Modified Pseudo Orthogonal M-sequence Sets for Synchronous Optical-CDMA

Yusuke Kozawa[1] and Hiromasa Habuchi[2]
[1,2]College of Engineering, Ibaraki University
4-12-1 Nakanarusawa Hitachi, Ibaraki 316-8511, Japan
E-mail : [1]07nm707r@hcs.ibaraki.ac.jp, [2]habuchi@mx.ibaraki.ac.jp

Abstract: In this paper, to increase normalized throughput of the optical code-division multiple-access (CDMA) system, optical-CDMA systems with multi-pulse pulse position modulation (MPPM) using the modified pseudo orthogonal M-sequence sets is proposed. The throughput performance and bit error rate (BER) are evaluated by theoretical analysis. Consequently, the normalized throughput of the proposed system can achieve 1.35 [bit/chip] when the number of slots per frame is 16. Furthermore, BER of the proposed system is superior to that of the conventional systems.

1. Introduction

Recently, there has increasing interest in optical communications such as fiber-optic communications and optical wireless communications. As a multiple access technique in the optical communication, optical code-division multiple-access (OCDMA) schemes have attracted much attention because these schemes allow multiple users to access the network asynchronously and simultaneously. In the OCDMA system using unipolar modulation, each user is assigned one particular optical spreading code. It is expected that the optical spreading code, which is assigned to each user, fulfills the following two conditions: (1) the relation between any two spreading codes is orthogonal and (2) the number of generated orthogonal codes, N, per spreading code length, L , is large, that is, $N/L \geq 1$. The OCDMA system with the spreading code satisfying these conditions can reduce the influence of multiple access interference (MAI) and can realize the high maximum data transmission rate (i.e., normalized throughput which is normalized by chip rate). In the conventional OCDMA systems using unipolar modulation, the optical orthogonal code (OOC), the extended prime code sequence and the pseudo orthogonal M-sequence are reported[1]-[3]. In particular, the number of generated orthogonal codes of the pseudo orthogonal M-sequence is equal to the code length. Therefore, when the OCDMA system uses the On-Off Keying (OOK) and the Multi-pulse PPM (MPPM)[4] as the unipolar modulation, the normalized throughput of the OCDMA system with the pseudo orthogonal M-sequence can achieve 1.0 [bit/chip]. However, it cannot exceed 1.0 [bit/chip] even if the number of pulses in a PPM frame increases because $N/L = 1$. Therefore, in order that the normalized throughput can exceed 1.0 [bit/chip], it is important to design the optical spreading code which can achieve $N/L > 1$.

In this paper, to realize high normalized throughput, we design the modified pseudo orthogonal M-sequence set[5]. This modified pseudo orthogonal M-sequence sets fulfills the conditions (1) and (2). Moreover, in this modified pseudo orthogonal M-sequence sets, two modified pseudo orthogonal M-sequences are generated by one primitive M-sequence. These two modified pseudo orthogonal M-sequences also consist of zero chips and positive chips. Therefore, the modified pseudo orthogonal M-sequence sets can achieve $N/L\simeq 2$. In this paper, we propose the OCDMA system using MPPM with the modified pseudo orthogonal M-sequence sets and show the normalized throughput of the proposed OCDMA system. Moreover, we analyze the bit error rate (BER) performance of the proposed system by taking into account the scintillation, background-noise, avalanche photo-diode (APD) noise, thermal noise and signal dependence noise.

2. Modified pseudo orthogonal M-sequence sets

In this section, we describe the modified pseudo orthogonal M-sequence sets. Let us consider how to design orthogonal M-sequences which are M-sequences with additional one chip. The primitive M-sequence of length L_M is defined as a sequence $(x_1, x_2, \cdots, x_{L_M})$ whose chips $x_i (i = 1, 2, \cdots, M)$ are either $+1$ or -1. Moreover, the primitive M-sequence consists of $(L_M - 1)/2$ negative chips and $(L_M + 1)/2$ positive chips. Therefore, the primitive orthogonal M-sequence consists of primitive M-sequence and additional one chip, that is, the primitive orthogonal M-sequences $OM_i (i = 1, 2, \cdots, L_M)$ are

$$\begin{cases} OM_1 = \{ & x_1 \quad x_2 \quad x_3 \quad \cdots \quad x_{L_M} \quad -1\} \\ OM_2 = \{ & x_2 \quad x_3 \quad \cdots \quad x_{L_M} \quad x_1 \quad -1\} \\ \quad\vdots\quad = \{ & \vdots \quad \} \\ OM_{L_M} = \{ & x_{L_M} \quad x_1 \quad x_2 \quad \cdots \quad x_{L_M-1} \quad -1\} \end{cases} \tag{1}$$

where $x_i = \pm 1$ $(i = 1, 2, \cdots, L_M)$. The cross-correlation value between any two orthogonal M-sequences is zero. In this orthogonal M-sequences, the sequences which replace -1 by zero are denoted $M_{Ai} (i = 1, 2, \cdots, L_M)$, and the sequences which replace $+1$ and -1 by zero and $+1$ respectively are denoted $M_{Bi} (i = 1, 2, \cdots, L_M)$. These $N = 2L_M$ sequences, that are M_{Ai} and M_{Bi}, are M-sequences with on-off signaling. The sequences length of OM, M_A, M_B is $L = L_M + 1$. For example, when $L_M = 7$ [chip], OM, M_A and M_B are expressed as

$$OM = \begin{bmatrix} OM_1 \\ OM_2 \\ OM_3 \\ OM_4 \\ OM_5 \\ OM_6 \\ OM_7 \end{bmatrix} = \begin{bmatrix} +1 & +1 & +1 & -1 & +1 & -1 & -1 & -1 \\ +1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 \\ +1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 \\ -1 & +1 & -1 & -1 & +1 & +1 & +1 & -1 \\ +1 & -1 & -1 & +1 & +1 & +1 & -1 & -1 \\ -1 & -1 & +1 & +1 & +1 & -1 & +1 & -1 \\ -1 & +1 & +1 & +1 & -1 & +1 & -1 & -1 \end{bmatrix} \tag{2}$$

$$M_A = \begin{bmatrix} M_{A1} \\ M_{A2} \\ M_{A3} \\ M_{A4} \\ M_{A5} \\ M_{A6} \\ M_{A7} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{3}$$

$$M_B = \begin{bmatrix} M_{B1} \\ M_{B2} \\ M_{B3} \\ M_{B4} \\ M_{B5} \\ M_{B6} \\ M_{B7} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \quad (4)$$

where the ith row vector of OM is OM_i, the ith row vector of M_A is M_{Ai} and the ith row vector of M_B is M_{Bi}. These sequences have the following characteristics

$$M_A OM^T = \left(\frac{L}{2}\right) E \quad (5)$$

$$M_B OM^T = \left(-\frac{L}{2}\right) E \quad (6)$$

$$OM = M_A - M_B \quad (7)$$

where E is a unit matrix and x^T is transposed matrix of x. Therefore, M_{Ai} or M_{Bi}, that are the on-off signaling, is used as the optical spreading code at the transmitter and OM_i is used as the reference spreading code at the receiver.

3. The proposed OCDMA system

In the MPPM system with the modified pseudo orthogonal M-sequence sets, ith user uses the two modified pseudo orthogonal M-sequences with on-off signaling (M_{Ai} and M_{Bi}) and the one primitive orthogonal M-sequence with polar signaling (OM_i). Therefore, when the length of pseudo orthogonal M-sequences is L [chip], the proposed system can achieve $L-1$ simultaneous users. The transmitter signal is generated by the combination of multi-pulse position and two modified pseudo orthogonal M-sequences with on-off signaling (M_{Ai} and M_{Bi}) at the transmitter. Thus, the $\binom{M}{r} \times 2^r$ transmitter signal patterns can be formed where r is the number of pulses in a frame and M is the number of slots in a frame. This means that the number of bits per frame is $\lfloor \log_2 \binom{M}{r} \rfloor + r$. For example, as shown in Fig.1, a transmitter signal can be expressed when $r = 2$ and $M = 4$.

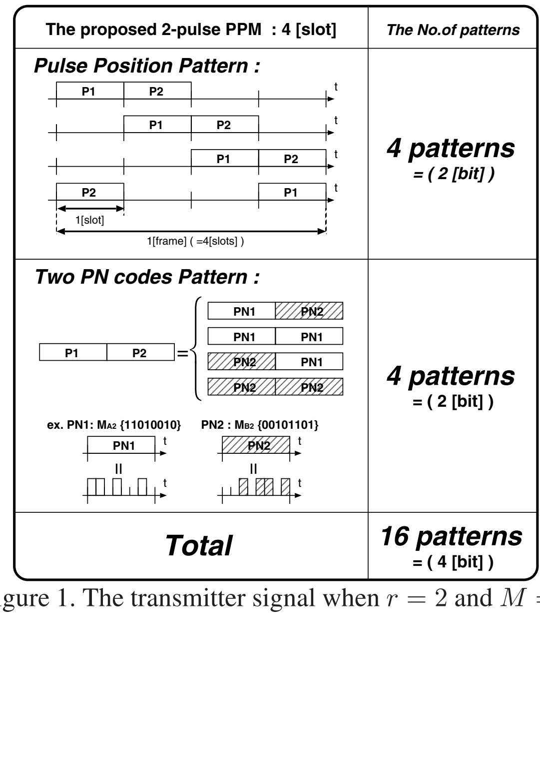

Figure 1. The transmitter signal when $r = 2$ and $M = 4$

Figure 2 shows the system model of the optical wireless MPPM with the modified pseudo orthogonal M-sequence sets. When source data is k [bit], correspond to k-2 [bit], the multi-pulse position is chosen by pulse position modulator. Two spreading codes are selected by switching correspond to the other 2[bit].

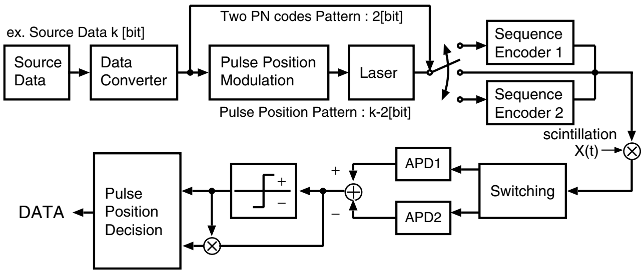

Figure 2. System model

At the receiver, high-speed switching and optical delay lines detect received optical power by the assigned primitive orthogonal M-sequence which are consisted of $+1$ and -1. However, it is difficult to deal in a negative optical power. In this paper, two APD (APD_1 and APD_2) are used. APD_1 is corresponds to M_{Ai} (i.e., a positive chips positions of OM_i). APD_2 is corresponds to M_{Bi} (i.e., a negative chips positions of OM_i). We define that $\mathbf{c} = (\mathbf{c_1}, \mathbf{c_2}, \cdots, \mathbf{c_L})$, where c_k is the received optical power at kth chip. The received optical power $\mathbf{r}$ is directed to multiply M_{Ai} and M_{Bi} for correlation operation. These correlation values $|\mathbf{cM_{Ai}^T}|$ and $|\mathbf{cM_{Bi}^T}|$ are converted into the electrical signal by APD_1 and APD_2, respectively. The difference between the output of APD_1 and output of APD_2 becomes the correlation value with reference sequence OM_i because $OM = M_A - M_B$ in Eq.(7). In the proposed system, the symbol is declared by absolute value and polarity of the correlation value of each slot. The PPM decoder declares multi-pulse position by comparing the absolute value of the correlation value of each slot. The spreading code is declared whether the polarity of the correlation value is positive or negative. Then the final output bits corresponding to the symbol are given.

4. Normalized throughput

We show the normalized throughput (i.e., maximum data transmission rate). The normalized throughput is the rate of the total message bits per one frame of simultaneous users to the frame length. The proposed system can achieve $L-1$ simultaneous users when the length of the modified pseudo orthogonal M-sequence is L [chip]. While, the conventional system can achieve L_M simultaneous users where L_M is the length of the conventional pseudo orthogonal M-sequence[3]. Therefore, the normalized throughput of the proposed system, C_{pro}, and that of the conventional system, C_{con}, are derived respectively, as

$$C_{pro} = \frac{L-1}{L} \times \frac{\left\lfloor \log_2 \begin{pmatrix} M \\ r \end{pmatrix} \right\rfloor + r}{M} \; [bit/chip] \quad (8)$$

$$C_{con} = \frac{\left\lfloor \log_2 \begin{pmatrix} M \\ r \end{pmatrix} \right\rfloor}{M} \; [bit/chip]. \quad (9)$$

Fig.3 shows the normalized throughput versus the number of pulses in a frame when $L = 16$ [chip] and $M = 8, 16$ [slot]. The normalized throughput of the proposed system is better than that of the conventional MPPM system with the pseudo orthogonal M-sequences and can achieve about 1.35 [bit/chip].

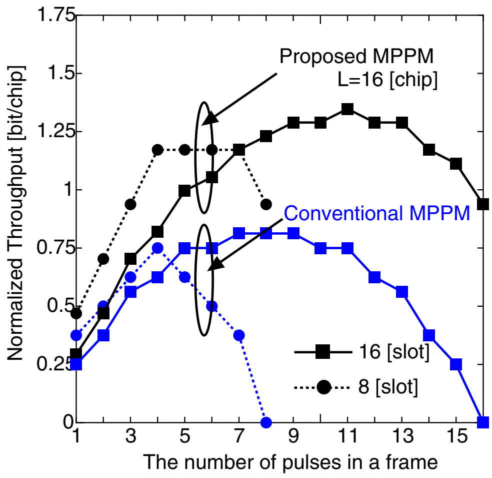

Figure 3. Normalized throughput versus the number of pulses in a frame for the proposed system ($L = 16$) and the conventional MPPM system considered with $M = 8, 16$ slots.

5. Theoretical analysis

In this section, we analyze the bit error (BER) rate performance of the proposed system using APD in the slot synchronous case. The output of APD is assumed to be a Gaussian distribution. The probability that a specified number of photons are absorbed from an incident optical field by an APD detector over a chip interval with T_c is given by a Poisson distribution[6].

In the optical wireless communication, we need to take into account the scintillation which influences the attenuation and the fluctuation of the received optical power. The scintillation X_i for ith user characterized by the stationary probability process. Its probability density function $p(X_i)$ can be written as [7]

$$p(X_i) = \frac{1}{\sqrt{2\pi\sigma_s^2 X_i}} \exp\left\{ -\left(\ln \mathrm{X_i} + \frac{\sigma_s^2}{2} \right)^2 / 2\sigma_s^2 \right\} \quad (10)$$

where the average of scintillation X is normalized to unity, and σ_s^2 is logarithm variance. The variance σ_s^2 is determined by the atmospheric state.

The average number of absorbed photons over T_c is

$$\lambda_s = \frac{\eta P_w}{hf} \quad (11)$$

where λ_s is the photon absorption rate, P_w is the received laser power, η is the APD efficiency, h is Planck's constant and f is the optical frequency. Through an avalanche multiplication process, the APD outputs some electrons in response to the absorption of λT_c primary photons on the average. Here, λ represents the total photon absorption rate due to signal, background light, and APD bulk leakage current. The total photon absorption rate λ of the proposed system is given by

$$\lambda = \begin{cases} \frac{L}{2}\lambda_s + \frac{L}{2}\lambda_b + \frac{I_b}{e} & \text{for a mark} \\ \frac{L}{2}\frac{\lambda_s}{M_e} + \frac{L}{2}\lambda_b + \frac{I_b}{e} & \text{for a space} \end{cases} \quad (12)$$

where λ_b is the photon absorption rate due to actual background light ($\lambda_b = \eta P_b / hf$ when the background noise per chip duration is P_b), I_b/e is represents the contribution of the APD bulk leakage current to the APD output and M_e is the extinction ratio of the laser diode output power in the mark and space states.

5.1 Bit error rate

We present the BER performance of the proposed system in the single-user case. In this paper, we define the BERs are $P_{SER}/2$ where the P_{SER} is the symbol error rate (SER). The symbol error rate, P_{SER}, is given by

$$P_{SER} = 1 - P_{suc} \quad (13)$$

where P_{suc} is the symbol correct rate of the proposed system. We define M is the number of PPM slots in a frame, r is the number of pulses in a frame, q_k is output of the correlation value in the kth slot ($k = 1, 2, \cdots, M$) and $|q_k|$ is the absolute value of the correlation value of kth slot.

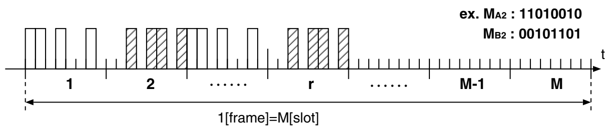

Figure 4. An example of a transmit signal pattern of the proposed system

When the desired user transmits using 1th slot, 2th slot, $\cdots$, $(r-1)$th slot and rth slot like Figure 4, the symbol correct rate of this case can be written as

$$P_{rob}\left[q_1 > 0, q_2 < 0, \cdots, q_r < 0, (|q_1|, \cdots, |q_r|) > (|q_{r+1}|, \cdots, |q_M|)\right].$$

In this case, the desired user transmits using M_{A2} in the 1th slot. Therefore, the polarity of the q_1 must be positive value. While, in the 2th slot, the polarity of the q_2 must be negative value.

The symbol correct rate at the Figure 4, by the symmetry property, can be expressed as

$$\begin{aligned} &P_{rob}\left[q_1 > 0, q_2 < 0, \cdots, q_r < 0, (|q_1|, \cdots, |q_r|) > (|q_{r+1}|, \cdots, |q_M|)\right] \\ &\quad = P_{rob}\left[(q_1, \cdots, q_r) > 0, (|q_1|, \cdots, |q_r|) > (|q_{r+1}|, \cdots, |q_M|)\right] \\ &\quad = P_{suc} \end{aligned} \quad (14)$$

Therefore, the symbol correct rate of the proposed system, P_{suc}, is given by

$$\begin{aligned} P_{suc} &= P_{rob}\left[(q_1, \cdots, q_r) > 0, (|q_1|, \cdots, |q_r|) > (|q_{r+1}|, \cdots, |q_M|)\right] \\ &= r! \int_0^\infty p(X_0) \int_0^\infty \frac{1}{\sqrt{2\pi\sigma_1^2(X_0)}} \exp\left\{ -\frac{(q_1 - \mu_1(X_0))^2}{2\sigma_1^2(X_0)} \right\} \\ &\quad \times \int_0^{q_1} \frac{1}{\sqrt{2\pi\sigma_2^2(X_0)}} \exp\left\{ -\frac{(q_2 - \mu_2(X_0))^2}{2\sigma_2^2(X_0)} \right\} \\ &\quad \times \cdots \times \int_0^{q_{r-1}} \frac{1}{\sqrt{2\pi\sigma_r^2(X_0)}} \exp\left\{ -\frac{(q_r - \mu_r(X_0))^2}{2\sigma_r^2(X_0)} \right\} \\ &\quad \times \prod_{i=r+1}^{M} \left[\int_{-q_r}^{q_r} \frac{1}{\sqrt{2\pi\sigma_i^2(X_0)}} \exp\left\{ -\frac{q_i^2}{2\sigma_i^2(X_0)} \right\} dq_i \right] \\ &\quad \times dq_r \cdots dq_2 \, dq_1 \, dX_0 \\ &< \int_0^\infty p(X_0) \prod_{i=r+1}^{M} \left[\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_i^2(X_0)}} \exp\left\{ -\frac{q_i^2}{2\sigma_i^2(X_0)} \right\} \right. \\ &\quad \left. \times \prod_{j=1}^{r} \left[\int_{|q_i|}^{\infty} \frac{1}{\sqrt{2\pi\sigma_j^2(X_0)}} \exp\left\{ -\frac{(q_j - \mu_j(X_0))^2}{2\sigma_j^2(X_0)} \right\} dq_j \right] dq_i \right] dX_0 \end{aligned} \quad (15)$$

where X_0 is the scintillation of desired user.

In Eq(15), the average and variance of the correlation value of 1th slot, $\mu_1(X_0)$, $\sigma_1^2(X_0)$ and the variance of the correlation value of $(r+1)$th slot, $\sigma_{r+1}^2(X_0)$ are derived, respectively, as

$$\mu_1(X_0) = GT_c \frac{L}{2}\left(\frac{M_e-1}{M_e}\right)\lambda_s X_0 \quad (16)$$

$$\sigma_1^2(X_0) = G^2 F_e T_c \left[\frac{L}{2}\left(\frac{M_e+1}{M_e}\right)\lambda_s X_0 + L\lambda_b + \frac{2I_b}{e}\right] + \frac{2I_s T_c}{e} + \sigma_{th}^2 \quad (17)$$

$$\sigma_{r+1}^2(X_0) = G^2 F_e T_c \left[L\frac{\lambda_s X_0}{M_e} + L\lambda_b + \frac{2I_b}{e}\right] + \frac{2I_s T_c}{e} + \sigma_{th}^2. \quad (18)$$

where G is the average APD gain, L is the length of the modified pseudo orthogonal M-sequences, I_s is the APD surface leakage current, F_e is the excess noise factor and σ_{th}^2 is the variance of thermal noise. F_e and σ_{th}^2 are given by

$$F_e = k_{eff}G + (1-k_{eff})\left(\frac{2G-1}{G}\right) \quad (19)$$

$$\sigma_{th}^2 = \frac{4k_B T_r T_c}{e^2 R_L} \quad (20)$$

where k_{eff} is the APD effective ionization ratio, k_B is Boltzmann's constant, T_r is the receiver noise temperature and R_L is the receiver load resistor.

6. Numerical results

In this section, we show the result from theoretical analysis of the BER performance in the slot synchronous case. In the multi-user case, we also present the BER performance of the proposed system and that of the conventional MPPM with the pseudo orthogonal M-sequences on the worst case condition. Table 1 shows the numerical conditions for evaluation. We use typical APD parameters[7][6] and assume that the slot duration is $1/156$ [μsec] and the length of modified pseudo orthogonal M-sequence, L, is 16 [chip].

Table 1. The numerical conditions

Name	Symbol	Value
Laser wavelength		830 [nm]
Background noise	P_b	-45 [dBm]
Quantum efficiency	η	0.6
Scintillation logarithm variance	σ_s^2	0.1
APD Gain	G	100
Effective ionization ratio	k_{eff}	0.02
Bulk leakage current	I_b	0.1 [nA]
Surface leakage current	I_s	10 [nA]
Modulation extinction ratio	M_e	100
Receiver noise temperature	T_r	1100[°K]
Receiver load resistor	R_L	1030[Ω]

Figure 5 shows the BER versus the received laser power per bit per frame P_{bit} for the proposed system ($M=4,\ r=2$), the binary antipodal DS/SS system, the OOK DS/SS system and the conventional MPPM system ($M=64,\ r=32$). The P_{bit} is given by

$$P_{bit} = \frac{rL}{2k}P_w \quad (21)$$

where k [bit] is the message bits per frame and P_w [dBm] is the received laser power per chip without scintillation. The four systems achieve the same normalized throughput (0.9375 [bit/slot]) and the same bit rate (156 [Mbps]). We define the BERs of the proposed system and the conventional MPPM system are $P_{SER}/2$. In Fig.5, the BER performance of the proposed system is the best in the four systems.

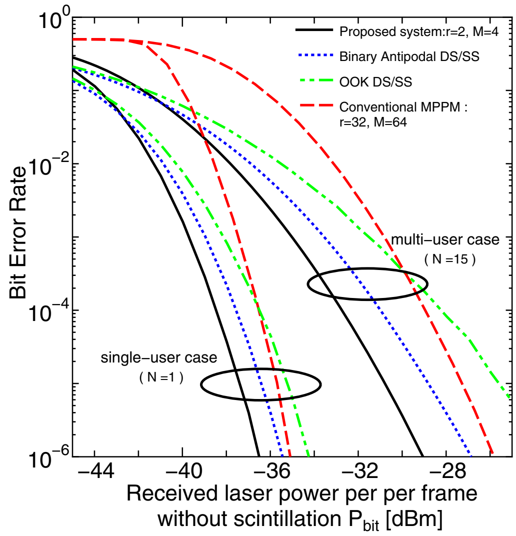

Figure 5. BER versus received laser optical power per bit per frame P_{bit} for the proposed system ($M=4$, $r=2$, $L=16$), the binary antipodal DS/SS system ($L=16$) ,the OOK DS/SS system ($L=16$) and the conventional MPPM system ($M=64$, $r=32$, $L_M=15$).

7. Conclusion

In this paper, in order to realize high normalized throughput, we have designed the modified orthogonal M-sequence sets and have proposed the OCDMA system using MPPM with the modified pseudo orthogonal M-sequence sets. Moreover, we show the result from theoretical analysis of the BER of the proposed system. Consequently, the normalized throughput of the proposed system is better than that of the conventional MPPM. Furthermore, in the same bit rate, the BER of the proposed system is better than that of the conventional systems.

Acknowledgment

This study was supported in part by Grant-in-Aid for Scientific Research (C) in Japan.

References

[1] Chung F.R.K,"Optical orthogonal codes:design,analysis,and applications",IEEE Trans.Inform.Theory,VOL35,No3,PP.595-604(1989)

[2] Shaar A.A,"Prime sequences:Quasioptical sequences for or channel code division multiplexing", Electron.Ltt, VOL19, PP.888-889(1983)

[3] Jen-Fa Huang and Dar-Zu Hsu,"Fiber-Grating-Based Optical CDMA Spectral Coding with Nearly Orthogonal M-Sequence Codes",IEEE Photonics.Ltt,VOL12,NO.9,PP.1252-1254(Sep 2000)

[4] Hisayoshi SUGIYAMA, Kiyoshi NOSU, "MPPM:A Method for Improving the Band-Utilization Efficiency in Optical PPM", IEEE Journal of Lightwave Technology, VOL LT-7, No3, PP.465-472(March 1989)

[5] Yusuke KOZAWA, Hiromasa HABUCHI, "Theoretical Analysis of Atmospheric Optical DS/SS with On-Off Orthogonal M-sequence Pairs ",ICICS07-P0686(Dec 2007)

[6] Hyuck M.Kwon,"Optical Orthogonal Code-Division Multiple-Access System Part1 : APD Noise and Thermal Noise", IEEE Trans. Comm, VOL42, No7, PP.2470-2479(July 1994)

[7] Tomoaki OHTSUKI, "Performance Analysis of Atmospheric Optical PPM CDMA systems", JOURNAL OF LIGHTWAVE TECHNOLOGY, VOL.21, No2, PP406-411(Feb 2003)