude  $A$  and phase  $\theta_0$  of the LOS component in (1) are supposed to be constant. Thus, the joint PDF  $p_{R\dot{R}}(z, \dot{z})$  can be obtained by making use of the statistics of the processes  $\mu_1(t)$  and  $\mu_2(t)$  and their time derivatives denoted by  $\dot{\mu}_1(t)$  and  $\dot{\mu}_2(t)$ , respectively. An expression for the joint PDF  $p_{\mu_1\mu_2\dot{\mu}_1\dot{\mu}_2}(x_1, x_2, \dot{x}_1, \dot{x}_2)$  under non-isotropic scattering conditions has recently been derived in [15]. This expression is reproduced below for convenience

$$p_{\mu_1\mu_2\dot{\mu}_1\dot{\mu}_2}(x_1, x_2, \dot{x}_1, \dot{x}_2) = \frac{1}{(2\pi)^2 \sqrt{D_1 D_2}} \times e^{-\frac{1}{2D_1}(\beta_2 x_1^2 + \sigma_1^2 \dot{x}_2^2 - 2b_1 x_1 \dot{x}_2)} e^{-\frac{1}{2D_2}(\beta_1 x_2^2 + \sigma_2^2 \dot{x}_1^2 + 2b_1 x_2 \dot{x}_1)} \quad (9)$$

where  $D_1 = \sigma_1^2 \beta_2 - b_1^2$  and  $D_2 = \sigma_2^2 \beta_1 - b_1^2$ . In (9),  $\beta_i$  is the variance of the Gaussian process  $\dot{\mu}_i(t)$  ( $i = 1, 2$ ), and  $b_1$  is the non-isotropy parameter. The result corresponding to isotropic scattering is obtained when  $b_1 = 0$ . The next step towards the derivation of  $p_{R\dot{R}}(z, \dot{z})$  is to perform the transformation of the Cartesian coordinates  $(x_1, x_2)$  to polar coordinates  $(z, \theta)$  according to

$$\begin{aligned} x_1 &= z \cos(\theta) - A \cos(\theta_0), & \dot{x}_1 &= \dot{z} \cos(\theta) - z \dot{\theta} \sin(\theta) \\ x_2 &= z \sin(\theta) - A \sin(\theta_0), & \dot{x}_2 &= \dot{z} \sin(\theta) + z \dot{\theta} \cos(\theta) \end{aligned} \quad (10)$$

to get the joint PDF  $p_{R\dot{R}\theta\dot{\theta}}(z, \dot{z}, \theta, \dot{\theta})$  as

$$\begin{aligned} p_{R\dot{R}\theta\dot{\theta}}(z, \dot{z}, \theta, \dot{\theta}) &= \frac{z^2}{(2\pi)^2 \sqrt{D_1 D_2}} \\ &\times e^{-\frac{b_1}{D_2}(z \sin(\theta) - A \sin(\theta_0))(z \cos(\theta) - z \dot{\theta} \sin(\theta))} \\ &\times e^{\frac{b_1}{D_1}(z \cos(\theta) - A \cos(\theta_0))(z \sin(\theta) + z \dot{\theta} \cos(\theta))} \\ &\times e^{-\frac{1}{2D_1}[\beta_2(z \cos(\theta) - A \cos(\theta_0))^2 + \sigma_1^2(z \sin(\theta) + z \dot{\theta} \cos(\theta))^2]} \\ &\times e^{-\frac{1}{2D_2}[\beta_1(z \sin(\theta) - A \sin(\theta_0))^2 + \sigma_2^2(z \cos(\theta) - z \dot{\theta} \sin(\theta))^2]} \end{aligned} \quad (11)$$

where  $0 \leq z < \infty$ ,  $-\infty < \dot{z} < \infty$ ,  $-\pi \leq \theta < \pi$ , and  $-\infty < \dot{\theta} < \infty$ . Finally, the integration of (11), with respect to  $\theta$  and  $\dot{\theta}$ , yields the following expression for  $p_{R\dot{R}}(z, \dot{z})$

$$\begin{aligned} p_{R\dot{R}}(z, \dot{z}) &= \frac{z}{(2\pi)^{\frac{3}{2}}} \int_0^{2\pi} \frac{1}{\sqrt{g(\theta)}} e^{\frac{1}{2D_1 D_2 g(\theta)}} f^2(z, \dot{z}, \theta) \\ &\times e^{-\frac{b_1 \dot{z}}{D_1 D_2} [D_1 \cos(\theta)(z \sin(\theta) - A \sin(\theta_0)) - D_2 \sin(\theta)(z \cos(\theta) - A \cos(\theta_0))]} \\ &\times e^{-\frac{1}{2D_2} [\beta_1 (z \sin(\theta) - A \sin(\theta_0))^2 + \sigma_2^2 (z \cos(\theta))^2]} \\ &\times e^{-\frac{1}{2D_1} [\beta_2 (z \cos(\theta) - A \cos(\theta_0))^2 + \sigma_1^2 (z \sin(\theta))^2]} d\theta \end{aligned} \quad (12)$$

where the functions  $g(\theta)$  and  $f(z, \dot{z}, \theta)$  are given by

$$g(\theta) = D_2 \sigma_1^2 \cos^2(\theta) + D_1 \sigma_2^2 \sin^2(\theta) \quad (13)$$

and

$$\begin{aligned} f(z, \dot{z}, \theta) &= \dot{z} \sin(\theta) \cos(\theta) (\sigma_1^2 D_2 - \sigma_2^2 D_1) - b_1 [D_1 \sin(\theta) \\ &\times (z \sin(\theta) - A \sin(\theta_0)) + D_2 \cos(\theta) (z \cos(\theta) - A \cos(\theta_0))] \end{aligned} \quad (14)$$

respectively. As in the isotropic Beckmann channel model [6], it can be noted that  $p_{R\dot{R}}(z, \dot{z}) \neq p_R(z) p_{\dot{R}}(\dot{z})$ . Hence, the processes  $R(t)$  and  $\dot{R}(t)$  are statistically dependent. For completeness, it is insightful to derive the PDF  $p_{\dot{R}}(\dot{z})$  of the rate of change  $\dot{R}(t)$  of the fading process  $R(t)$ . An expression for  $p_{\dot{R}}(\dot{z})$  can be obtained from (12) according to

$$p_{\dot{R}}(\dot{z}) = \int_0^\infty p_{R\dot{R}}(z, \dot{z}) dz. \quad (15)$$

Substituting (12) into (15) and using [10, Eq. (3.462.5)] yield

$$\begin{aligned} p_{\dot{R}}(\dot{z}) &= \frac{1}{(2\pi)^{\frac{3}{2}}} \int_0^{2\pi} \frac{1}{2q(\theta) \sqrt{g(\theta)}} e^{\frac{1}{2D_1 D_2 g(\theta)}} l^2(\dot{z}, \theta) \\ &\times e^{-\frac{1}{2D_1} [\beta_2 A^2 \cos^2(\theta_0) + \sigma_1^2 \dot{z}^2 \sin^2(\theta)]} \\ &\times e^{-\frac{1}{2D_2} [\beta_1 A^2 \sin^2(\theta_0) + \sigma_2^2 \dot{z}^2 \cos^2(\theta)]} \\ &\times e^{-\frac{b_1 A \dot{z}}{D_1 D_2} [D_2 \cos(\theta_0) \sin(\theta) - D_1 \sin(\theta_0) \cos(\theta)]} \\ &\times \left( 1 - h(\dot{z}, \theta) \sqrt{\frac{\pi}{q(\theta)}} e^{\frac{h^2(\dot{z}, \theta)}{q(\theta)}} \left( 1 - \operatorname{erf} \left[ \frac{h(\dot{z}, \theta)}{\sqrt{q(\theta)}} \right] \right) \right) d\theta \end{aligned} \quad (16)$$

where the functions  $l(\dot{z}, \theta)$ ,  $q(\theta)$ , and  $h(\dot{z}, \theta)$  are expressed by

$$\begin{aligned} l(\dot{z}, \theta) &= D_2 \cos(\theta) (\sigma_1^2 \dot{z} \sin(\theta) + b_1 A \cos(\theta_0)) \\ &- D_1 \sin(\theta) (\sigma_2^2 \dot{z} \cos(\theta) - b_1 A \sin(\theta_0)) \end{aligned} \quad (17)$$

$$q(\theta) = \frac{(\sigma_2^2 \cos^2(\theta) + \sigma_1^2 \sin^2(\theta))}{2g(\theta) (\beta_1 \cos^2(\theta) + \beta_2 \sin^2(\theta))^{-1}} - \frac{b_1^2}{2g(\theta)} \quad (18)$$

and

$$\begin{aligned} h(\dot{z}, \theta) &= \frac{-1}{2g(\theta)} \left[ b_1 \dot{z} \cos(\theta) \sin(\theta) (\sigma_2^2 - \sigma_1^2) - A b_1^2 \cos(\theta - \theta_0) \right. \\ &\left. + \frac{A (\beta_1 \cos^2(\theta) + \beta_2 \sin^2(\theta))}{(\sigma_2^2 \cos(\theta) \cos(\theta_0) + \sigma_1^2 \sin(\theta) \sin(\theta_0))^{-1}} \right]. \end{aligned} \quad (19)$$

respectively. The derived expressions in (11), (12), and (16) can be shown to reduce to the special cases listed in Table I.

#### IV. RATE AND AVERAGE DURATION OF OUTAGES

In this section, we confine our attention to the derivation of analytical expressions for the ARO and the ADO. The ARO (or the LCR)  $N_R(r)$  of the process  $R(t)$  can be obtained by using the Rice's formula [19] according to

$$N_R(r) = \int_0^\infty \dot{z} p_{R\dot{R}}(r, \dot{z}) d\dot{z}. \quad (20)$$

Substituting (12) in (20), and solving the integral over  $\dot{z}$  using [10, Eq. (3.462.5)], the ARO  $N_R(r)$  can be written as

$$\begin{aligned} N_R(r) &= \frac{r}{(2\pi)^{\frac{3}{2}} \sigma_1^2 \sigma_2^2} \int_0^{2\pi} \sqrt{g(\theta)} \\ &\times e^{-\frac{\beta_2}{2D_1} (z \cos(\theta) - A \cos(\theta_0))^2} e^{-\frac{\beta_1}{2D_2} (z \sin(\theta) - A \sin(\theta_0))^2} \\ &\times e^{\frac{b_1^2 [D_1 \sin(\theta)(z \sin(\theta) - A \sin(\theta_0)) + D_2 \cos(\theta)(z \cos(\theta) - A \cos(\theta_0))]^2}{2D_1 D_2 g(\theta)}} \\ &\times \left( 1 - \sqrt{\pi} k(r, \theta) e^{k^2(r, \theta)} (1 - \operatorname{erf}[k(r, \theta)]) \right) d\theta \end{aligned} \quad (21)$$

where

$$\begin{aligned} k(r, \theta) &= \frac{b_1}{\sqrt{2\sigma_1^2 \sigma_2^2 g(\theta)}} \left[ r \cos(\theta) \sin(\theta) (\sigma_1^2 - \sigma_2^2) \right. \\ &\left. + A (\sigma_2^2 \sin(\theta) \cos(\theta_0) - \sigma_1^2 \cos(\theta) \sin(\theta_0)) \right]. \end{aligned} \quad (22)$$

Notice that (21) reduces to the special cases given in Table I. Finally, it remains to deduce the ADO (or ADF) denoted by  $T_R(r)$ . The ADO  $T_R(r)$ , which is defined as the average time during which  $R(t)$  remains below the threshold level  $r$ , is obtained by using both the OP and the ARO according to

$$T_R(r) = \frac{F_R(r)}{N_R(r)}. \quad (23)$$

Thus, by substituting (8) and (21) in (23), the ADO  $T_R(r)$  of the non-isotropic Beckmann channel model can be evaluated.

We conclude this section by mentioning that the extraction of the outage performance metrics in terms of the signal-to-noise ratio (SNR) in noise-limited systems is straightforward. The instantaneous SNR per bit is defined as  $\eta(t) = R^2(t) E_b / N_0$ , where  $E_b$  is the energy per bit and  $N_0$  is the noise PSD. If we impose on the Beckmann model that  $E\{R^2(t)\} = 1$ , then the ARO  $N_\eta(\eta_0)$  and the ADO  $T_\eta(\eta_0)$  are obtained from (21) and (23), respectively, by setting  $r = \sqrt{\eta_0 / \bar{\eta}}$ , with  $\eta_0$  being the SNR threshold and  $\bar{\eta}$  denoting the average SNR.

#### V. NUMERICAL AND SIMULATION RESULTS

In this section, numerical examples in comparison with the corresponding simulation data are presented for the derived statistics of the non-isotropic Beckmann fading channel model. We consider an M2M multipath propagation scenario, where both the transmitter and the receiver can be in motion. The angle of departure (AOD) and angle of arrival (AOA) of the multipath components are both assumed to follow the von Mises distribution given by [9]

$$p_{\alpha_i}(\alpha_i) = \frac{\exp(k_i \cos(\alpha_i - \phi_i))}{2\pi I_0(k_i)} \quad (24)$$

TABLE I. SPECIAL CASES OF THE STATISTICS OF THE NON-ISOTROPIC BECKMANN FADING CHANNEL MODEL.

Channel Model	Conditions		Related Available Results				
			$p_{R\dot{R}\theta\theta}(z, \dot{z}, \theta, \dot{\theta})$	$p_{R\dot{R}}(z, \dot{z})$	$p_{\dot{R}}(\dot{z})$	$N_R(r)$	
Beckmann fading	$\sigma_1^2 \neq \sigma_2^2$ , $\beta_1 \neq \beta_2$ , and $A \neq 0$	Non-Isotropic	$b_1 \neq 0$	Eq. (11)	Eq. (12)	Eq. (16)	Eq. (21)
		Isotropic	$b_1 = 0$	[6, Eq. (7)]	[6, Eq. (15)]	can be obtained from Eq. (16)	[6, Eq. (16)]
Rice fading	$\sigma_1^2 = \sigma_2^2$ , $\beta_1 = \beta_2$ , and $A \neq 0$	Non-Isotropic	$b_1 \neq 0$	[11, Eq. (20)]	[11, Eq. (27)]	can be obtained from Eq. (16)	[11, Eq. (29)]
		Isotropic	$b_1 = 0$	[16, Eq. (37)]	[16, Eq. (41)]	[16, Eq. (43)]	[17, Eq. (4.8)]
Hoyt fading	$\sigma_1^2 \neq \sigma_2^2$ , $\beta_1 \neq \beta_2$ , and $A = 0$	Non-Isotropic	$b_1 \neq 0$	[15, Eq. (5)]	[15, Eq. (6)]	[15, Eq. (7)]	[15, Eq. (11)]
		Isotropic	$b_1 = 0$	[18, Eq. (10)]	[18, Eq. (11)]	[18, Eq. (12)]	[18, Eq. (13)]
Rayleigh fading	$\sigma_1^2 = \sigma_2^2$ , $\beta_1 = \beta_2$ , and $A = 0$	Non-Isotropic	$b_1 \neq 0$	[17, Eq. (4.5)]	[17, Eq. (4.6)]	[20, Eq. (29)]	[12, Eq. (3.1.31)]
		Isotropic	$b_1 = 0$	[12, Eq. (1.3.33)]	[12, Eq. (1.3.34)]	[12, Eq. (1.3.34)]	[12, Eq. (1.3.35)]

where  $I_0(\cdot)$  is the zeroth-order modified Bessel function of the first kind [10, Eq. (8.406.1)] and the subscript  $i \in \{T, R\}$  is used to distinguish between the transmitter ( $T$ ) and the receiver ( $R$ ). Therefore, in (24)  $\alpha_T$  ( $\alpha_R$ ) denotes the AOD (AOA) and  $\phi_T$  ( $\phi_R$ ) represents the mean value of the random variable  $\alpha_T$  ( $\alpha_R$ ). Furthermore,  $k_T$  ( $k_R$ ) controls the angular spread of the AOD  $\alpha_T$  (AOA  $\alpha_R$ ). For the special case given by  $k_i = 0$ , we obtain the isotropic scattering environment in which the corresponding AOD and AOA are uniformly distributed, i.e.,  $p_{\alpha_i}(\alpha_i) = 1/2\pi$ ,  $i \in \{T, R\}$ . Recall that the derived statistical metrics are functions of the first and second spectral moments defined by the parameters  $b_1$ ,  $\beta_1$ , and  $\beta_2$ . Expressions for these parameters are given by [15]

$$b_1 = 2\pi \sqrt{\sigma_1^2 \sigma_2^2} f_{R,\max} \left( \frac{a \cos(\phi_T) I_1(k_T)}{I_0(k_T)} + \frac{\cos(\phi_R) I_1(k_R)}{I_0(k_R)} \right) \quad (25)$$

$$\beta_1 = 2\pi^2 \sigma_1^2 f_{R,\max}^2 \left( 4a \cos(\phi_T) \cos(\phi_R) \frac{I_1(k_T) I_1(k_R)}{I_0(k_T) I_0(k_R)} + \frac{a^2 \cos(2\phi_T) I_2(k_T)}{I_0(k_T)} + a^2 + \frac{\cos(2\phi_R) I_2(k_R)}{I_0(k_R)} + 1 \right) \quad (26)$$

and

$$\beta_2 = \frac{\sigma_2^2}{\sigma_1^2} \beta_1 \quad (27)$$

where  $a = f_{T,\max}/f_{R,\max}$ , in which  $f_{T,\max}$  ( $f_{R,\max}$ ) represents the maximum Doppler frequency caused by the motion of the transmitter (receiver). In (25) and (26),  $I_m(\cdot)$ ,  $m = 1, 2$ , denotes the  $m$ th-order modified Bessel function of the first kind. For the case of isotropic scattering, i.e.,  $k_T = k_R = \phi_T = \phi_R = 0$ , it can be noticed that  $b_1 = 0$  and  $\beta_1 = 2\pi^2 \sigma_1^2 f_{R,\max}^2 (1 + a^2)$ .

The concept of the sum-of-cisoids model was used to simulate the Gaussian processes  $\mu_1(t)$  and  $\mu_2(t)$ . Concerning the determination of the corresponding simulation parameters, we employed the generalized method of equal areas [21]. For the design of the simulation model, we used the following parameters: the number of cisoids is  $N = 25$ , the maximum Doppler frequency experienced at the receiver is fixed to  $f_{R,\max} = 90$  Hz. If not otherwise stated, the variances of the Gaussian processes  $\mu_1(t)$  and  $\mu_2(t)$  are set to  $\sigma_1^2 = 1$  and  $\sigma_2^2 = 0.2$ , respectively.

Fig. 1 shows the OP (or the CDF)  $P_{\text{out}}(r)$  in (8) for various values of the amplitude  $A$  of the LOS component and the

variances  $\sigma_1^2$  and  $\sigma_2^2$ . The good match between the theoretical and simulation results of the OP confirms the validity of the derived expression in (8). Fig. 2 illustrates the theoretical and simulated PDFs  $p_{\dot{R}}(\dot{z})$  of the rate of change  $\dot{R}(t)$  for different values of  $A$  under non-isotropic conditions. Again, as can be seen, there is an excellent agreement between theory and simulation which demonstrates the correctness of the derived expression in (16). From this figure, it can be seen that the amplitude  $A$  has a significant influence on the shape of the PDF  $p_{\dot{R}}(\dot{z})$ . Fig. 3 depicts the ARO (or the LCR)  $N_R(r)$  for  $A = 1$  and different non-isotropic scattering scenarios. As expected, the angular spread, controlled by  $k_T$  and  $k_R$ , has an important impact on the behaviour of the LCR  $N_R(r)$ . Finally, Fig. 4 illustrates the influence of the amplitude  $A$  of the LOS component on the ADO  $T_R(r)$  of non-isotropic Beckmann fading channel model, for  $a = 1$ ,  $k_T = k_R = 10$ , and  $\phi_T = \phi_R = 0$ . As can be observed, increasing the value of  $A$  results in a significant decrease in the ADO.

## VI. CONCLUSION

In this paper, we investigated the outage statistics of the Beckmann fading channel model under non-isotropic scattering conditions. Specifically, expressions for the OP, ARO, and ADO have been derived together with the PDF of the rate of change of the Beckmann process. It has been shown that all the obtained new analytical results encompass a variety of known results from related multipath fading channel model, under isotropic and non-isotropic conditions, which appear as special cases of the general Beckmann fading channel model. Moreover, the validity of the determined theoretical results has also been checked by means of computer simulations of non-isotropic M2M propagation scenarios, where an excellent agreement between theory and simulation has been observed.

## REFERENCES

- [1] M. Pätzold, *Mobile Radio Channels*, 2nd ed. John Wiley, 2011.
- [2] P. Beckmann and A. Spizzichino, *The Scattering of Electromagnetic Waves from Rough Surfaces*, Norwood, MA: Artech House, 1987.
- [3] Y. Xie and Y. Fang, "A general statistical channel model for mobile satellite systems," *IEEE Trans. Veh. Technol.*, vol. 49, no. 3, pp. 744–752, May 2000.
- [4] O. Filio-Rodríguez, V. Kontorovich, S. Primak, and F. Ramos-Alarcon, "Collaborative spectrum sensing for cognitive radio: Diversity combining approach," *Wireless Sensor Network*, vol. 3, no. 1, pp. 24–37, Jan. 2011.
- [5] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, Wiley-Interscience, 2nd ed., 2005.



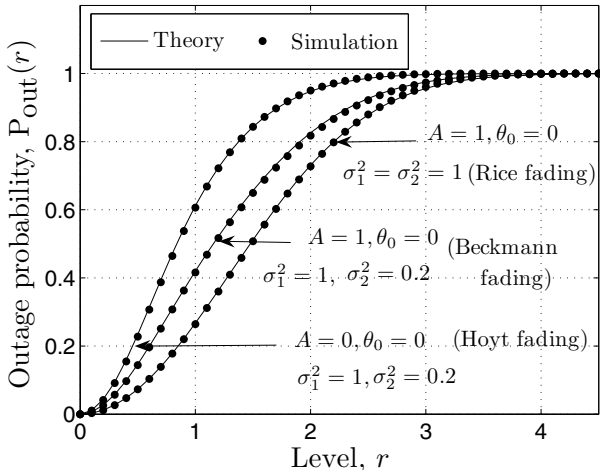


Fig. 1. The OP  $P_{\text{out}}(r)$  of the non-isotropic Beckmann fading channel model for different values of  $A$ ,  $\sigma_1^2$ , and  $\sigma_2^2$ .

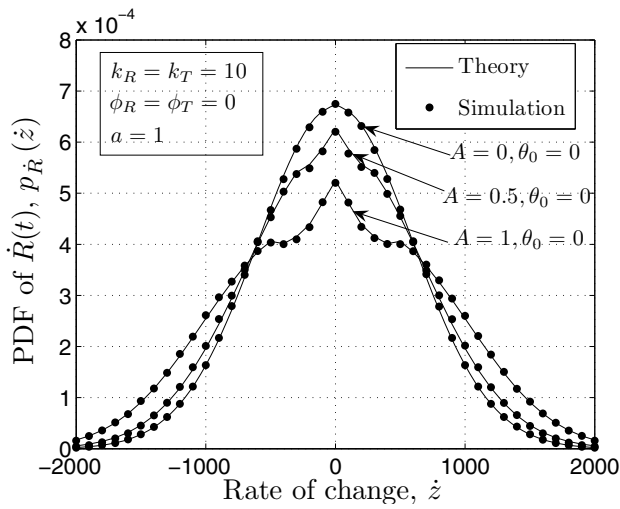


Fig. 2. The PDF  $p_{\dot{R}}(\dot{z})$  of  $\dot{R}(t)$  of the non-isotropic Beckmann fading channel model for different values of  $A$ .

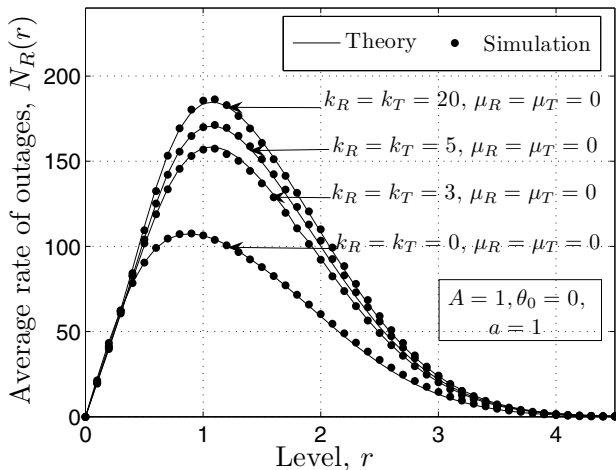


Fig. 3. The ARO  $N_R(r)$  of the non-isotropic Beckmann fading channel model for different values of  $k_T$ ,  $k_R$ ,  $\phi_T$ , and  $\phi_R$ .

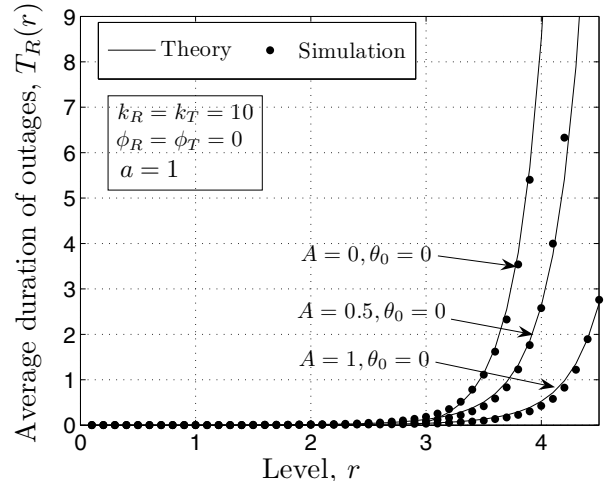


Fig. 4. The ADO  $T_R(r)$  of the non-isotropic Beckmann fading channel model for different values of  $A$ .

*Proc. IEEE 59th Vehicular Technology Conference (VTC'04-Spring)*, Milan, Italy, vol. 1, pp. 162–165, May 2004.

- [7] M. D. Petković and M. Č. Stefanović, “On the phase crossing statistics and random FM noise in generalized Rice fading channels,” *Journal of Electrical Engineering*, vol. 63, no. 1, pp. 41–46, 2012.
- [8] P. Dharmawansa, N. Rajatheva, and C. Tellambura, “Envelope and phase distribution of two correlated Gaussian variables,” *IEEE Trans. Commun.*, vol. 57, no. 4, pp. 915–921, Apr. 2009.
- [9] A. Abdi, J. A. Barger, and M. Kaveh, “A parametric model for the distribution of the angle of arrival and the associated correlation function and power spectrum at the mobile station,” *IEEE Trans. Veh. Technol.*, vol. 51, no. 3, pp. 425–434, May 2002.
- [10] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, New York: Academic Press, 7th ed., 2007.
- [11] M. Pätzold, U. Killat, and F. Laue, “An extended Suzuki model for land mobile satellite channels and its statistical properties,” *IEEE Trans. Veh. Technol.*, vol. 47, no. 2, pp. 617–630, May 1998.
- [12] W. C. Jakes, *Microwave Mobile Communications*, New York: Wiley, 1974.
- [13] A. Annamalai, C. Tellambura and J. Matyjas, “A new twist on the generalized Marcum  $Q$ -function  $Q_M(a, b)$  with fractional-order  $M$  and its applications,” *Proc. 6th IEEE Consumer Communications and Networking Conference (CCNC'09)*, pp. 1–5, Jan. 2009.
- [14] J. F. Paris and D. Morales-Jiménez, “Outage probability analysis for Nakagami- $q$  (Hoyt) fading channels under Rayleigh interference,” *IEEE Trans. Wireless. Commun.*, vol. 9, no. 4, pp. 1272–1276, Apr. 2010.
- [15] W. Dahech, N. Hajri, N. Youssef, M. Pätzold, and T. Kawabata, “Level-crossing rate and average duration of fades in non-isotropic Hoyt fading channels with applications to selection combining diversity,” *Proc. IEEE 82nd Vehicular Technology Conference (VTC'15-Fall)*, Boston, USA, Sep. 2015, accepted for publication.
- [16] S. A. Fasolo and R. S. Duque, “On time derivative of Yacoub’s  $\kappa - \mu$  envelope,” *15th Virginia Tech/MPRG Symposium on Wireless Personal Communications*, Blacksburg, pp. 71–78, Jun. 2005.
- [17] S. O. Rice, “Statistical properties of a sine wave plus random noise,” *Bell Syst. Tech. J.*, vol. 27, pp. 109–157, Jan. 1948.
- [18] N. Youssef, C. X. Wang, and M. Pätzold, “A study on the second order statistics of Nakagami-Hoyt mobile fading channels,” *IEEE Trans. Veh. Technol.*, vol. 54, no. 4, pp. 1259–1265, Jul. 2005.
- [19] S. O. Rice, “Mathematical analysis of random noise,” *Bell Syst. Tech. J.*, vol. 23, pp. 282–332, Jul. 1944.
- [20] A. Abdi, “On the second derivative of a Gaussian process envelope,” *IEEE Trans. Inf. Theory*, vol. 48, no. 5, pp. 1226–1231, May 2002.
- [21] C. A. Gutiérrez and M. Pätzold, “The generalized method of equal areas for the design of sum-of-cisoids simulators for mobile Rayleigh fading channels with arbitrary Doppler spectra,” *Wirel. Commun. Mob. Comput.*, vol. 13, no. 10, pp. 951–966, Jul. 2013.

- [6] N. Youssef, C. X. Wang, M. Pätzold, I. Jaafar, and S. Tabbane, “On the statistical properties of generalized Rice multipath fading channels,”