

# Efficient Maximum Likelihood Approach to Channel Estimation for Space-Time Coded Systems

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**Abstract:** In wireless communication systems, there is a performance degradation due to fading. Orthogonal space-time block coded (OSTBC) systems are known to be useful to improve the performance in fading channels. In OSTBC systems, the knowledge of the channel is required to detect symbols. This paper proposes a channel estimation method using M-algorithm. The proposed method can drastically reduce a computational complexity of the maximum likelihood approach and provide sufficient estimate accuracy. Simulation results show the effectiveness of the proposed method.

## 1. Introduction

With increasing demands in wireless communication systems such as cellular phones, WiMAX, and wireless LAN, a trend is to provide high-speed data transmission in fading channels. Orthogonal space-time block coded (OSTBC) [1] systems which provide a larger diversity gain are used to improve the transmission performance.

An OSTBC system requires the knowledge of the channel to detect symbols. Thus, channel estimation is a critical issue. There are currently many channel estimation methods. Although methods using training data can achieve high estimation accuracy, the use of the training data reduces frequency efficiency. Blind channel estimation methods which require no training data are attractive. A method based on second-order statistics [2] can reduce the required amount of data, however, it tends to have lower estimation accuracy. Whereas a method based on maximum likelihood [3] which jointly detects symbols and estimates a channel can achieve high estimation accuracy. However, its computational complexity is prohibitive for a practical implementation. Thus, at this moment, the decisive method has not been established.

In this paper, we consider the computational complexity reduction of the maximum likelihood approach. The computational complexity of the maximum likelihood estimation increases exponentially with both the constellation size and the number of blocks decoded at a time. We propose to use the M-algorithm whose effectiveness has been shown in various applications, e.g., multiuser detection in CDMA systems [4]. We apply the M-algorithm to OSTBC and show its effectiveness by examining both the estimate accuracy and computational complexity by computer simulation.

## 2. OSTBC systems

An OSTBC system is shown in Fig.1 (a). Consider a wireless communication system equipped with  $t$  transmit and  $r$

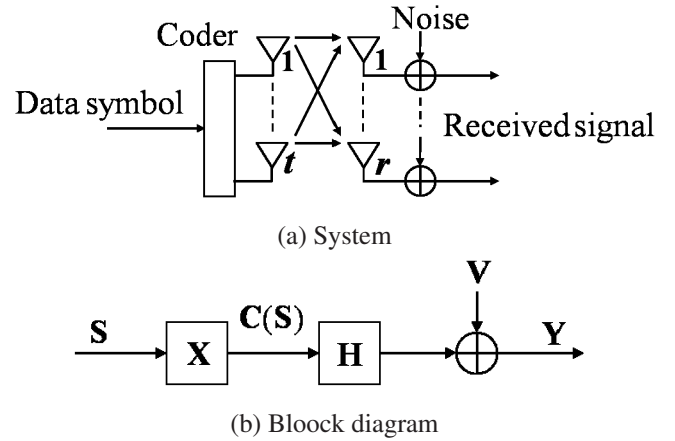


Figure 1. Orthogonal space-time block coded system.

receive antennae. Let the  $p$ th transmitted symbol block be  $\mathbf{s}^{(p)}$  which consists of  $K$  symbols  $\{s_k^{(p)}; k = 1, \dots, K\}$ . The  $k$ th data symbol of the  $p$ th block is drawn from a set  $C$ . Coded transmission matrix  $\mathbf{C}(\mathbf{s}^{(p)}) = \sum_{k=1}^K \mathbf{X}_k s_k^{(p)}$  is transmitted by  $t$  transmit antennae over  $T$  epochs, where  $\mathbf{X}_k$  is a  $t \times T$  orthogonal matrix determined by the used code.

For example,  $K = 2, t = 2, T = 2$ ,

$$\mathbf{C}(\mathbf{s}^{(p)}) = \begin{bmatrix} s_1^{(p)} & -s_2^{(p)} \\ s_2^{(p)} & s_1^{(p)} \end{bmatrix},$$

then its basis matrices are

$$\mathbf{X}_1 = \begin{bmatrix} x_1 & 0 \\ 0 & x_1 \end{bmatrix}, \mathbf{X}_2 = \begin{bmatrix} 0 & -x_2 \\ x_2 & 0 \end{bmatrix}.$$

The signal through a channel is received at  $r$  receive antennae. The channel can be represented by an  $r \times t$  matrix  $\mathbf{H}$ , whose element  $h_{j,i}$  is the channel gain from the  $i$ th transmit antenna to the  $j$ th receive antenna. The received signal of the  $p$ th block is written as an  $r \times T$  matrix  $\mathbf{Y}^{(p)} = \mathbf{H}\mathbf{C}(\mathbf{s}^{(p)}) + \mathbf{V}^{(p)}$ , where the matrix  $\mathbf{V}^{(p)}$  represents noise. Assuming that the channel remains static during the transmission of  $\mathbf{S} = [\mathbf{s}^{(1)} \dots \mathbf{s}^{(P)}]$ , the received signal can be written as  $\mathbf{Y} = [\mathbf{Y}^{(1)} \dots \mathbf{Y}^{(P)}] = \mathbf{H}\mathbf{C}(\mathbf{s}) + \mathbf{V}$  where  $\mathbf{C}(\mathbf{S}) = [\mathbf{C}(\mathbf{s}^{(1)}) \dots \mathbf{C}(\mathbf{s}^{(P)})]$  and  $\mathbf{V} = [\mathbf{V}^{(1)} \dots \mathbf{V}^{(P)}]$ . Figure 1 (b) shows the block diagram of the OSTBC system.

If the receiver has the knowledge of the channel, the symbols can be detected as follows:

$$\hat{s}_k^{(p)} = \text{Re}\{\text{tr}\{\mathbf{Y}^p \mathbf{X}_k^H \mathbf{H}^H\}\} \quad (1)$$

for  $k = 1, \dots, K$ ,  $p = 1, \dots, P$ , where  $(\cdot)^H$  represents the complex conjugate transpose of a matrix. If the channel is unknown, the channel should be estimated for symbol detection. Next we consider estimating the channel by a maximum likelihood approach.

### 3. Joint symbol detection and channel estimation

In the maximum likelihood estimation [3], the symbols are detected as

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s} \in C^{KP}} \|\mathbf{R}\mathbf{s}\|_2^2 \quad (2)$$

where  $\mathbf{s} = [s_1 \dots s_{KP}]^T = [\mathbf{s}^{(1)T} \dots \mathbf{s}^{(P)T}]^T$  is a symbol vector of length  $KP$  and an upper triangular matrix  $\mathbf{R}$  is determined as follows:

**Step1** Obtain  $\mathbf{G}$  whose  $(p, q)$ th submatrix is  $\mathbf{G}_{p,q}$  whose  $(k, l)$ th component is given by

$$[\mathbf{G}_{p,q}]_{k,l} = \text{Re}\{\text{tr}\{\mathbf{Y}^{(p)} \mathbf{X}_k^H \mathbf{X}_l \mathbf{Y}^{(q)H}\}\}$$

for  $k, l = 1, \dots, K$ ,  $p, q = 1, \dots, P$ .

**Step2** Set  $\alpha$  to a larger value than the greatest eigenvalue of  $\mathbf{G}$ .

**Step3** Decompose  $\mathbf{R}^T \mathbf{R} = \alpha \mathbf{I} - \mathbf{G}$  by Cholesky decomposition.

Note that (2) does not require the knowledge of the channel. The computational complexity of (2) is proportional to the number of the candidates  $|C|^{KP}$  where  $|C|$  is the number of the elements in  $C$ .

After  $\hat{\mathbf{s}}$  is determined, the channel estimation is obtained by

$$\hat{\mathbf{H}} = \mathbf{Y} \mathbf{C}^H (\hat{\mathbf{S}}) \{\mathbf{C} (\hat{\mathbf{S}}) \mathbf{C}^H (\hat{\mathbf{S}})\}^{-1}. \quad (3)$$

We can decode the symbols using  $\hat{\mathbf{H}}$  as long as the change of  $\mathbf{H}$  is little.

We propose to apply the M-algorithm [4] to reduce the computational complexity. The M-algorithm is a pruning technique that limits the number of symbol candidates and corresponds to a suboptimal tree search. This algorithm selects paths corresponding to smaller path metrics and retains at most  $M$  paths at each level of the tree. At the last level of the tree, the path with the smallest metric is selected and its corresponding symbol candidate is used as the final decision  $\hat{\mathbf{s}}$ .

The metric of the  $m$ th stage is

$$e_m = \|\mathbf{R}_m \mathbf{s}_m\|_2^2, \quad m = 1, \dots, KP \quad (4)$$

where  $\mathbf{R}_m = \mathbf{R}(KP - m + 1 : KP, KP - m + 1 : KP)$  and  $\mathbf{s}_m$  is a vector of length  $m$  consisted of a combination of a symbol candidate vector  $\mathbf{s}_{m-1} \in C^{m-1}$  and  $C$ . In each stage, we calculate about  $|C| \times M$  candidates.

The proposed algorithm is summarized as follows:

**Step1** Obtain  $\mathbf{R}$  and set  $m = 1$ .

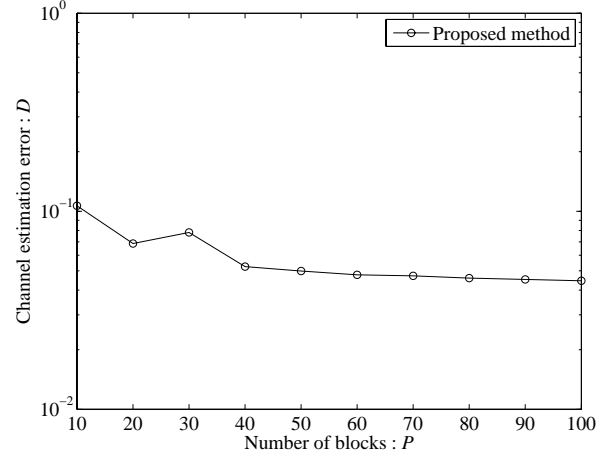


Figure 2. Influence of the number of blocks on the channel estimation accuracy.

**Step2** Generate symbol candidates  $\mathbf{s}_m$  and compute metrics of the candidates

$$e_m = \|\mathbf{R}_m \mathbf{s}_m\|_2^2.$$

**Step3** if  $m < KP$

Select and retain  $\min(|C|^m, M)$  candidates corresponding to smaller metrics.

$m = m + 1$  and go to **Step2**.

else

Select the smallest metric corresponding symbol candidate.

### 4. Simulation results

We show simulation results to evaluate the performance of the proposed method. We set  $t = 3$ ,  $r = 2$ ,  $T = 4$ ,  $K = 4$  and use the following code

$$\mathbf{C}(\mathbf{s}^{(p)}) = \begin{bmatrix} s_1^{(p)} & -s_2^{(p)} & -s_3^{(p)} & -s_4^{(p)} \\ s_2^{(p)} & s_1^{(p)} & s_4^{(p)} & s_3^{(p)} \\ s_3^{(p)} & s_4^{(p)} & s_1^{(p)} & -s_2^{(p)} \end{bmatrix}. \quad (5)$$

The channel gains are set zero-mean complex Gaussian random variables. The results are averaged over 100 different channels.

First, the influence of the number of blocks  $P$  is evaluated. We set  $M = 6$  and Signal to noise ratio (SNR) is 5 dB. Figure 2 shows the channel estimation accuracy as a function of  $P$ . The performance measure is the estimation error defined by

$$D = \frac{\|\mathbf{H} - \hat{\mathbf{H}}\|_F^2}{\|\mathbf{H}\|_F^2}.$$

It can be observed that the estimation error goes through the floor when  $P$  is more than 40 blocks.

We suppose that the time variation of  $\mathbf{H}$  is little and the symbols of 1000 blocks are decoded by using  $\hat{\mathbf{H}}$ . Symbol error rate (SER) is shown in Fig.3. SER is saturated within 40 blocks like in Fig. 2.

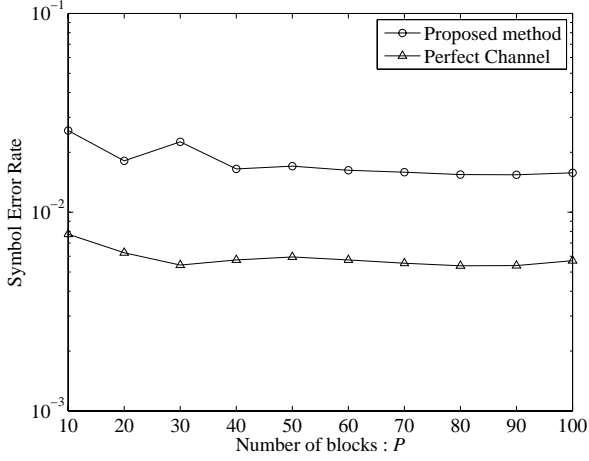


Figure 3. Influence of the number of blocks on SER.

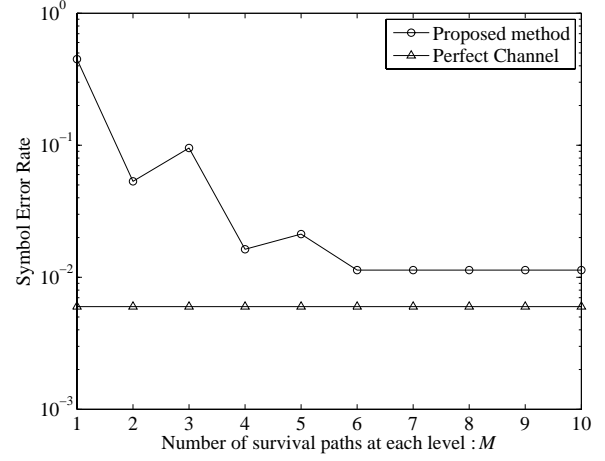


Figure 5. Influence of the survival paths at each level on SER.

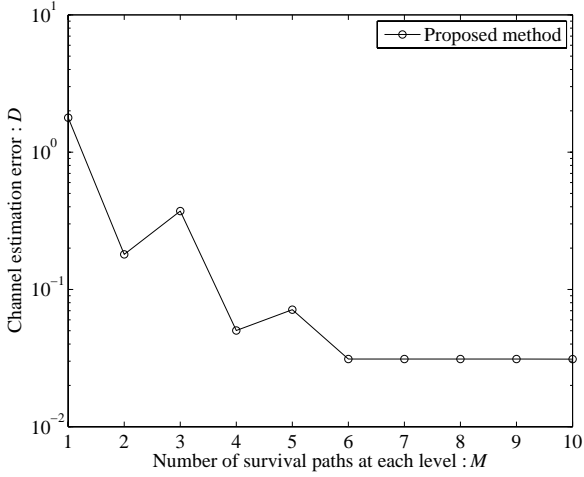


Figure 4. Influence of the number of the survival paths at each level on the channel estimation accuracy.

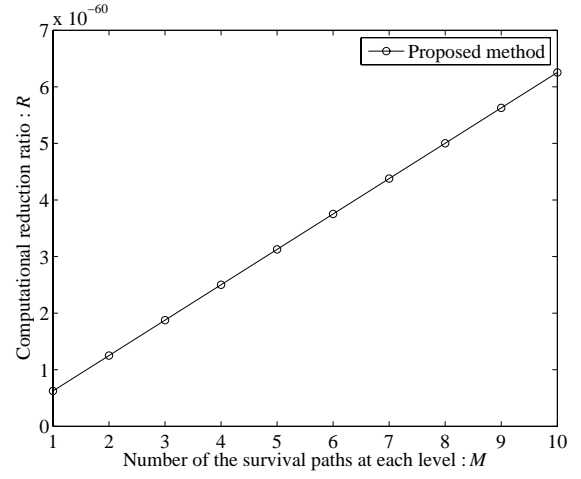


Figure 6. Computational complexity.

Next, we evaluate the number of survival paths at each level  $M$ . We set  $P = 50$  and  $\text{SNR} = 5$  dB and compute the channel estimation accuracy, the SER and the computational complexity. Both the estimation accuracy and the SER are sufficiently small when  $M$  is more than 6.

We compare the computational complexity of the M-algorithm with that of full search. The computational complexity is evaluated by the complexity reduction ratio defined by

$$R = \frac{\text{the number of multiplication needed by M-algorithm}}{\text{the number of multiplication needed by full search}}$$

Figure 6 shows the complexity reduction ratio as a function of  $M$ , the number of survival paths at each level. It can be observed that the M-algorithm greatly reduces the computational complexity.

Figure 7 shows the channel estimation accuracy versus the SNR, when  $M = 6$  and  $P = 50$  from past results. We see that the estimation error is less than  $10^{-2}$  when SNR is larger than

6 dB.

In Fig. 8, we plot the SER versus the SNR. It can be observed that the SER obtained by the proposed method is closer to the achievable SER with the perfect channel knowledge as SNR becomes larger.

We show additional results using another OSTBC in Figs. 9 and 10. We set  $t = 5$ ,  $r = 3$ ,  $T = 8$ ,  $K = 8$ ,  $M = 6$ ,  $P = 50$ , and

$$\mathbf{C}(\mathbf{s}^{(p)}) = \begin{bmatrix} s_1^{(p)} & -s_2^{(p)} & -s_3^{(p)} & -s_4^{(p)} & -s_5^{(p)} & -s_6^{(p)} & -s_7^{(p)} & -s_8^{(p)} \\ s_2^{(p)} & s_1^{(p)} & -s_4^{(p)} & s_3^{(p)} & -s_6^{(p)} & s_5^{(p)} & s_8^{(p)} & -s_7^{(p)} \\ s_3^{(p)} & s_4^{(p)} & s_1^{(p)} & -s_2^{(p)} & -s_7^{(p)} & -s_8^{(p)} & s_5^{(p)} & s_6^{(p)} \\ s_4^{(p)} & -s_3^{(p)} & s_2^{(p)} & s_1^{(p)} & -s_8^{(p)} & s_7^{(p)} & -s_6^{(p)} & s_5^{(p)} \\ s_5^{(p)} & s_6^{(p)} & s_7^{(p)} & s_8^{(p)} & s_1^{(p)} & -s_2^{(p)} & -s_3^{(p)} & -s_4^{(p)} \end{bmatrix}$$

We see that the proposed method still provides sufficient performance.

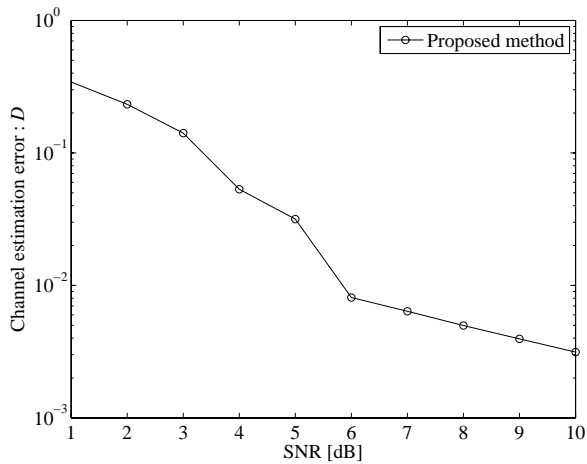


Figure 7. Channel estimation accuracy versus SNR.

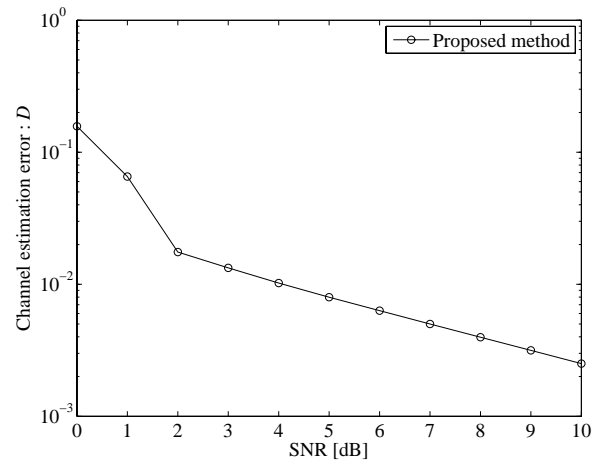


Figure 9. Channel estimation accuracy versus SNR (another code).

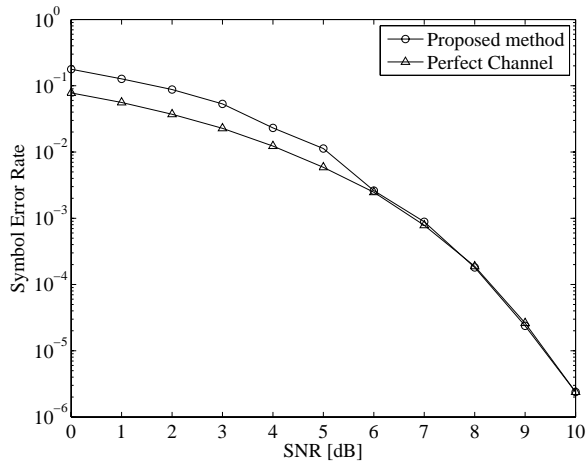


Figure 8. SER versus SNR.

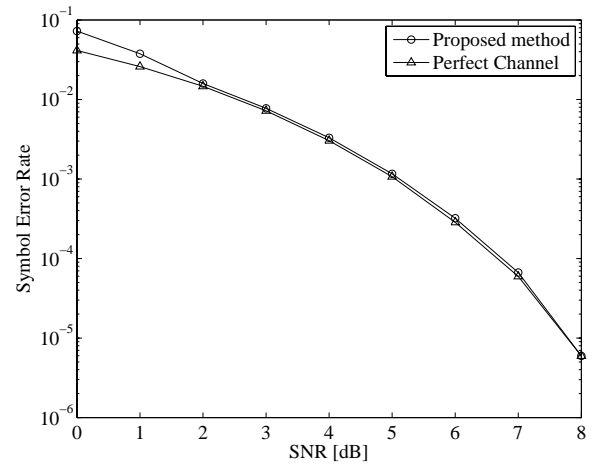


Figure 10. SER versus SNR (another code).

## 5. Conclusion

We proposed to apply the M-algorithm to OSTBC and evaluate its performance. We showed that the computational complexity is greatly reduced while the channel estimation accuracy remains acceptable.

### Acknowledgment

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