Generalization of Even-Shift Orthogonal Sequences to Multi-Dimension

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Abstract: The even-shift orthogonal sequence (E-sequence) is a binary sequence, whose out-of-phase aperiodic auto-correlation function takes zero at any even shift. This paper considers the generalization of E-sequence to multi-dimension. It is shown that multi-dimensional E-sequences can be constructed by multi-dimensional complementary sequences. Especially a logic function generating multi-dimensional E-sequences of power-of-two length is formulated, which can give multi-dimensional E-sequences with a good correlation property that almost $3/4$ of all shifts is zero.

1. Introduction

Even-shift orthogonal sequences (E-sequences) are binary sequences whose out-of-phase aperiodic auto-correlation function takes zero at any even shift[1] - [4]. E-sequences have been applied for spread spectrum communication.

In this paper we consider the generalization of E-sequences to multi-dimension. We clarify the construction of multi-dimensional E-sequences of $S2^\ell$ with $S = 1, 10, 26$ and $\ell \geq 1$, and formulate a logic function generating them of length 2^ℓ. In addition, their aperiodic auto-correlation properties are investigated.

In section 2, multi-dimensional E-sequences are introduced. In section 3, it is shown that multi-dimensional E-sequences can be derived from multi-dimensional complementary sequences of half length, which mean a pair of binary sequences and the sum of these aperiodic auto-correlation function takes zero except all zero shifts [4][17]. In section 4, we formulate a logic function of multi-dimensional E-sequences, and investigate correlation properties of multi-dimensional E-sequences.

2. Definition of multi-dimensional E-sequences

Let e be a multi-dimensional (n-dimensional) binary sequence of length $L = L_1 \times L_2 \times \cdots \times L_n$, expressed as

$$e = \{e_{i_1,i_2,\cdots,i_j,\cdots,i_n} \in \{1,-1\} | 0 \leq i_j < L_j\}.$$

The aperiodic auto-correlation function of e is written as

$$C_{ee}(\tau_1, \tau_2 \cdots, \tau_n) = \sum_{i_1=0}^{L_1-1} \sum_{i_2=0}^{L_2-1} \cdots \sum_{i_n=0}^{L_n-1} e_{i_1,i_2,\ldots,i_n} e_{i_1+\tau_1,i_2+\tau_2,\ldots,i_n+\tau_n},$$

where $e_{i_1,i_2,\cdots,i_j,\cdots,i_n} = 0$ for $i_j < 0$ and $i_j \geq L_j$. If the out-of-phase aperiodic auto-correlation function is zero at even shift,i.e.,

$$C_{ee}(2m_1, 2m_2, \cdots 2m_n) = \begin{cases} L & \text{for } m_1 = m_2 = \cdots = m_n = 0, \\ 0 & \text{otherwise.} \end{cases}$$

e is called a multi-dimensional E-sequence, or an n-dimensional E-sequence.

3. Construction of multi-dimensional E-sequences

In this section, we show that n-dimensional E-sequences can be constructed from n-dimensional complementary sequences of half length.

Let a and b be n-dimensional binary sequences of length $L = L_1 \times L_2 \times \cdots \times L_n$, expressed by

$$a = \{a_{i_1,i_2,\cdots,i_j,\cdots,i_n} \in \{1,-1\} | \, 0 \leq i_j < L_j\},$$
$$b = \{b_{i_1,i_2,\cdots,i_j,\cdots,i_n} \in \{1,-1\} | \, 0 \leq i_j < L_j\}.$$

If the sum of their aperiodic auto-correlation functions can be written as

$$C_{aa}(\tau_1, \tau_2, \cdots, \tau_n) + C_{bb}(\tau_1, \tau_2, \cdots, \tau_n) = \begin{cases} 2L & \text{for } \tau_1 = \tau_2 = \cdots = \tau_n = 0, \\ 0 & \text{otherwise,} \end{cases}$$

the pair of the sequences, $[a, b]$, is called multi-dimensional (n-dimensional) complementary sequences or complementary arrays [17]. They can be constructed as shown in the following theorems.

[Theorem 1] Let $[a, b]$ be n-dimensional complementary sequences.

1. Interchanging a and b gives an n-dimensional complementary sequence. i.e., $[b, a]$.
2. Inversion of a gives an n-dimensional complementary sequence, i.e., $[-a, b]$.
3. Interchanging some axes gives n-dimensional complementary sequences, $[\{a_{i_{k_1},i_{k_2},\cdots,i_{k_n}}\}, \{b_{i_{k_1},i_{k_2},\cdots,i_{k_n}}\}]$ with $1 \leq k_m (\neq k_j) \leq n$.
4. Reversing of a is a mate of b, i.e.,
$$[\{a_{L_1-i_1-1, L_2-i_2-1,\cdots,L_n-i_n-1,}\}, b].$$
5. Reversing at some axio gives complementary arrays, $[\{a_{i_1,\cdots,L_j-i_j-1,\cdots,i_n}\}, \{b_{i_1,\cdots,L_j-i_j-1,\cdots,i_n}\}]$.

Note that Theorem 1 can produce a lot of n-dimensional complementary sequences. For example, use of 2, 3, 2, and 3 in Theorem 1, in order, gives n-dimensional complementary sequences, $[-a, -b]$.

[Theorem 2] Let $[a, b]$ be n-dimensional complementary sequences of length L. We have $(n+1)$-dimensional complementary sequences $[\hat{a}, \hat{b}]$ of length $2L$, which are written as

$$\hat{a}_{i_1,i_2,\cdots,i_n,i_{n+1}} = \begin{cases} a_{i_1,i_2,\cdots,i_n} & \text{for } i_{n+1} = 0, \\ b_{i_1,i_2,\cdots,i_n} & \text{for } i_{n+1} = 1, \end{cases}$$

$$\hat{b}_{i_1,i_2,\cdots,i_n,i_{n+1}} = \begin{cases} a_{i_1,i_2,\cdots,i_n} & \text{for } i_{n+1} = 0, \\ -b_{i_1,i_2,\cdots,i_n} & \text{for } i_{n+1} = 1. \end{cases}$$

Multi-dimensional complementary sequences can be easily produced by interleaving and concatenation methods, as well as 1-dimensional complementary sequences [4][5].

[Theorem 3] Let $[a,b]$ be n-dimensional complementary sequences of length L. We have n-dimensional complementary sequences $[\hat{a},\hat{b}]$ of length $2L$ expressed by

$$\hat{a}_{i_1,\cdots,i'_j,\cdots,i_n} = \begin{cases} a_{i_1,\cdots,i_j,\cdots,i_n} & \text{for } i'_j = 2i_j, \\ b_{i_1,\cdots,i_j,\cdots,i_n} & \text{for } i'_j = 2i_j+1, \end{cases}$$

$$\hat{b}_{i_1,\cdots,i'_j,\cdots,i_n} = \begin{cases} a_{i_1,\cdots,i_j,\cdots,i_n} & \text{for } i'_j = 2i_j, \\ -b_{i_1,\cdots,i_j,\cdots,i_n} & \text{for } i'_j = 2i_j+1. \end{cases}$$

[Theorem 4] Let $[a,b]$ be n-dimensional complementary sequences of length L. We have n-dimensional complementary sequences $[\hat{a},\hat{b}]$ of length $2L$ expressed by

$$\hat{a}_{i_1,\cdots,i'_j,\cdots,i_n} = \begin{cases} a_{i_1,\cdots,i_j=i'_j,\cdots,i_n} \\ \qquad \text{for } 0 \le i'_j < L_j, \\ b_{i_1,\cdots,i_j=i'_j-L_j,\cdots,i_n} \\ \qquad \text{for } L_j \le i'_j < 2L_j, \end{cases}$$

$$\hat{b}_{i_1,\cdots,i'_j,\cdots,i_n} = \begin{cases} a_{i_1,\cdots,i_j=i'_j,\cdots,i_n} \\ \qquad \text{for } 0 \le i'_j < L_j, \\ -b_{i_1,\cdots,i_j=i'_j-L_j,\cdots,i_n} \\ \qquad \text{for } L_j \le i'_j < 2L_j. \end{cases}$$

The construction methods in Theorems 3 and 4 are well-known as interleaving and concatenation methods, respectively. We note that n-dimensional complementary sequences of length $L = S2^\ell$ with $S = 1, 10$ or 26 can be derived from 1-dimensional complementary sequences of length S, called kernels [1][4].

We give a special construction method of n-dimensional complementary sequences of length 2^ℓ, as the following conjecture.

[Conjecture 1] Let $[a,b]$ be n-dimensional complementary sequences of length $L = 2^\ell$. Let $K = 2^k \le L$. We have n-dimensional complementary sequences $[\hat{a},\hat{b}]$ of length $2L$ expressed by

$$\hat{a}_{i_1,\cdots,i'_j,\cdots,i_n} = \begin{cases} a_{i_1,\cdots,i_j=m_2K+m_1,\cdots,i_n} \\ \qquad \text{for } i'_j = 2m_2K + m_1, \\ b_{i_1,\cdots,i_j=m_2K+m_1,\cdots,i_n} \\ \qquad \text{for } i'_j = (2m_2+1)K + m_1, \end{cases}$$

$$\hat{b}_{i_1,\cdots,i'_j,\cdots,i_n} = \begin{cases} a_{i_1,\cdots,i_j=m_2K+m_1,\cdots,i_n} \\ \qquad \text{for } i'_j = 2m_2K + m_1, \\ -b_{i_1,\cdots,i_j=m_2K+m_1,\cdots,i_n} \\ \qquad \text{for } i'_j = (2m_2+1)K + m_1, \end{cases}$$

with $0 \le m_1 < K$, $0 \le m_2 < L/K$, $0 \le i'_j < 2L_j$, and $0 \le l_k < L_k$.

Note that combination of Theorems 1-4 and Conjecture 1 can give a lot of n-dimensional complementary sequences.

[Theorem 5] Let $[a,b]$ be n-dimensional complimentary sequences of length $L = L_1 \times \cdots \times L_j/2 \times \cdots \times L_n$ and $[\hat{a},\hat{b}]$ be n-dimensional complimentary sequences of length $2L = L_1 \times \cdots \times L_j \times \cdots \times L_n$. Each of $\hat{a}$ and $\hat{b}$ given by applying Theorem 3 are E-sequences. It is proved as follows. The aperiodic auto-correlation functions of the above n-dimensional complementary sequences $\hat{a}$ and $\hat{b}$ can be expressed by

$$\begin{aligned} & C_{\hat{a}\hat{a}}(\tau_1,\tau_2,\cdots,2m_j,\cdots,\tau_n) \\ &= C_{\hat{b}\hat{b}}(\tau_1,\tau_2,\cdots,2m_j,\cdots,\tau_n) \\ &= C_{aa}(\tau_1,\tau_2,\cdots,m_j,\cdots,\tau_n) + C_{bb}(\tau_1,\tau_2,\cdots,m_j,\cdots,\tau_n) \\ &= \begin{cases} 2L \ (\tau 1 = \tau_2 = \cdots = m_j = \cdots = \tau_n = 0) \\ 0 \ otherwise. \end{cases} \end{aligned}$$

Since $C_{\hat{a}\hat{a}}(\cdot)$ and $C_{\hat{b}\hat{b}}(\cdot)$ are zero if either of $\tau_1,\tau_2,\cdots,\tau_n$ is even, we have the following theorem.

We show some example for the construct of E-sequence.

[Example 1] Let a and b be 2-dimensional complimentary sequences of length 8×2 as expressed by

$$a = \begin{pmatrix} + & + & + & - & + & - & + & + \\ + & + & + & - & - & + & - & - \end{pmatrix},$$

$$b = \begin{pmatrix} + & + & + & - & + & - & + & + \\ - & - & - & + & + & - & + & + \end{pmatrix},$$

where $+$ and $-$ are 1 and -1 respectively. Theorem 3 gives 2-dimensional E-sequences of length 8×4 .

$$\hat{a} = \begin{pmatrix} + & + & + & - & + & - & + & + \\ + & + & + & - & + & - & + & + \\ + & + & + & - & - & + & - & - \\ - & - & - & + & + & - & + & + \end{pmatrix}$$

$$\hat{b} = \begin{pmatrix} + & + & + & - & + & - & + & + \\ - & - & - & + & - & + & - & - \\ + & + & + & - & - & + & - & - \\ + & + & + & - & - & + & - & - \end{pmatrix}$$

The aperiodic auto-correlation functions of $\hat{a}$ and $\hat{b}$ can be written as

$$C_{\hat{a}\hat{a}}(\tau_1,\tau_2) = \begin{pmatrix} 32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -3 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & 2 & 1 & 0 & 1 & 2 & 1 \end{pmatrix},$$

$$C_{\hat{b}\hat{b}}(\tau_1,\tau_2) = \begin{pmatrix} 32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -2 & 3 & 0 & -1 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & -2 & -1 & 0 & -1 & -2 & -1 \end{pmatrix}.$$

[Example 2] Let a and b be 3-dimensional complimentary sequences of length $4 \times 4 \times 2$ as expressed by

$$a = \left(\begin{array}{cccc|cccc} + & + & + & - & + & + & + & - \\ + & + & + & - & - & - & - & + \\ + & - & + & + & + & - & + & + \\ - & + & - & - & + & - & + & + \end{array}\right),$$

$$b = \left(\begin{array}{cccc|cccc} + & + & + & - & - & - & - & + \\ + & + & + & - & + & + & + & - \\ + & - & + & + & - & + & - & - \\ - & + & - & - & - & + & - & - \end{array}\right).$$

Theorem 3 gives 3-dimensional E-sequences $[\hat{a}, \hat{b}]$ of length $4 \times 4 \times 4$

$$\hat{a} = \left(\begin{array}{cccc|cccc|} + & + & + & - & + & + & + & - \\ + & + & + & - & - & - & - & + \\ + & + & - & + & + & + & - & + \\ + & + & - & + & - & - & + & - \end{array}\right.$$

$$\left.\begin{array}{cccc|cccc} + & + & + & - & + & + & + & - \\ - & - & - & + & + & + & + & - \\ - & - & + & - & - & - & + & - \\ + & + & - & + & - & - & + & - \end{array}\right)$$

$$\hat{b} = \left(\begin{array}{cccc|cccc|} + & + & + & - & - & - & - & + \\ + & + & + & - & + & + & + & - \\ + & + & - & + & - & - & + & - \\ + & + & - & + & + & + & - & + \end{array}\right.$$

$$\left.\begin{array}{cccc|cccc} + & + & + & - & - & - & - & + \\ - & - & - & + & - & - & - & + \\ - & - & + & - & + & + & - & + \\ + & + & - & + & + & + & - & + \end{array}\right)$$

The aperiodic auto-correlation functions of $\hat{a}$ and $\hat{b}$ are

$$C_{\hat{a}\hat{a}}(\tau_1, \tau_2, \tau_3) = \left(\begin{array}{cccc|cccc|} 64 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 & 6 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 & -1 & -2 & -1 \end{array}\right.$$

$$\left.\begin{array}{cccc|cccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 & -2 & -4 & -2 \\ 0 & 0 & 0 & 0 & 0 & -1 & -2 & -1 \end{array}\right),$$

$$C_{\hat{b}\hat{b}}(\tau_1, \tau_2, \tau_3) = \left(\begin{array}{cccc|cccc|} 64 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -5 & -6 & -1 \\ 0 & 0 & 0 & 0 & 0 & -2 & -4 & -2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 \end{array}\right.$$

$$\left.\begin{array}{cccc|cccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 2 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 \end{array}\right).$$

4. Logic functions of E-sequences

We consider the function of n-dimensional E-sequences of length 2^ℓ. From Theorem 4 and the reference [17], we can have

[Theorem 6] Let $e_{i_1,i_2,\cdots,i_j,\cdots,i_n}$ be an n-dimensional E-sequences of length $L = L_1 \times L_2 \times \cdots \times L_n$. Let $f(\cdot)$ be the logic function. Let $\vec{i_j} = (i_{j_1}, i_{j_2}, \cdots i_{j_{\ell_j}}) \in V_{\ell_j}$ be a binary vector of order ℓ_j, whose elements are coefficients expressed by binary expansion of an integer $i_j (0 \leq i_j < L_j = 2^{\ell_j})$, i.e.,

$$i_j = i_{j_1} 2^0 + i_{j_2} 2^1 + \cdots + i_{j_{\ell_j}} 2^{\ell_j - 1}.$$

The n-dimensional E-sequences e can be written as

$$e_{i_1,i_2,\cdots,i_j,\cdots,i_n} = (-1)^{f(\vec{i_1}, \vec{i_2}, \cdots, \vec{i_n})},$$

The functions $f(\cdot)$ is logic function defined by

$$\begin{aligned} f(\vec{i_1}, \vec{i_2}, \quad \cdots, \quad & \vec{i_n}) \\ = \quad & \lambda_1\lambda_2 \oplus \lambda_1\lambda_3 \oplus \cdots \oplus \lambda_{\ell-1}\lambda_\ell \\ \oplus \quad & c_{1,1} i_{1,1} \oplus c_{1,2} i_{1,2} \oplus \cdots \oplus c_{1,\ell_1} i_{1,\ell_1} \\ \oplus \quad & c_{2,1} i_{2,1} \oplus c_{2,2} i_{2,2} \oplus \cdots \oplus c_{2,\ell_2} i_{2,\ell_2} \\ \vdots \quad & \\ \oplus \quad & c_{n,1} i_{n,1} \oplus c_{n,2} i_{n,2} \oplus \cdots c_{n,\ell_n} i_{n,\ell_n} \oplus d, \end{aligned}$$

where $\oplus$ denotes addition of module 2, i.e., EXOR, $c_{1,1}, \cdots, c_{j,\ell_j} \in \{0,1\}$ and $d \in \{0,1\}$ are parameter to give different E-sequences, $\lambda_k (1 \leq k \leq \ell = \sum_{j=1}^{n} \ell_j)$ $i_{j_1}, i_{j_2}, \cdots, i_{j_{\ell_j}}$ of $\vec{i_j}$ with $\lambda_k \neq \lambda_m$ for $k \neq m$, and either of λ_1 or λ_ℓ must be i_{j_1}.

[Example 3] Let

$$f(\vec{i_1}, \vec{i_2}) = i_{12} i_{11} \oplus i_{11} i_{13} \oplus i_{13} i_{22} \oplus i_{22} i_{21}, \qquad (1)$$

be a logic function, where we set that $\lambda_1 = i_{12}$, $\lambda_2 = i_{11}$, $\lambda_3 = i_{13}$, $\lambda_4 = i_{22}$, $\lambda_5 = i_{21}$, $c_{1,1} = c_{1,2} = c_{1,3} = c_{2,1} = c_{2,2} = 0$ and $d = 0$.

Table 1. The truth table of function (1)

$f(\vec{i_1}, \vec{i_2})$		i_{12}	0	0	0	0	1	1	1	1
		i_{11}	0	0	1	1	0	0	1	1
		i_{10}	0	1	0	1	0	1	0	1
i_{21}	i_{20}									
0	0		0	0	0	1	0	1	0	0
0	1		0	0	0	1	0	1	0	0
1	0		0	0	0	1	1	0	1	1
1	1		1	1	1	0	0	1	0	0

Converting 0 and 1 in Table 1 into $+$ and $-$ respectively gives the 2-dimensional E-sequence in Example 1.

[Example 4] Let a logic function be

$$f(\vec{i_1}, \vec{i_2}) = i_{11} i_{12} \oplus i_{12} i_{13} \oplus i_{13} i_{22} \oplus i_{22} i_{21},$$

where $\lambda_1 = i_{j_1}, \lambda_\ell = i_{k1} (j \neq k)$. We can make following E-sequences of length 8×4.

$$e = \begin{pmatrix} + & + & + & - & + & + & - & + \\ + & + & + & - & + & + & - & + \\ + & + & + & - & - & - & + & - \\ - & - & - & + & + & + & - & + \end{pmatrix}$$

which its aperiodic auto-correlation function is written as

$$C_{ee}(\tau_1, \tau_2) = \begin{pmatrix} 32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 5 & 0 & 1 & 0 & 1 \end{pmatrix}.$$

Note that the out-of-phase aperiodic auto-correlation function takes zero if either τ_1 or τ_2 is even.

This is a good correlation property that almost $3/4$ of the correlation values is zero. We show one more example of 3-dimensional E-sequences.

[Example 5] If

$$f(\overrightarrow{i_1}, \overrightarrow{i_2}, \overrightarrow{i_3}) = i_{21}i_{22} \oplus i_{12}i_{22} \oplus i_{22}i_{32} \oplus i_{32}i_{11} \oplus i_{11}i_{31},$$

E-sequence of length 8×4 is given as

$$e = \left(\begin{array}{cccc|cccc|} + & + & + & + & + & - & + & - \\ + & + & - & - & + & - & - & + \\ + & + & - & - & + & - & - & + \\ + & + & + & + & + & - & + & - \end{array} \right.$$
$$\left. \begin{array}{cccc|cccc} + & - & + & - & + & + & + & + \\ + & - & - & + & + & + & - & - \\ - & + & + & - & - & - & + & + \\ - & + & - & + & - & - & - & - \end{array} \right),$$

with

$$C_{ee}(\tau_1, \tau_2, \tau_3) = \left(\begin{array}{cccc|cccc} 64 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & -3 & -2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & 1 & -2 & -1 \end{array} \right.$$
$$\left. \begin{array}{cccc|cccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & -3 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & -3 & -2 & -1 \end{array} \right).$$

Note that the out-of-phase aperiodic auto-correlation function takes zero if either τ_2 or τ_3 is even. So that we can produce some n-dimensional E-sequences with a good correlation property that almost $3/4$ of all shifts is zero.

5. Conclusion

In this paper, we have considered the generalization of E-sequences to the multi-dimension. We have clarified that n-dimensional E-sequences can be constructed from n-dimensional complementary sequences of half length. We have derived the logic function of n-dimensional E-sequences of length 2^ℓ from the logic function of n-dimensional complementary sequences. In addition, the logic function can produce multi-dimensional E-sequences with the good correlation property that almost $3/4$ of all shifts is zero.

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