Energy-base Deadlock-free Supervisory Control of Quantitative Discrete Event Systems under Partial Observation

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Abstract: We study the control of partially observed non-terminating quantitative DESs under the fixed-initial-credit energy objective. We model the control using a two-player game played between the supervisor and the DES on a weighted automaton. The DES aims to execute the events so that its energy level goes below zero, while the supervisor aims to maintain the energy level above zero. We show that the proposed problem is reducible to finding a winning strategy in a turn-based reachability game.

Keywords—**Supervisory control, Discrete event system, Partial observation, Optimal control, Energy game**

1. Introduction

The supervisory control of qualitative discrete event systems (DESs) was introduced by Ramadge and Wonham in [1], where a DES is modeled by an automaton whose events are partitioned into controllable and uncontrollable ones. Based on the observed generated sequence, the supervisor controls the DES by disabling or enabling any of the controllable events at the current state of the system. The control objective is to synthesize a supervisor that controls the DES to generate a specified $*-$language.

In real situations, the supervisor may not be able to observe some events in the generated sequences. Therefore, the control under partial observation were introduced in [2], where the set of events of the DES is partition into the set of observable and unobservable ones. Takai et al. [3] introduced a framework of supervisory control under partial event and state observation by mapping each event and state to the corresponding masked event and masked state that are observed by a supervisor.

Supervisory control for $\omega-$language specification was studied in [4], [5]. The $\omega-$language is suitable for modeling the sequential behaviors of non-terminating systems. Optimal control of quantitative non-terminating DESs modeled by weighted automata was considered in [6]. The control cost is evaluated by the sum of the weights assigned to transitions along the generated infinite trajectory. However, partially observed non-terminating DESs were not considered.

Controller design problems can be formulated as two-player games played between the controller and the system. In [7], a control problem for partially observed DESs was transformed into the problem of satisfying a μ-calculus formula. The control problem under budget constraints were studied under a two-player game setting in [8]. In [9], the minimum attention controller synthesis for omega-regular objectives was consider using a two-player game automaton.

In this paper, we study the control of partially observed non-terminating quantitative DESs under the energy objective where the initial credit energy is given [10]. We model the control of the DES using a two-player game played between the supervisor and the DES on a weighted automaton. The DES aims to execute the events so that its energy level goes below zero after a finite number of events occur. On the other hand, the supervisor aims to maintain the energy level above zero. The fixed-initial-credit energy problem is to compute a supervisor under which the supervised DES contains no deadlock and the energy level of the DES never goes below zero. We show that the proposed problem is reducible to finding a winning strategy in a turn-based reachability game [10], [11].

The rest of the paper is organized as follows. Section 2 introduces quantitative DESs and provides the basic notations. Section 3 introduces supervisory control under partial observation based on two-player game setting. Section 4 formulates the control problem and provides algorithms. Finally, Section 5 presents the conclusions.

2. Quantitative Discrete Event Systems

In this paper, $\mathbb{N}$ denotes the set of natural numbers including zero, and $\mathbb{Z}$ denotes the set of integers.

We consider a quantitative DES modeled by a weighted automaton $G =< X, \Sigma, \delta, x_{G0}, w >$, where X is a finite set of states, Σ is a finite set of events, $\delta \subseteq X \times \Sigma \times X$ is a set of transition relations, $x_{G0} \in X$ is the initial state, and $w : \delta \to \mathbb{Z}$ is a function that assigns a weight to each transition. The event set Σ is partitioned into two disjoint sets: uncontrollable events set Σ_u and controllable events set Σ_c. Likewise, δ is partitioned into $\delta_c = \delta \cap (X \times \Sigma_c \times X)$ and $\delta_u = \delta \setminus \delta_c$. For each $x \in X$, let $\Sigma(x) = \{\sigma \in \Sigma | \exists x' \in X, (x, \sigma, x') \in \delta\}$. A state $x \in X$ such that $\Sigma(x) = \emptyset$ is called a *dead* state.

A *run* (*resp. history*) generated by the DES G is an infinite (*resp.* a finite) sequence $r = x_0\sigma_1x_1 \ldots \in X(\Sigma X)^\omega$ (*resp.* $h = x_0\sigma_1x_1 \ldots x_n \in X(\Sigma X)^*$) such that $(x_i, \sigma_{i+1}, x_{i+1}) \in \delta$ for each $i \in \mathbb{N}$ (*resp.* $i \in \{0, 1, \ldots, n\}$). For a history $h = x_0\sigma_1x_1 \ldots x_n$, $last(h)$ denotes the last state x_n. For a set of histories H, $last(H) = \bigcup_{h\in H}\{last(h)\}$. The history h is also called a *cycle* if $x_0 = x_n$. For each $i \in \mathbb{N}$, we denote the prefix $x_0\sigma_1x_1 \ldots x_i \in X(\Sigma X)^*$ of the run r by $r[i]$. Likewise, for a history $h = x_0\sigma_1x_1 \ldots x_n$ and an integer $i \in \{0, 1, \ldots, n\}$, $h[i]$ denotes the prefix $x_0\sigma_1x_1 \ldots x_i$. For each state $x \in X$, $Run(G, x)$ and $His(G, x)$ are the sets of all runs and histories generated by the DES G starting from x, respectively. A run, or a history, is *initialized* if it starts from the initial state x_{G0}. Let $Run(G) = Run(G, x_{G0})$ and $His(G) = His(G, x_{G0})$ be the sets of all runs and histories generated by G starting from the initial state, respectively. For a history $h = x_0\sigma_1x_1 \ldots x_n \in \bigcup_{x\in X} His(G, x)$, the *energy level* $EL(h) = \sum_{i=0}^{n-1} w(x_i, \sigma_{i+1}, x_{i+1})$.

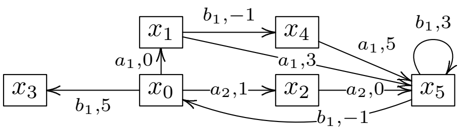

Figure 1. A partially observed DES, where x_0 is the initial state, $\Sigma_u = \{a_1, a_2\}$, $\Sigma_c = \{b_1\}$, $M_Y(x_0) = M_Y(x_3) = y_0$, $M_Y(x_1) = M_Y(x_2) = y_1$, $M_Y(x_4) = M_Y(x_5) = y_2$, $M_\Lambda(a_1) = M_\Lambda(a_2) = a$, and $M_\Lambda(b_1) = b$. The label of each transition represents the corresponding event and weight.

3. The Control under Partial Observation

Let Y and Λ be the set of masked states and masked events, respectively [3]. A surjective function $M_Y : X \to Y$ (*resp.* $M_\Lambda : \Sigma \to \Lambda$) maps each state (*resp.* event) to its masked state (*resp.* masked event). A supervisor cannot observe the states of the DES and the generated events, but can observe their masked states and masked events. We assume that $M_Y(x_{G0}) = y_{G0}$ and $M_Y^{-1}(y_{G0}) = \{x_{G0}\}$. An observation function $M_O : \bigcup_{x \in X} His(G, x) \to Y(\Lambda Y)^*$ is defined as follows: for each $h = x_0\sigma_1 x_1 \ldots \sigma_n x_n \in \bigcup_{x \in X} His(G, x)$,

$$M_O(h) = \begin{cases} y_{G0}, & \text{if } h = x_{G0}, \\ M_O(h[n-1])M_\Lambda(\sigma_n)M_Y(x_n) & \text{otherwise.} \end{cases}$$

Let $M_O(H) = \{M_O(h) | h \in H\}$ for each $H \subseteq His(G)$.

A supervisor observes generated masked sequences of the histories and control the DES by disabling any of the controllable events at the current state of the system. An event set $\gamma \subseteq \Sigma$ such that $\Sigma_u \subseteq \gamma$ is called a *control pattern*, which represents control-enabled events by the supervisor. Let $\Gamma = \{\gamma \in 2^\Sigma | \Sigma_u \subseteq \gamma\}$ be the set of all control patterns. If the DES generates a history h, the supervisor observes the sequence $M_O(h)$ and assigns a control pattern $\gamma \in \Gamma$. Then, only events included in γ are enabled at the state $last(h)$.

We consider the control process as a two-player game played between the supervisor and the DES, where a strategy of the supervisor (*resp.* the DES) is a function $\pi_S : M_O(His(G)) \to \Gamma$ (*resp.* $\pi_D : His(G) \times \Gamma \to \delta$). We impose the following condition on π_D: $\forall h \in His(G), \forall \gamma \in \Gamma$ such that $\Sigma(last(h)) \cap \gamma \neq \emptyset$, if $\pi_D(h, \gamma) = (x, \sigma, x')$, then $x = last(h)$ and $\sigma \in \gamma$. Let Π_S and Π_D be the sets of strategies of the supervisor and the DES, respectively. For each generated history h, the supervisor first selects the control pattern $\gamma = \pi_S(M_O(h))$, then the DES executes the transitions $(x, \sigma, x') = \pi_D(h, \gamma)$. This process is repeated so that the following run or history, which is denoted by $seq(G, \pi_s, \pi_d)$, is generated:

1) A run $r = x_0\sigma_1 x_1 \ldots$ such that $\forall i \in \mathbb{N}$, $\sigma_{i+1} \in \pi_S(M_O(r[i]))$ and $(x_i, \sigma_{i+1}, x_{i+1}) \in \pi_D(r[i], \pi_S(M_O(r[i])))$.
2) A history $h = x_0\sigma_1 x_1 \ldots x_n$ such that $\Sigma(x_n) \cap \pi_S(M_O(h)) = \emptyset$, and $\forall i \in \{0, 1, \ldots, n-1\}$, $\sigma_{i+1} \in \pi_S(M_O(h[i]))$ and $(x_i, \sigma_{i+1}, x_{i+1}) \in \pi_D(h[i], \pi_S(M_O(h[i])))$.

The history in case 2) is called a *deadlock*, since the DES is forced to terminate at the dead state x_n. For each $\pi_S \in \Pi_s$, let $Run(G, \pi_S) = \{seq(G, \pi_S, \pi_D) \in Run(G) | \pi_D \in \Pi_D\}$ and $Dead(G, \pi_S) = \{seq(G, \pi_S, \pi_D) \in His(G) | \pi_D \in \Pi_D\}$.

4. Fixed-initial-credit Energy Problem

In this paper, we consider the following control problem.

Definition 1: For a given credit $c_0 \in \mathbb{N}$, the *fixed-initial-credit energy problem* is to find a strategy $\pi_s \in \Pi_S$ such that 1) $Run(G, \pi_S) \neq \emptyset$ and $Dead(G, \pi_S) = \emptyset$, and 2) for each $h \in His(G, \pi_S)$, $c_0 + EL(h) \geq 0$.

We show that this problem is reducible to computing a winning strategy in a turn-based reachability game [11].

An *observation function* is a function $o : X \to \mathbb{Z} \cup \{\bot\}$, which indicates the set of possible current states of the DES and their energy levels. For each state $x \in X$, if $o(x) \in \mathbb{Z}$, then x is a possible current state and its energy is $o(x)$, otherwise x is not the current state. Denoted by $supp(o) = \{x \in X | o(x) \neq \bot\}$ is the set of all possible current states indicated by the function o. The function o is said to be *non-negative* if $o(x) \geq 0$ for each $x \in supp(o)$. Let O be the set of all observation functions of the DES G. Let $\preceq$ be a relation on O such that for each $o_1, o_2 \in O$, $o_1 \preceq o_2$ if 1) $supp(o_1) = supp(o_2)$ and 2) $o_1(x) \leq o_2(x)$ for each $x \in supp(o_1)$.

For a control pattern $\gamma \in \Gamma$, a masked event $\lambda \in \Lambda$, and a masked state $y \in Y$, o_2 is the (γ, λ, y)-*successor* of o_1 if
1. $supp(o_2) = \{x_2 \in X | \exists (x_1, \sigma, x_2) \in \delta, x_1 \in supp(o_1), \sigma \in \gamma \cap M_\Lambda^{-1}(\lambda), M_Y(x_2) = y\}$,
2. for each $x_2 \in supp(o_2), o_2(x_2) = min\{o_1(x_1) + w(x_1, \sigma, x_2) | (x_1, \sigma, x_2) \in \delta, x_1 \in supp(o_1), \sigma \in \gamma \cap M_\Lambda^{-1}(\lambda)\}$, and
3. for each $x_1 \in supp(o_1)$, there exists $(x_1, \sigma, x_2) \in \delta$ such that $x_2 \in supp(o_2)$ and $\sigma \in \gamma \cap M_\Lambda^{-1}(\lambda)$.

Namely, the set of possible current states changes from $supp(o_1)$ to $supp(o_2)$ if the supervisor selects the control pattern γ, and observes the masked state y and the masked event λ. The condition 3 guarantees that the selected control pattern γ enables at least one event at each state in $supp(o_1)$. Moreover, the observation o_2 indicates the worst-case energy level of each state in $supp(o_2)$. For any observation $o \in O$, $succ(o, \gamma, \lambda, y)$ denotes the (γ, λ, y)-*successor* of o.

Then, we construct a game automaton $H =< Q_H = Q_S \cup Q_D, \Sigma_H = \Gamma \cup (\Lambda \times Y), \delta_H = \delta_S \cup \delta_D, o_{H0} >$, where $Q_S \subseteq O((\Lambda \times Y)O)^*$ (*resp.* $Q_D \subseteq O((\Lambda \times Y)O)^*\Gamma$) is the set of states of the supervisor (*resp.* the DES). $\delta_S \subseteq Q_S \times \Gamma \times Q_D$ (*resp.* $\delta_D \subseteq Q_D \times (\Lambda \times Y) \times Q_S$) is the set of out-going transitions from the supervisor's (*resp.* the DES's) states, and $o_{H0} \in Q_S$ is the initial state. The construction of H is shown in Algorithm 1. This algorithm is modified from the algorithm for solving the fixed initial credit problem proposed in [10].

Figure 2 shows the game automaton constructed from the DES in Figure 1 using Algorithm 1 where the initial credit $c_0 = 0$. The dead states in this automaton are h_2, h_4, h_5, and h_6. Notice that $h_2 = o_0(b, y_0)o_2$, and $supp(o_2)$ contains the dead state x_3. Moreover, $h_4 = o_0(a, y_1)o_1(b, y_2)o_4$, and the function o_4 is not non-negative because $o_4(x_4) = -1$. Algorithm 1 also returns the state set $Q^+ \subseteq Q_H$ and the function PRE $: Q^+ \to \mathbb{N}$. The set $Q^+ = \{q = o_0(\lambda_1, y_1)o_1 \ldots (\lambda_n, y_n)o_n \in Q_S | o_n$ is non-negative, $\Sigma_H(q) = \emptyset$, and there exists an integer $m \in \{0, 1, \ldots, n-1\}$ such that $o_m \preceq o_n\}$. For each $q = o_0(\lambda_1, y_1)o_1 \ldots (\lambda_n, y_n)o_n \in Q^+$, PRE$(q)$ is the index

Algorithm 1 $H(G)$

Require: $G =< X, \Sigma, \delta, x_{G0}, w >$

$Q^+ \leftarrow \emptyset$

$o_{H0}(x) \leftarrow \begin{cases} c_0 & \text{if } x = x_{G0}, \\ \perp & \text{otherwise,} \end{cases}$, and add o_{H0} in Q_S

for all $q_s = o_0(\lambda_1, y_1)o_1 \dots (\lambda_n, y_n)o_n \in Q_S$ **do**

  if o_n is non-negative **and** PRE(q_s)$= -1$ **then**

    for all $(\gamma, \lambda, y) \in \Gamma \times \Lambda \times Y$ such that exists $o = succ(o_n, \gamma, M_\Lambda(\sigma), M_Y(x'))$ **do**

      add $q_s\gamma$ in Q_D and $(q_s, \gamma, q_s\gamma)$ in δ_S

      if $q'_s = q_s(\lambda, y)o \notin Q_s$ **then**

        add q'_s in Q_s

      end if

      add $(q_s\gamma, (\lambda, y), q'_s)$ in δ_D

    end for

  else if o_n is non-negative **then**

    add $q_s \in Q^+$

  end if

end for

return $H =< Q_H = Q_S \cup Q_D, \Sigma_H = \Gamma \cup (\Lambda \times Y), \delta_H = \delta_S \cup \delta_D, o_{H0} >, Q^+$

function PRE($o_0(\lambda_1, y_1)o_1 \dots (\lambda_n, y_n)o_n$)

  if $\exists m \in \{0, 1, \dots, n-1\}, o_m \preceq o_n$ **then**

    return m

  else

    return -1

  end if

end function

$m < n$ such that $o_m \preceq o_n$. For the example in Figure 2, $Q^+ = \{h_5, h_6\}$.

Theorem 2: The state set Q is finite.

Proof: For every infinite sequences $o_0o_1 \dots \in O^\omega$ such that o_k is non-negative and $supp(o_k) = supp(o_{k+1})$ for each $k \in \mathbb{N}$, there exist $i, j \in \{0, 1, \dots, n\}$ such that $i < j$ and $o_i \preceq o_j$ by Dickson's lemma. Since the state set X is finite, $\{supp(o)|o \in O\} = 2^X$ is also finite. Therefore, for every $o_0o_1 \dots \in O^\omega$ such that o_k is non-negative, there exist $i, j \in \{0, 1, \dots, n\}$ such that $i < j$ and $o_i \preceq o_j$.

Then, we prove this theorem using a contradiction. Suppose that Q is infinite. Since X and Σ are finite, the number of outgoing transitions at each state in Q is also finite. By König's lemma, there exists $q_0^S\gamma_1 q_1^D(\lambda_1, y_1)q_1^S \dots \in Run(H)$. From the construction of H, there exist $o_0(\lambda_1, y_1)o_1 \dots \in O((\Lambda \times Y)O)^\omega$ such that $o_0(\lambda_1, y_1)o_1 \dots (\lambda_k, y_k)o_k \in Q_S$ for each $k \in \mathbb{N}$, all observations $o_0, o_1, \dots$ are non-negative, and there is no $i, j \in \mathbb{N}$ such that $i < j$ and $o_i \preceq o_j$. However, this is a contradiction to Dickson's lemma in the above discussion. ■

From Theorem 2, the construction of H always terminates. Then, from the construction of H, it can be easily shown that $\bigcup_{q \in Q} His(H, q)$ does not contain any cycle. Moreover, each dead state q of H (i.e., each $q \in Q$ such that $\Sigma_H(q) = \emptyset$) is included in Q_s.

Then, we consider a turn-based reachability game played on H between the supervisor and the DES. A strategy of the supervisor (*resp.* the DES) is a function $\phi_S : Q_S \to \delta_S$ (*resp.* $\phi_D : Q_D \to \delta_D$). We restrict that for each $q_d \in Q_D$, $\phi_D(q_d)$ is an out-going transition from q_d. Let Φ_s and Φ_D be the set of strategies of the supervisor and the DES for the game H, respectively. Both players play the game by alternately selecting an out-going transition from the current state of the game in their turns. Recall that we have $Q^+ = \{q = o_0(\lambda_1, y_1)o_1 \dots (\lambda_n, y_n)o_n \in Q_S | o_n$ is non-negative, $\Sigma_H(q) = \emptyset$, and $\exists m \in \{0, 1, \dots, n-1\}$ such that $o_m \preceq o_n.\}$. The objective of the supervisor is to reach any state in Q^+, while the objective of the DES is to prevent the game from entering Q^+. Since $\bigcup_{q \in Q} His(H, q)$ does not contain any cycle, this game always terminate at a dead state. Let $seq(H, \phi_s, \phi_d) \in His(H)$ be the history generated under strategies ϕ_s and ϕ_d, and $Dead(H, \phi_s) = \{h = seq(H, \phi_s, \phi_d) | \phi_d \in \Phi_d, \Sigma_H(last(h)) = \emptyset.\}$.

Definition 3: The Q^+*-reachability problem* is to find $\phi_s \in \Phi_s$ such that $last(Dead(H, \phi_s)) \subseteq Q^+$.

Let $H' =< Q_H \setminus Q^+, \Sigma_H, \delta'_H, o_{H0} >$ be the automaton modified from H as follows: for each $(q_d, (\lambda, y), q_s) \in \delta_H$, $(q_d, (\lambda, y), \text{PRE}(q_s)) \in \delta'_H$ if $q_s \in Q^+$, and $(q_d, (\lambda, y), q_s) \in \delta'_H$ otherwise. Then, from a given strategy $\phi_s \in \Phi_s$ for the game H, we define a strategy $\pi_{\phi_s} \in \Pi_S$ for the game G as follows. For each $h = y_0\lambda_1 y_1 \dots \lambda_n y_n \in M_0(His(G))$, 1) if there exists a history $h_H = q_0^S\gamma_1 q_1^D(\lambda_1, y_1)q_1^S \dots \gamma_n q_n^D (\lambda_n, y_n)q_n^S \in His(H')$ such that $\gamma_i = \phi_s(q_i^S)$ for each $i \in \{0, 1, \dots, n-1\}$, then $\pi_{\phi_s}(h) = \phi_s(q_n^s)$; 2) otherwise, $\pi_{\phi_s}(h) = \Sigma$. The modified automaton of the game automaton in Figure 2 is illustrated in Figure 3

Theorem 4: There exists $\pi_s \in \Pi_S$ that satisfies the fixed-initial-credit energy problem if and only if there exists $\phi_s \in \Phi_s$ that satisfies the Q^+-reachability problem. Moreover, for a given strategy $\phi_s \in \Phi_s$ that satisfies the Q^+-reachability problem, the strategy $\pi_{\phi_s} \in \Pi_S$ satisfies the fixed-initial-credit energy problem.

Proof: $(\rightarrow)$ Let $\pi_s \in \Pi_s$ be a strategy of the supervisor for the game G that satisfies the fixed-initial-credit energy problem. Let $\phi_s \in \Phi_s$ be a strategy of the supervisor for the

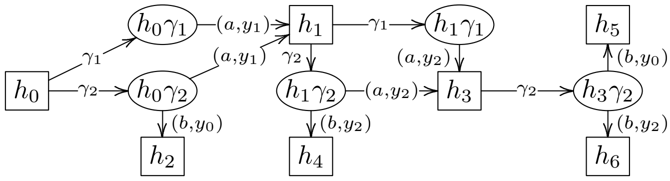

Figure 2. The game automaton constructed from the DES in Figure 1 using Algorithm 1. The control patterns are $\gamma_1 = \{a_1, a_2\}$ and $\gamma_2 = \{a_1, a_2, b_1\}$. Observations o_0, o_1, o_2, o_3, and o_4 are defined as follows. $supp(o_0) = \{x_0\}$, $o_0(x_0) = c_0 = 0$. $supp(o_1) = \{x_1, x_2\}$, $o_1(x_1) = 0, o_1(x_2) = 1$. $supp(o_2) = \{x_3\}, o_2(x_3) = 5$. $supp(o_3) = \{x_5\}$, $o_3(x_5) = 1$. $supp(o_4) = \{x_4\}$, $o_4(x_4) = -1$. Then, we have $h_0 = o_0$, $h_1 = h_0(a, y_1)o_1$, $h_2 = h_0(b, y_0)o_2$, $h_3 = h_1(a, y_2)o_3$, $h_4 = h_1(b, y_2)o_4$, $h_5 = h_3(b, y_0)o_0$, $h_6 = h_3(b, y_2)o_3$.

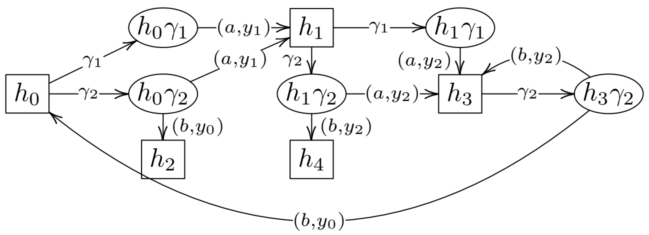

Figure 3. The automaton modified from one in Figure 2.

game H defined as follows.
1. $\phi_s(q_{H0}) = \pi_s(y_{G0})$.
2. For each $q = o_0(\lambda_1, y_1)o_1 \ldots (\lambda_n, y_n)o_n \in Q_S$,

$$\phi_s(q) = \pi_s(y_{G0}\lambda_1 y_1 \ldots \lambda_n y_n).$$

From the construction of H, ϕ_s is well defined. Then, we will show that $last(Dead(H, \phi_s)) \subseteq Q^+$ using a contradiction. Obviously, $Dead(H, \phi_s) \subseteq Q_s$. Suppose that there exists $h_H = o_0(\lambda_1, y_1)o_1 \ldots (\lambda_n, y_n)o_n \in Dead(H, \phi_s)$ such that $last(h_H) \notin Q^+$. Then, at least one of the following cases holds.
1. $last(h_H)$ is not non-negative.
2. There does not exist a transition $(\gamma, \lambda, y, o) \in \Gamma \times \Lambda \times Y \times \times O$ such that o is the $(\gamma, \lambda, y)-$successor of o_n.
For case 1, from the construction of H, there exists $h_G = x_0\sigma_1 x_1 \ldots x_n \in His(G, \pi_s)$ such that $M_O(h_G) = y_{G0}\lambda_1 y_1 \ldots \lambda_n y_n$, and $c_0 + EL(h_G) < 0$. For case 2, from the construction of H, there exists $h_G = x_0\sigma_1 x_1 \ldots x_n \in His(G, \pi_s)$ such that $M_O(h_G) = y_{G0}\lambda_1 y_1 \ldots \lambda_n y_n$ and $\Sigma(x_n) = \emptyset$. Therefore, $Dead(G, \pi_s) \neq \emptyset$. Both cases are contradictions to Definition 1.

($\leftarrow$) Let $\phi_s \in \Phi_s$ be a strategy of the supervisor for the game H that satisfies the Q^+-reachability problem. We show that the strategy $\pi_{\phi_s} \in \Pi_s$ satisfies the fixed-initial-credit energy problem using a contradiction. Suppose that π_{ϕ_s} does not satisfy the problem. By Definition 1, we have $Dead(G, \pi_{\phi_s}) \neq \emptyset$ or $\exists h \in His(G, \pi_{\phi_s}), c_0 + EL(h) < 0$.

First, consider the case where $Dead(G, \pi_{\phi_s}) \neq \emptyset$. Let $h_G = x_0\sigma_1 x_1 \ldots \sigma_n x_n \in Dead(G, \pi_{\phi_s})$. From the construction of H and H', there exists $h_H = q_0^S\gamma_1 q_1^D(\lambda_1, y_1)q_1^S \ldots \gamma_n q_n^D(\lambda_n, y_n)q_n^S \in His(H', \phi_s)$ such that $\Sigma_H(q_n^S) = \emptyset$ and $M_O(h_G) = y_{G0}\lambda_1 y_1 \ldots \lambda_n y_n$. However, since $q_n^s \in Q'_H$ and $\Sigma_H(q_n^S) = \emptyset$, we have $q_n^s \notin Q^+$. Thus, $h_H \in His(H, \phi_s)$ is the history that visits a dead state which is not in Q^+. This is a contradiction to Definition 3.

Next, consider the case where there exists $h_G = x_0\sigma_1 x_1 \ldots \sigma_n x_n \in His(G, \pi_{\phi_s})$, $c_0 + EL(h) < 0$. From the construction of H and H', there exists $h_H = q_0^S\gamma_1 q_1^D(\lambda_1, y_1)q_1^S \ldots \gamma_n q_n^D(\lambda_n, y_n)q_n^S \in His(H', \phi_s)$ such that $o = last(q_n^S) \in O$ is not non-negative and $M_O(h_G) = y_{G0}\lambda_1 y_1 \ldots \lambda_n y_n$. From the construction of H', h_H must also be included in $His(H, \phi_s)$. Obviously, $q_n^S \notin Q^+$. This is a contradiction to Definition 3. ■

From Theorem 4, the fixed-initial-credit energy problem can be solved by algorithms for computing a wining positional strategy of the first player turn-based reachability games [11].

5. Conclusions

We studied the supervisory control of partially observed non-terminating quantitative DESs under the fixed-initial-credit energy objective. Partial observation is modeled by mapping each event and state of the DES to the corresponding masked event and masked state that are observed by a supervisor. An optimal control action is represented by a winning strategy of a two-player game played between the supervisor and the DES on a weighted automaton. The fixed-initial-credit energy problem is to synthesize a supervisor under which the supervised DES does not contain a deadlock and the energy level of the system never goes below zero. Then, the proposed problem was reduced to computing a winning strategy in a turn-based reachability game. It is future work to consider the other supervisory control problems of non-terminating DESs under partial observation.

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References

[1] P. J. Ramadge and W. M. Wonham, "Supervisory control of a class of discrete-event process," *SIAM J. Control Optim.*, vol. 25, no. 1, pp. 206–230, Jan. 1987.

[2] F. Lin and W. M. Wonham, "On observability of discrete event systems," *Information Sciences*, vol. 44, no.3, pp. 173–198, 1998.

[3] S. Takai, T. Ushio and S. Kodama, "A new class of supervisors for partially observed discrete event systems," *Proc. ACC*, pp. 2631–2635, 1995.

[4] J. G. Thistle and W. M. Wonham, "Supervision of infinite behavior of discrete-event systems," *SIAM J. Control Optim.*, vol. 32, no. 4, pp. 1098–1113, 1994.

[5] P. J. Ramadge, "Some tractable supervisory control problems for discrete-event systems modeled by Büchi automata," *IEEE Trans. Automat. Control*, vol. 34, no. 1, pp. 10–19, 1989.

[6] S. Pruekprasert, T. Ushio, and T. Kanazawa, "Quantitative Supervisory Control Game for Discrete Event Systems," *to appear in IEEE Transactions on Automatic Control*, doi: 10.1109/TAC.2015.2513901.

[7] A. Arnold, A. Vincent, and I. Walukiewicz, "Games for synthesis of controllers with partial observation," *Theoretical Computer Science*, vol. 303, no. 1, pp. 7–34, 2003.

[8] K. Chatterjee, R. Majumdar, and T. A. Henzinger, "Controller synthesis with budget constraints," *Lecture Notes in Computer Science*, vol. 4981, pp. 72–86, 2008.

[9] K. Chatterjee, R. Majumdar, "Minimum attention controller synthesis for omega-regular objectives," *Lecture Notes in Computer Science*, vol. 6919, pp 145–159, 2011.

[10] A. Degorre, L. Doyen, R. Gentilini, J. Raskin, and S. Torunczyk, "Energy and Mean-Payoff Games with Imperfect Information," *Proc. CSL'10*, pp. 260–274, 2010.

[11] L. Alfaro, T. A. Henzinger, and O. Kupferman, "Concurrent reachability games," *Theoretical Computer Science*, vol. 386, nol 3, pp. 188–217, 2007.