

# Generalization of Even-Shift Orthogonal Sequences to Multi-Dimension

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**Abstract:** The even-shift orthogonal sequence (E-sequence) is a binary sequence, whose out-of-phase aperiodic auto-correlation function takes zero at any even shift. This paper considers the generalization of E-sequence to multi-dimension. It is shown that multi-dimensional E-sequences can be constructed by multi-dimensional complementary sequences. Especially a logic function generating multi-dimensional E-sequences of power-of-two length is formulated, which can give multi-dimensional E-sequences with a good correlation property that almost 3/4 of all shifts is zero.

## 1. Introduction

Even-shift orthogonal sequences (E-sequences) are binary sequences whose out-of-phase aperiodic auto-correlation function takes zero at any even shift [1] - [4]. E-sequences have been applied for spread spectrum communication.

In this paper we consider the generalization of E-sequences to multi-dimension. We clarify the construction of multi-dimensional E-sequences of  $S2^\ell$  with  $S = 1, 10, 26$  and  $\ell \geq 1$ , and formulate a logic function generating them of length  $2^\ell$ . In addition, their aperiodic auto-correlation properties are investigated.

In section 2, multi-dimensional E-sequences are introduced. In section 3, it is shown that multi-dimensional E-sequences can be derived from multi-dimensional complementary sequences of half length, which mean a pair of binary sequences and the sum of these aperiodic auto-correlation function takes zero except all zero shifts [4][17]. In section 4, we formulate a logic function of multi-dimensional E-sequences, and investigate correlation properties of multi-dimensional E-sequences.

## 2. Definition of multi-dimensional E-sequences

Let  $e$  be a multi-dimensional ( $n$ -dimensional) binary sequence of length  $L = L_1 \times L_2 \times \dots \times L_n$ , expressed as

$$e = \{e_{i_1, i_2, \dots, i_j, \dots, i_n} \in \{1, -1\} | 0 \leq i_j < L_j\}.$$

The aperiodic auto-correlation function of  $e$  is written as

$$C_{ee}(\tau_1, \tau_2, \dots, \tau_n) = \sum_{i_1=0}^{L_1-1} \sum_{i_2=0}^{L_2-1} \dots \sum_{i_n=0}^{L_n-1} e_{i_1, i_2, \dots, i_n} e_{i_1+\tau_1, i_2+\tau_2, \dots, i_n+\tau_n},$$

where  $e_{i_1, i_2, \dots, i_j, \dots, i_n} = 0$  for  $i_j < 0$  and  $i_j \geq L_j$ . If the out-of-phase aperiodic auto-correlation function is zero at even shift, i.e.,

$$C_{ee}(2m_1, 2m_2, \dots, 2m_n) = \begin{cases} L & \text{for } m_1=m_2=\dots=m_n=0, \\ 0 & \text{otherwise.} \end{cases}$$

$e$  is called a multi-dimensional E-sequence, or an  $n$ -dimensional E-sequence.

## 3. Construction of multi-dimensional E-sequences

In this section, we show that  $n$ -dimensional E-sequences can be constructed from  $n$ -dimensional complementary sequences of half length.

Let  $a$  and  $b$  be  $n$ -dimensional binary sequences of length  $L = L_1 \times L_2 \times \dots \times L_n$ , expressed by

$$a = \{a_{i_1, i_2, \dots, i_j, \dots, i_n} \in \{1, -1\} | 0 \leq i_j < L_j\}, \\ b = \{b_{i_1, i_2, \dots, i_j, \dots, i_n} \in \{1, -1\} | 0 \leq i_j < L_j\}.$$

If the sum of their aperiodic auto-correlation functions can be written as

$$C_{aa}(\tau_1, \tau_2, \dots, \tau_n) + C_{bb}(\tau_1, \tau_2, \dots, \tau_n) = \begin{cases} 2L & \text{for } \tau_1 = \tau_2 = \dots = \tau_n = 0, \\ 0 & \text{otherwise,} \end{cases}$$

the pair of the sequences,  $[a, b]$ , is called multi-dimensional ( $n$ -dimensional) complementary sequences or complementary arrays [17]. They can be constructed as shown in the following theorems.

**[Theorem 1]** Let  $[a, b]$  be  $n$ -dimensional complementary sequences.

1. Interchanging  $a$  and  $b$  gives an  $n$ -dimensional complementary sequence. i.e.,  $[b, a]$ .
2. Inversion of  $a$  gives an  $n$ -dimensional complementary sequence, i.e.,  $[-a, b]$ .
3. Interchanging some axes gives  $n$ -dimensional complementary sequences,  $[\{a_{i_{k_1}, i_{k_2}, \dots, i_{k_n}}\}, \{b_{i_{k_1}, i_{k_2}, \dots, i_{k_n}}\}]$  with  $1 \leq k_m (\neq k_j) \leq n$ .
4. Reversing of  $a$  is a mate of  $b$ , i.e.,

$$[\{a_{L_1-i_1-1, L_2-i_2-1, \dots, L_n-i_n-1}\}, b].$$

5. Reversing at some axis gives complementary arrays,  $[\{a_{i_1, \dots, L_j-i_j-1, \dots, i_n}\}, \{b_{i_1, \dots, L_j-i_j-1, \dots, i_n}\}]$ .

Note that Theorem 1 can produce a lot of  $n$ -dimensional complementary sequences. For example, use of 2, 3, 2, and 3 in Theorem 1, in order, gives  $n$ -dimensional complementary sequences,  $[-a, -b]$ .

**[Theorem 2]** Let  $[a, b]$  be  $n$ -dimensional complementary sequences of length  $L$ . We have  $(n+1)$ -dimensional complementary sequences  $[\hat{a}, \hat{b}]$  of length  $2L$ , which are written as

$$\hat{a}_{i_1, i_2, \dots, i_n, i_{n+1}} = \begin{cases} a_{i_1, i_2, \dots, i_n} & \text{for } i_{n+1} = 0, \\ b_{i_1, i_2, \dots, i_n} & \text{for } i_{n+1} = 1, \end{cases} \\ \hat{b}_{i_1, i_2, \dots, i_n, i_{n+1}} = \begin{cases} a_{i_1, i_2, \dots, i_n} & \text{for } i_{n+1} = 0, \\ -b_{i_1, i_2, \dots, i_n} & \text{for } i_{n+1} = 1. \end{cases}$$

Multi-dimensional complementary sequences can be easily produced by interleaving and concatenation methods, as well as 1-dimensional complementary sequences [4][5].

**[Theorem 3]** Let  $[a, b]$  be  $n$ -dimensional complementary sequences of length  $L$ . We have  $n$ -dimensional complementary sequences  $[\hat{a}, \hat{b}]$  of length  $2L$  expressed by

$$\begin{aligned} \hat{a}_{i_1, \dots, i'_j, \dots, i_n} &= \begin{cases} a_{i_1, \dots, i_j, \dots, i_n} & \text{for } i'_j = 2i_j, \\ b_{i_1, \dots, i_j, \dots, i_n} & \text{for } i'_j = 2i_j + 1, \end{cases} \\ \hat{b}_{i_1, \dots, i'_j, \dots, i_n} &= \begin{cases} a_{i_1, \dots, i_j, \dots, i_n} & \text{for } i'_j = 2i_j, \\ -b_{i_1, \dots, i_j, \dots, i_n} & \text{for } i'_j = 2i_j + 1. \end{cases} \end{aligned}$$

**[Theorem 4]** Let  $[a, b]$  be  $n$ -dimensional complementary sequences of length  $L$ . We have  $n$ -dimensional complementary sequences  $[\hat{a}, \hat{b}]$  of length  $2L$  expressed by

$$\begin{aligned} \hat{a}_{i_1, \dots, i'_j, \dots, i_n} &= \begin{cases} a_{i_1, \dots, i_j=i'_j, \dots, i_n} & \text{for } 0 \leq i'_j < L_j, \\ b_{i_1, \dots, i_j=i'_j-L_j, \dots, i_n} & \text{for } L_j \leq i'_j < 2L_j, \end{cases} \\ \hat{b}_{i_1, \dots, i'_j, \dots, i_n} &= \begin{cases} a_{i_1, \dots, i_j=i'_j, \dots, i_n} & \text{for } 0 \leq i'_j < L_j, \\ -b_{i_1, \dots, i_j=i'_j-L_j, \dots, i_n} & \text{for } L_j \leq i'_j < 2L_j. \end{cases} \end{aligned}$$

The construction methods in Theorems 3 and 4 are well-known as interleaving and concatenation methods, respectively. We note that  $n$ -dimensional complementary sequences of length  $L = S2^\ell$  with  $S = 1, 10$  or  $26$  can be derived from 1-dimensional complementary sequences of length  $S$ , called kernels [1][4].

We give a special construction method of  $n$ -dimensional complementary sequences of length  $2^\ell$ , as the following conjecture.

**[Conjecture 1]** Let  $[a, b]$  be  $n$ -dimensional complementary sequences of length  $L = 2^\ell$ . Let  $K = 2^k \leq L$ . We have  $n$ -dimensional complementary sequences  $[\hat{a}, \hat{b}]$  of length  $2L$  expressed by

$$\begin{aligned} \hat{a}_{i_1, \dots, i'_j, \dots, i_n} &= \begin{cases} a_{i_1, \dots, i_j=m_2K+m_1, \dots, i_n} & \text{for } i'_j = 2m_2K + m_1, \\ b_{i_1, \dots, i_j=m_2K+m_1, \dots, i_n} & \text{for } i'_j = (2m_2 + 1)K + m_1, \end{cases} \\ \hat{b}_{i_1, \dots, i'_j, \dots, i_n} &= \begin{cases} a_{i_1, \dots, i_j=m_2K+m_1, \dots, i_n} & \text{for } i'_j = 2m_2K + m_1, \\ -b_{i_1, \dots, i_j=m_2K+m_1, \dots, i_n} & \text{for } i'_j = (2m_2 + 1)K + m_1, \end{cases} \end{aligned}$$

with  $0 \leq m_1 < K$ ,  $0 \leq m_2 < L/K$ ,  $0 \leq i'_j < 2L_j$ , and  $0 \leq l_k < L_k$ .

Note that combination of Theorems 1-4 and Conjecture 1 can give a lot of  $n$ -dimensional complementary sequences.

**[Theorem 5]** Let  $[a, b]$  be  $n$ -dimensional complimentary sequences of length  $L = L_1 \times \dots \times L_j/2 \times \dots \times L_n$  and  $[\hat{a}, \hat{b}]$  be  $n$ -dimensional complimentary sequences of length  $2L = L_1 \times \dots \times L_j \times \dots \times L_n$ . Each of  $\hat{a}$  and  $\hat{b}$  given by applying Theorem 3 are E-sequences. It is proved as follows.

The aperiodic auto-correlation functions of the above  $n$ -dimensional complementary sequences  $\hat{a}$  and  $\hat{b}$  can be expressed by

$$\begin{aligned} C_{\hat{a}\hat{a}}(\tau_1, \tau_2, \dots, 2m_j, \dots, \tau_n) &= C_{\hat{b}\hat{b}}(\tau_1, \tau_2, \dots, 2m_j, \dots, \tau_n) \\ &= C_{aa}(\tau_1, \tau_2, \dots, m_j, \dots, \tau_n) + C_{bb}(\tau_1, \tau_2, \dots, m_j, \dots, \tau_n) \\ &= \begin{cases} 2L & (\tau_1 = \tau_2 = \dots = m_j = \dots = \tau_n = 0) \\ 0 & \text{otherwise.} \end{cases} \end{aligned}$$

Since  $C_{\hat{a}\hat{a}}(\cdot)$  and  $C_{\hat{b}\hat{b}}(\cdot)$  are zero if either of  $\tau_1, \tau_2, \dots, \tau_n$  is even, we have the following theorem.

We show some example for the construct of E-sequence.

**[Example 1]** Let  $a$  and  $b$  be 2-dimensional complimentary sequences of length  $8 \times 2$  as expressed by

$$\begin{aligned} a &= \begin{pmatrix} + & + & + & - & + & - & + & + \\ + & + & + & - & - & + & - & - \end{pmatrix}, \\ b &= \begin{pmatrix} + & + & + & - & + & - & + & + \\ - & - & - & + & + & - & + & + \end{pmatrix}, \end{aligned}$$

where  $+$  and  $-$  are 1 and  $-1$  respectively. Theorem 3 gives 2-dimensional E-sequences of length  $8 \times 4$ .

$$\begin{aligned} \hat{a} &= \begin{pmatrix} + & + & + & - & + & - & + & + \\ + & + & + & - & + & - & + & + \\ + & + & + & - & - & + & - & - \\ - & - & - & + & + & - & + & + \end{pmatrix} \\ \hat{b} &= \begin{pmatrix} + & + & + & - & + & - & + & + \\ - & - & - & + & - & + & - & - \\ + & + & + & - & - & + & - & - \\ + & + & + & - & - & + & - & - \end{pmatrix} \end{aligned}$$

The aperiodic auto-correlation functions of  $\hat{a}$  and  $\hat{b}$  can be written as

$$\begin{aligned} C_{\hat{a}\hat{a}}(\tau_1, \tau_2) &= \begin{pmatrix} 32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -3 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & 2 & 1 & 0 & 1 & 2 & 1 \end{pmatrix}, \\ C_{\hat{b}\hat{b}}(\tau_1, \tau_2) &= \begin{pmatrix} 32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -2 & 3 & 0 & -1 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -2 & -1 & 0 & -1 & -2 & -1 \end{pmatrix}. \end{aligned}$$

**[Example 2]** Let  $a$  and  $b$  be 3-dimensional complimentary sequences of length  $4 \times 4 \times 2$  as expressed by

$$a = \begin{pmatrix} + & + & + & - & | & + & + & + & - \\ + & + & + & - & | & - & - & - & + \\ + & - & + & + & | & + & - & + & + \\ - & + & - & - & | & + & - & + & + \end{pmatrix},$$

$$b = \left( \begin{array}{ccc|ccc} + & + & + & - & - & - & + \\ + & + & + & - & + & + & - \\ + & - & + & + & - & + & - \\ - & + & - & - & - & + & - \end{array} \right).$$

Theorem 3 gives 3-dimensional E-sequences  $[\hat{a}, \hat{b}]$  of length  $4 \times 4 \times 4$

$$\hat{a} = \left( \begin{array}{ccc|ccc} + & + & + & - & + & + & + & - \\ + & + & + & - & - & - & - & + \\ + & + & - & + & + & + & - & + \\ + & + & - & + & - & - & + & - \\ + & + & + & - & + & + & + & - \\ - & - & - & + & + & + & + & - \\ - & - & + & - & - & - & + & - \\ + & + & - & + & - & - & + & - \end{array} \right)$$

$$\hat{b} = \left( \begin{array}{ccc|ccc} + & + & + & - & - & - & - & + \\ + & + & + & - & + & + & + & - \\ + & + & - & + & - & - & + & - \\ + & + & - & + & + & + & - & + \\ + & + & + & - & - & - & - & + \\ - & - & - & + & - & - & - & + \\ - & - & + & - & + & + & - & + \\ + & + & - & + & + & + & - & + \end{array} \right)$$

The aperiodic auto-correlation functions of  $\hat{a}$  and  $\hat{b}$  are

$$C_{\hat{a}\hat{a}}(\tau_1, \tau_2, \tau_3) = \left( \begin{array}{cccc|cccc} 64 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 & 6 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 & -1 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 & -2 & -4 & -2 \\ 0 & 0 & 0 & 0 & 0 & -1 & -2 & -1 \end{array} \right),$$

$$C_{\hat{b}\hat{b}}(\tau_1, \tau_2, \tau_3) = \left( \begin{array}{cccc|cccc} 64 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -5 & -6 & -1 \\ 0 & 0 & 0 & 0 & 0 & -2 & -4 & -2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 2 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 \end{array} \right).$$

#### 4. Logic functions of E-sequences

We consider the function of  $n$ -dimensional E-sequences of length  $2^\ell$ . From Theorem 4 and the reference [17], we can have

**[Theorem 6]** Let  $e_{i_1, i_2, \dots, i_j, \dots, i_n}$  be an  $n$ -dimensional E-sequences of length  $L = L_1 \times L_2 \times \dots \times L_n$ . Let  $f(\cdot)$  be the logic function. Let  $\vec{i}_j = (i_{j1}, i_{j2}, \dots, i_{j\ell_j}) \in V_{\ell_j}$  be a binary vector of order  $\ell_j$ , whose elements are coefficients expressed

by binary expansion of an integer  $i_j (0 \leq i_j < L_j = 2^{\ell_j})$ , i.e.,

$$i_j = i_{j1}2^0 + i_{j2}2^1 + \dots + i_{j\ell_j}2^{\ell_j-1}.$$

The  $n$ -dimensional E-sequences  $e$  can be written as

$$e_{i_1, i_2, \dots, i_j, \dots, i_n} = (-1)^{f(\vec{i}_1, \vec{i}_2, \dots, \vec{i}_n)},$$

The functions  $f(\cdot)$  is logic function defined by

$$\begin{aligned} f(\vec{i}_1, \vec{i}_2, \dots, \vec{i}_n) &= \lambda_1 \lambda_2 \oplus \lambda_1 \lambda_3 \oplus \dots \oplus \lambda_{\ell-1} \lambda_\ell \\ &\oplus c_{1,1} i_{1,1} \oplus c_{1,2} i_{1,2} \oplus \dots \oplus c_{1,\ell_1} i_{1,\ell_1} \\ &\oplus c_{2,1} i_{2,1} \oplus c_{2,2} i_{2,2} \oplus \dots \oplus c_{2,\ell_2} i_{2,\ell_2} \\ &\vdots \\ &\oplus c_{n,1} i_{n,1} \oplus c_{n,2} i_{n,2} \oplus \dots \oplus c_{n,\ell_n} i_{n,\ell_n} \oplus d, \end{aligned}$$

where  $\oplus$  denotes addition of module 2, i.e., EXOR,  $c_{1,1}, \dots, c_{j,\ell_j} \in \{0, 1\}$  and  $d \in \{0, 1\}$  are parameter to give different E-sequences,  $\lambda_k (1 \leq k \leq \ell = \sum_{j=1}^n \ell_j)$   $i_{j1}, i_{j2}, \dots, i_{j\ell_j}$  of  $\vec{i}_j$  with  $\lambda_k \neq \lambda_m$  for  $k \neq m$ , and either of  $\lambda_1$  or  $\lambda_\ell$  must be  $i_{j1}$ .

**[Example 3]** Let

$$f(\vec{i}_1, \vec{i}_2) = i_{12} i_{11} \oplus i_{11} i_{13} \oplus i_{13} i_{22} \oplus i_{22} i_{21}, \quad (1)$$

be a logic function, where we set that  $\lambda_1 = i_{12}, \lambda_2 = i_{11}, \lambda_3 = i_{13}, \lambda_4 = i_{22}, \lambda_5 = i_{21}, c_{1,1} = c_{1,2} = c_{1,3} = c_{2,1} = c_{2,2} = 0$  and  $d=0$

Table 1. The truth table of function (1)

$f(\vec{i}_1, \vec{i}_2)$	$i_{12}$	$i_{11}$	$i_{13}$	$i_{21}$	$i_{20}$				
						0	0	0	0
						1	1	1	1
						0	0	1	1
						0	1	0	1
						0	1	0	0
						0	0	1	0
						1	1	0	1
						1	1	1	0
						0	1	0	0
						0	0	1	1
						1	1	0	1
						1	0	1	1
						0	1	0	0

Converting 0 and 1 in Table 1 into + and - respectively gives the 2-dimensional E-sequence in Example 1.

**[Example 4]** Let a logic function be

$$f(\vec{i}_1, \vec{i}_2) = i_{11} i_{12} \oplus i_{12} i_{13} \oplus i_{13} i_{22} \oplus i_{22} i_{21},$$

where  $\lambda_1 = i_{j1}, \lambda_\ell = i_{k1} (j \neq k)$ . We can make following E-sequences of length  $8 \times 4$ .

$$e = \left( \begin{array}{cccc|cccc} + & + & + & - & + & + & - & + \\ + & + & + & - & + & + & - & + \\ + & + & + & - & - & - & + & - \\ - & - & - & + & + & + & - & + \end{array} \right)$$

which its aperiodic auto-correlation function is written as

$$C_{ee}(\tau_1, \tau_2) = \begin{pmatrix} 32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 5 & 0 & 1 & 0 & 1 \end{pmatrix}.$$

Note that the out-of-phase aperiodic auto-correlation function takes zero if either  $\tau_1$  or  $\tau_2$  is even.

This is a good correlation property that almost 3/4 of the correlation values is zero. We show one more example of 3-dimensional E-sequences.

**[Example 5]** If

$$f(\vec{i}_1, \vec{i}_2, \vec{i}_3) = i_{21}i_{22} \oplus i_{12}i_{22} \oplus i_{22}i_{32} \oplus i_{32}i_{11} \oplus i_{11}i_{31},$$

E-sequence of length  $8 \times 4$  is given as

$$e = \begin{pmatrix} + & + & + & + & + & - & + & - \\ + & + & - & - & + & - & - & + \\ + & + & - & - & + & - & - & + \\ + & + & + & + & + & - & + & - \\ + & - & + & - & + & + & + & + \\ + & - & - & + & + & + & - & - \\ - & + & + & - & - & - & + & + \\ - & + & - & + & - & - & - & - \end{pmatrix},$$

with

$$C_{ee}(\tau_1, \tau_2, \tau_3) = \begin{pmatrix} 64 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & -3 & -2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & -3 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & -3 & -2 & -1 \end{pmatrix}.$$

Note that the out-of-phase aperiodic auto-correlation function takes zero if either  $\tau_2$  or  $\tau_3$  is even. So that we can produce some  $n$ -dimensional E-sequences with a good correlation property that almost 3/4 of all shifts is zero.

## 5. Conclusion

In this paper, we have considered the generalization of E-sequences to the multi-dimension. We have clarified that  $n$ -dimensional E-sequences can be constructed from  $n$ -dimensional complementary sequences of half length. We have derived the logic function of  $n$ -dimensional E-sequences of length  $2^\ell$  from the logic function of  $n$ -dimensional complementary sequences. In addition, the logic function can produce multi-dimensional E-sequences with the good correlation property that almost 3/4 of all shifts is zero.

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