Efficient Ray Tracing Algorithm with the Avoidance of Duplicate Image Generation

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***Abstract*—In image-based ray tracing, the possible permutations of reflectors (walls, doors, and sides of pieces of furniture) are all chosen to exhaustively generate images of a signal source and, for each generated image, the existence of an actual ray path that leaves the image and reaches the destination is investigated. In a typical indoor environment, where the reflectors are arranged parallel or perpendicular to each other, permutations of reflectors generate many image duplicates; that is, different permutations of reflectors generate the same image. Focusing on this fact, this paper proposes a novel technique for accelerating the image-based ray tracing. The key idea of the proposal is to choose a limited set of the permutations of reflectors to generate necessary and sufficient images without duplication. The proposed technique avoids the redundant ray-path search to reduces the run time of ray tracing without degrading the accuracy. The simulation experiments verify that the proposed technique makes the ray tracing much less demanding of computation.**

I. Introduction

In indoor environments, the multiple echoes of the transmission signal often reaches the receiver through various paths because of many reflectors, such as walls, doors, and sides of pieces of furniture. These multiple echoes arriving with various delays may cancel or distort the received signal. This is often called *multipath problem*. There has been a strong need for predicting radio-wave propagation to characterize the multipath problem in indoor environments with the significant growth of indoor wireless communications like wireless LANs.

Ray-tracing techniques have been widely applied to predict radio-wave propagation using site-specific information. Ray tracing approximates the scattering of electromagnetic waves on the walls or other indoor structures by simple reflection, refraction, and diffraction. Ray tracing techniques are categorized into two types of methods; ray-launching method and image-based method. The ray-launching method [1]–[4] launches a bundle of rays from the signal source in a solid angle and finds true ray paths to a given destination by looking for rays arriving at the receiver. The image-based method [5]–[12] searches possible ray paths to the destination by constructing the images of the signal source. The former offers lower computational complexity than the latter, but it is less precise because finding all paths to the destination is difficult especially if the receiver is far from the signal source. The latter is more precise because it exhaustively takes into account of all paths to the destination up to a given number of reflections but its computational complexity increases exponentially with the number of indoor reflectors.

Reducing the computational time of the ray-tracing method is a challenging problem, and several acceleration techniques have been proposed in the literature. Intersection calculations dominate the run time of ray tracing, and a common algorithm to reduce the number of intersection tests is the intersection of rays with a tree-like list of possible interaction objects, rather than the whole database of objects. This reduction can be conducted, for example, using the concept of bounding volumes, which are simple geometric objects that surrounds the objects of the environments in a tree-like manner [13]. Acceleration techniques using Binary Space Partitioning (BSP) algorithm [14], Space Volumetric Partitioning (SVP) [15], or visible trees [16] are categorized into this type. Imai proposed a ray-tracing acceleration technique, which selects indoor structures where rays are likely to reflect employing the genetic algorithm [17]. Azpilicueta *et al.* used a neural network to predict the necessary and sufficient number of rays to be launched for ray-launching methods [18].

In the image-based method [19], an one-reflection image of a signal source is generated for each reflector (wall, door, or furniture). Then, a two-reflection image is generated for each combination of one-reflection images and reflectors, and so forth. This image generation is equivalent to the reflector-permutation based generation, where a permutation of reflectors is first chosen and the image corresponding to the chosen reflector permutation is generated. There is the one-to-one correspondence between rays and images and, for each generated image, the image-based method searches the existence of an actual ray path that leaves the image and reaches the destination. In a typical indoor environment, where the reflectors are arranged parallel or perpendicular to each other, permutations of reflectors generate many image duplicates; that is, different permutations of reflectors generate the same image. Focusing on this fact, this paper proposes a novel technique for accelerating the image-based ray tracing. The key idea of the proposal is to construct a limited set of the permutations of reflectors that generate necessary and sufficient images without duplication. This technique avoids redundant ray-path search and thus significantly reduces the run time of the ray tracing. The simulation experiments verify that the proposed technique is much less demanding of computation.

The algorithm proposed in this paper is a revised version of the algorithm proposed in our previous paper [20]. The algorithm in the previous paper is not applicable to cases where some of reflectors are not arranged parallel nor perpendicular to other reflectors, but the algorithm in this paper can be applied without exception.

This paper is organized as follows. In Sec. II, we explains the outline of the proposal. In Sec. III, we show several numerical experiments to shows the effectiveness of the proposal. In Sec. IV, we conclude the paper.

II. Proposal

A. Image-based ray tracing method

Consider an indoor environment, in which N reflectors (walls, doors, windows, and sides of pieces of furniture) of wireless signals exist. Image-based ray tracing method first selects a permutation of the reflectors to generate an image and investigates whether an actual ray path exists or not for the generated image. Let w_i denote the ith reflector ($i = 1, \ldots, N$). Ray-paths with up to n_{th} times of reflection are searched in the following three steps.

Step 1: Choose a permutation composed of n reflectors $(w_{p_1}, \ldots, w_{p_n})$ $(n \leq n_{th})$.
Step 2: Generate the image corresponding to the chosen permutation.
Step 3: Verify the existence of an actual ray path that leaves the image, reflects on the reflectors in the order of $(w_{p_1}, \ldots, w_{p_n})$, and finally reaches the destination.

Permutation $(w_{p_1}, w_{p_2}, \ldots, w_{p_n})$ expresses the order of reflections; the firstly used reflector is w_{p_1}, the secondly used reflector is w_{p_2}, and so forth. Note that each reflector can be used multiple times for constructing a permutation; that is, w_{p_i} and w_{p_j} can be the same reflector. The procedure explained in the above is repeated for all permutations of reflectors satisfying $n \leq n_{th}$. The number of permutations of reflectors to explore the existence of ray exponentially increases as n_{th} increases.

B. Key Idea

In indoor environments, the walls and doors are often arranged parallel or perpendicular to each other (Fig.1). If the reflectors are perpendicular to each other, the image of a signal source corresponding to a multiple-times reflection does not depend on the order of reflection. For example, as shown in Fig. 2, a two-reflection image generated by permutation (w_1, w_2) is the same as the one generated by permutation (w_2, w_1). An actual ray exists only for permutation (w_1, w_2); the first reflector is wall 1, the second reflector is wall 2, and the ray reaches the receiver. As this simple example indicates, if the reflectors are arranged perpendicular to each other, more than one permutation of the reflectors generate the same image and, among those, at most one permutation has the corresponding actual path of a ray. Since ray-path search – verification of the existence of an actual ray path – is conducted for each generated image, the duplicate image generation causes the redundant ray-path search.

Once an image is generated, the ray path leaving the generated image and reaching the destination can be traced without using the information on the permutation of reflectors (the order of reflectors). Figure 2 shows the procedure of ray-path tracing without using the information about the order of reflectors. First, connect the image and the receiver with a straight line and find the intersection of the line with one of walls. The obtained intersection ($X2$) is the reflecting point closest to the receiver. Once the closest reflecting point ($X2$) is determined, the ray path until reaching $X2$ is obtained. The intersection of the ray path until reaching $X2$ with one of walls gives the reflecting point ($X1$) just prior to $X2$. By repeating this operation, a ray path connecting the sender and the receiver is traced. Thus, we do not need to find permutations that have the corresponding actual ray path; rather, we just need to exhaustively generate all images without duplication.

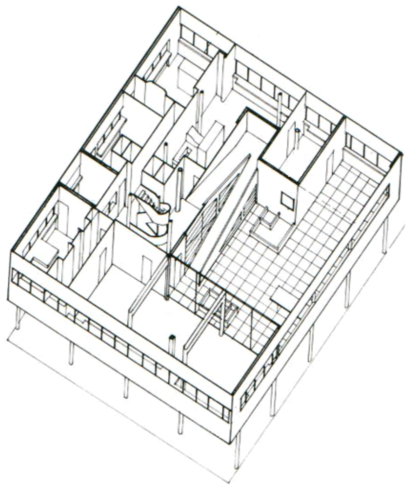

Fig. 1. An Example of the Indoor Environments [21].

Based on this observation, this paper proposes an algorithm for choosing a limited number of permutations of reflectors so as to exhaustively generate images without duplication. The algorithm avoids the redundant ray-path search and accelerates the image-based ray tracing without degrading the accuracy of prediction.

C. Case 1: All Reflectors are Arranged Parallel or Perpendicular

First, we consider the simplest case where all reflectors are arranged parallel or perpendicular. First, we classify all reflectors into the following three sets (Fig. 3).

Set 1: set of reflectors parallel to the y-z plane
Set 2: set of reflectors parallel to the x-z plane
Set 3: set of reflectors parallel to the x-y plane

Let $I(w_i)$ denote the ID of a set that reflector w_i belongs to ($I(w_i)$ = 1,2, or 3). For example, in Fig. 3, $I(w_2) = I(w_5) = 1$, $I(w_1) = I(w_6) = 2$, and $I(w_3) = I(w_4) = 3$.

Let construct the permutations of n reflectors $(w_{p_1}, w_{p_2}, \ldots, w_{p_n})$ to exhaustively generate n-reflection images. When the number of reflectors is N, the number of permutation is $N(N-1)^{n-1}$ under the constraint $w_{p_i} \neq w_{p_{i+1}}$ for $i = 1, \ldots, n-1$. This constraint comes from the consideration that a ray does not consecutively reflect on the same reflector. Even with this constraint, the number of permutation exponentially grows with n. The

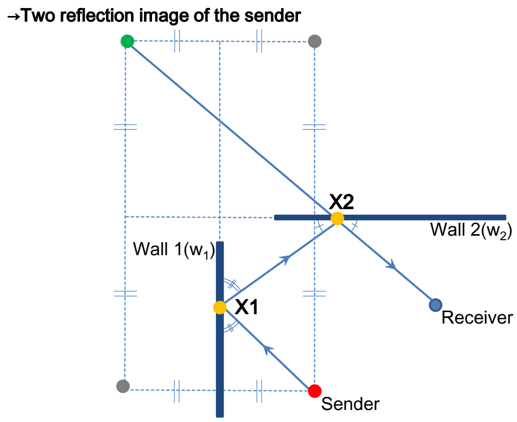

Fig. 2. Duplicate Image Generation of the Signal Sender.

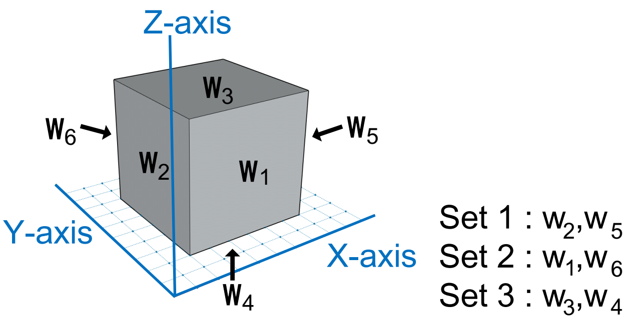

Fig. 3. Classification of Reflectors (Case 1).

proposed algorithm conducts the ray-path search only for the permutations satisfying the following additional constraint:

$$I(w_{p_i}) \leq I(w_{p_{i+1}}), \quad \text{for } i = 1, \ldots, n-1. \tag{1}$$

For example, in Fig. 3, (w_2, w_1, w_6) and (w_1, w_6, w_3) are chosen for generating the images, but (w_1, w_3, w_6) is not chosen. This is because the image of (w_1, w_3, w_6) is the same with the one of (w_1, w_6, w_3).

Constraint (1) greatly reduces the number of permutation. For example, in the case of Fig. 3, the number of possible permutations of n walls is $6 \cdot 5^{n-1}$, while the number of permutations satisfying (1) is $4n^2 + 2$ [20]. That is, the exponential-order dependence on n is alleviated to the polynomial order dependence by the introduction of (1). We have mathematically proven that permutations satisfying (1) exhaustively generate all images without duplication [20].

D. Case 2: General Case

Next, consider general cases where some of reflectors are not arranged parallel nor perpendicular to other reflectors (Fig. 4). In these cases, all reflectors are classified into the following four sets:

Set 1: set of reflectors parallel to the y-z plane
Set 2: set of reflectors parallel to the x-z plane
Set 3: set of reflectors parallel to the x-y plane
Set 4: set of reflectors not belonging to Sets 1 to 3.

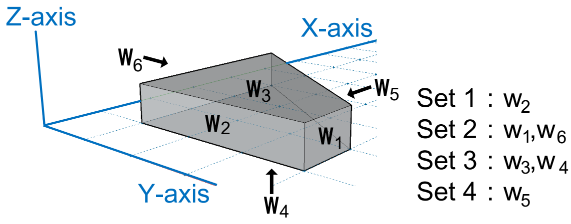

Fig. 4. Classification of Reflectors (Case 2).

TABLE I. Number of Permutations (Fig. 4)

n_{th}	w/o constraint (2)	w/ constrain (2)	Ratio
1	6	6	1.0000
2	36	28	0.7778
3	186	102	0.5484
4	936	352	0.3761
5	4686	1198	0.2557
6	23436	4060	0.1732
7	117186	13742	0.1173
8	585936	46496	0.0794
9	2929686	157302	0.0537
10	14648436	532156	0.0363

The proposed algorithm conducts the ray-path search only for the permutations satisfying the following constraint:

$$\text{If } I(w_{p_i}) \neq 4,\ I(w_{p_i}) \leq I(w_{p_{i+1}}), \quad \text{for } i = 1, \ldots, n-1, \tag{2}$$

For example, permutation (w_1, w_5, w_6) is chosen for image generation in Fig. 4, while it is not in Fig. 3.

The number of permutations satisfying (2) can be recursively obtained. To show this, let $g(n, k)$ denote the number of permutations composed of n reflectors, $(w_{p_1}, \ldots, w_{p_n})$, that satisfies (2) and $I(w_{p_n}) = k$. It is easy to see that

$$\begin{aligned} g(n+1,1) &= (N_1 - 1)g(n,1) + N_1 g(n,4), \\ g(n+1,2) &= (N_2 - 1)g(n,2) + N_2(g(n,1) + g(n,4)), \\ g(n+1,3) &= (N_3 - 1)g(n,3) \\ &\quad + N_3(g(n,1) + g(n,2) + g(n,4)), \\ g(n+1,4) &= (N_4 - 1)g(n,4) \\ &\quad + N_4(g(n,1) + g(n,2) + g(n,3)), \end{aligned} \tag{3}$$

and $g(1, i) = N_i$ for $i = 1, \ldots, 4$, where N_i is the number of reflectors belonging to Set i. The total number of permutations of n reflectors satisfying (2), $g(n)$, is given as

$$g(n) = \sum_{k=1}^{4} g(n,k).$$

Table I shows the number of permutations composed of up to n_{th} reflectors when the reflectors are arranged as shown in Fig. 4. Table I shows that constraint (2) significantly reduces the number of permutations of reflectors to be explored for ray-path search. Since the ray-path search dominate the run time of ray tracing, imposing constraint (2) should largely reduce the run time, which will be verified in the next section.

Remark. *The number of permutation required for the ray-path search can be further reduced by considering the direction of the ray. When wall 3 is chosen as the first reflector as shown in*

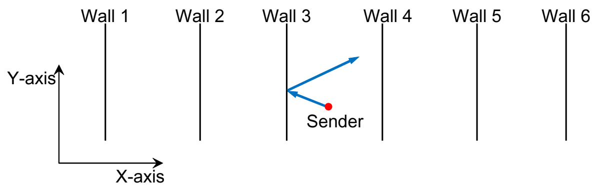

Fig. 5. How to find the wall that exists in the direction of the ray.

TABLE II. SIMULATION CONDITIONS

	Detail
Height of the transmitter	2 m
Height of the receiver	1.8 m
Frequency	2.4 GHz
Antenna gain	1
Antenna directionality	Non-directional antenna
Polarized wave	Transverse magnetic
Material of the structure	Concrete
Thickness of the structure	10 cm

Fig. 5, the second reflector should be chosen among walls 4, 5 and 6 because they are in the direction of the ray after the ray is first reflected on wall 3. Thus, the existence of the ray for permutation (Wall 3, Wall 1, . . .) or (Wall 3, Wall 2, . . .) does not need to be checked. Since we have reduced the number of permutations in the ray-path search as noted above, the numbers of permutations in Tables IV and VI in Sec. III are smaller than those obtained by recursive formula (3).

III. EVALUATION

A. Simulation Conditions

In order to validate the efficiency of the proposed acceleration technique, we implemented a ray-tracing program by C language, which works in a three-dimensional space, and evaluated the run time required for predicting the radio wave propagation in two scenarios: Scenario 1 (Fig. 6), where all walls are arranged parallel or perpendicular to each other, and Scenario 2 (Fig. 7), where several walls are not parallel nor perpendicular to other walls. Simulation conditions are summarized in Table II. The implemented program run on a Linux machine (OS ubuntu10.04 LTS).

B. Simulation Results: Scenario 1

First, we evaluated the run time required for the evaluation of the signal strength at the received point (R_x) when the signal was transmitted from the source (T_x) in Fig. 6. (Fir reference, the signal strength of the environment is depicted in Fig. 8). In the evaluation of signal strength, we considered only rays reaching the received point with up to n_{th} times of reflections and penetrations. The results are summarized in Table III, where the results without using the proposed acceleration technique is also shown for the comparison. The table shows that the proposed acceleration technique significantly reduces the run time when the allowable number of reflections and penetrations, n_{th}, is large. In particular, the run time is reduced to its one-tenth or less when $n_{th} \geq 5$.

The proposed acceleration techniques aims at reducing the number of ray-path searches because the ray-path search dominate the run time of ray tracing. We summarized the

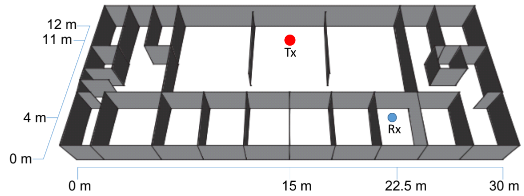

Fig. 6. View of the Scenario for Simulation (Scenario 1).

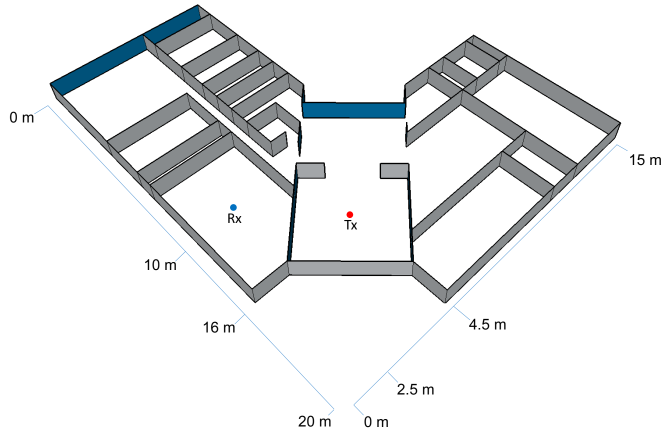

Fig. 7. View of the Scenario for Simulation (Scenario 2).

numbers of ray path searches conducted during the evaluation of the signal strength in Table IV, showing that the number of ray-path searches was more largely reduced as n_{th} increases. We also see that the reduction ratio of the run time was almost equal to the reduction ratio of the number of ray searches (Fig. 9). This result verifies that the ray-path search dominates the run time of ray tracing, and aiming at reducing number of ray-path search is very reasonable for making the ray tracing light weight.

C. Simulation Results: Scenario 2

Next, we show the results of Scenario 2. Figure 10 shows the strength of the signal in the floor when the signal was transmitted from the point T_x in Fig. 7. The run time required for estimating the strength of the signal at the received point (R_x) is summarized in Table V. The proposed acceleration technique is effective also for the environments where some of walls are not arranged parallel nor perpendicular to other walls like Scenario 2 although the reduction effect is less significant than Scenario 1. Table VI summarizes results in terms of the number of ray-path searches. The table shows that the number of ray-path searches is significantly reduced when n_{th} is large, and that the reduction ratio of the number of ray searches is also consistent with the reduction ratio of the run time as the case of Scenario 1 (Fig. 11).

IV. CONCLUSION

In this paper, we have proposed a novel acceleration technique for image-based ray tracing. The key idea of the proposal is to choose a limited set of the permutations of reflectors to generate necessary and sufficient images without

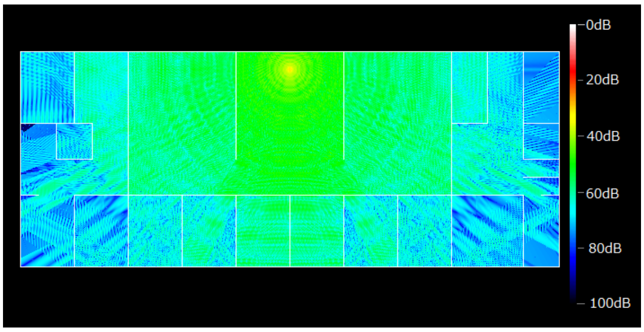

Fig. 8. Color Map of Signal Strength (Scenario 1).

TABLE III. RUN TIME (SCENARIO 1)

n_{th}	w/o Acceleration [s]	w/ Acceleration [s]	Ratio
1	0.0011	0.0010	0.9091
2	0.0020	0.0019	0.9500
3	0.0347	0.0205	0.5908
4	0.8608	0.1890	0.2196
5	29.3780	2.2130	0.0753
6	1008.8200	27.8170	0.0276
7	34145.1860	344.1110	0.0101

TABLE IV. NUMBER OF RAY-PATH SEARCHES (SCENARIO 1).

n_{th}	w/o Acceleration	w/ Acceleration	Ratio
1	31	31	1.0000
2	961	595	0.6191
3	28861	8479	0.2938
4	865861	107032	0.1236
5	25975861	1274227	0.0491
6	779275861	14734766	0.0189
7	23378275861	167506800	0.0072

duplication. The proposed technique avoids the redundant ray-path search and thus significantly reduces the run time of ray tracing. The simulation experiments verify that the proposed technique makes the ray tracing much less demanding of computation.

In our previous paper [20], we proposed the other acceleration technique, which excludes walls or other building structures far away from the signal source when constructing the permutations of reflectors for ray-path search. Applying this acceleration technique to environments where some of reflectors are not arranged parallel nor perpendicular to other reflectors remains as a future work.

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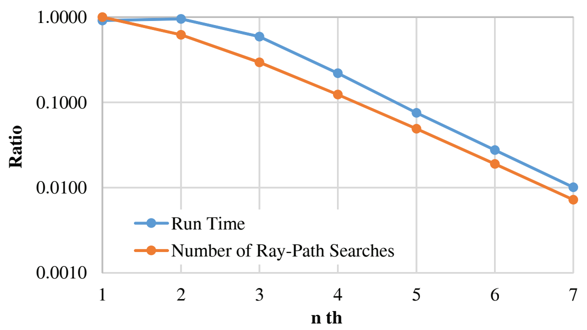

Fig. 9. Ratio of Run Time and Number of Ray-Path Searches (Scenario 1).

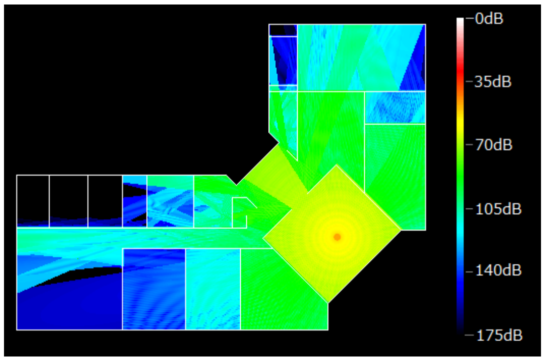

Fig. 10. Color Map of Signal Strength (Scenario 2).

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TABLE V. RUN TIME (SCENARIO 2).

n_{th}	w/o Acceleration [s]	w/ Acceleration [s]	Ratio
1	0.0010	0.0010	1.0000
2	0.0021	0.0020	0.9524
3	0.0560	0.0435	0.7768
4	1.9880	1.2183	0.6128
5	88.0770	39.6980	0.4507
6	4775.0000	1276.9810	0.2674

TABLE VI. NUMBER OF RAY-PATH SEARCHES (SCENARIO 2).

n_{th}	w/o Acceleration	w/ Acceleration	Ratio
1	40	40	1.0000
2	1522	1162	0.7635
3	57838	33120	0.5726
4	2197846	946078	0.4305
5	83518150	27026420	0.3236
6	3173689702	772012701	0.2433

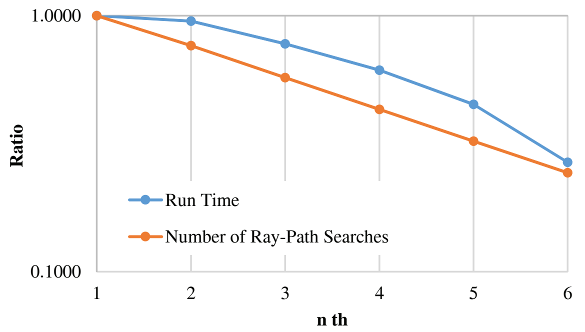

Fig. 11. Ratio of Run Time and Number of Ray-Path Searches (Scenario 2).

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