An Improved Block Tomlinson-Harashima Precoder for Multi-user MIMO Systems

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Abstract: In this paper, we propose two algorithms which improve the error performance of the Block Tomlinson-Harashima precoding (BTHP) system. The proposed expanded ML (eML) algorithm provides an approach to apply the ML receiver to the BTHP system. The proposed block QR (bQR) algorithm makes the effective MIMO channel from which the proposed eML can obtain spatial diversity gain. By applying the proposed methods, BTHP can obtain the spatial diversity and considerable error performance improvement.

1. Introduction

Because of their huge potential gain in channel capacity, there are many efforts to use the multi-input multi-output (MIMO) techniques at the multi-user (MU) communication systems. In the MU MIMO systems, multiple mobile stations with multiple antennas communicate with a base station. The base station has multiple antennas as many as total number of antennas of simultaneously supported mobiles. In the uplink, the base station with full channel state information (CSI) can apply a single user (SU) MIMO receiver to estimate the transmitted signals from all mobile stations. In the downlink, however, because a mobile station has fewer antennas than the base station and cannot cooperate with the others, a user cannot receive the data alone. Thus, there are many attempts to help a user to receive the data successfully by pre-eliminating the multi-user interference (MUI) [3][4].

The ideal approach to solve the MUI problem with full CSI is the dirty paper coding (DPC) [1]. Using perfect CSI and complete knowledge of the transmitted signals, DPC can pre-cancel interference at the transmitter without capacity loss. However, because DPC requires infinitely large codebook and cannot be implemented, several practical interference pre-cancelation techniques have been proposed. Tomlinson-Harashima precoder (THP) is a main example of nonlinear practical approaches which can reduce channel capacity loss relatively to linear techniques like channel inversion [2]. Because of its simple structure, THP is an attractive method for MU MIMO precoder. Although THP can transmit signals without MUI regardless of the number of user's antennas, when a user receives multiple data streams, it cannot guarantee spatial diversity gain. Block THP (BTHP), the combination of channel decomposition which creates block triangular effective channel and vector-level THP, can achieve the spatial diversity because it eliminates MUI only, but the conventional approach [5] cannot provide spatial diversity because of its transceiver structure.

In this paper, we propose two algorithms that can be applied to BTHP system, and obtain the spatial diversity gain from multiple antennas of each mobile. We assume that the base station has full CSI for all mobiles and sufficiently many transmit antennas, and mobiles have CSI of their own effective channel. We propose a maximum likelihood (ML) receiver which can be applied in BTHP system, and a channel decomposition method that forms a block triangular effective channel from which the proposed receiver can obtain the spatial diversity gain. Through the proposed algorithms, BTHP can obtain full spatial diversity gain.

This paper is organized as follows. Next section provides the system model and assumptions. In Section 3, we describe the proposed eML receiver and bQR algorithm. In section 4, we evaluate the error performance of the proposed algorithms by computer simulations and draw some conclusions in section 5.

2. System Model and Assumptions

In this paper, a downlink MU MIMO communication system is considered. Base station has N_T antennas and K users have $N_{R_1}, N_{R_2}, \ldots, N_{R_K}$ antennas respectively. We assume that N_T is equal to or larger than $N_R = \sum_{k=1}^{K} N_{R_k}$.

In this system, a received signal vector of the k^{th} user is given by

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{s} + \mathbf{n}_k, \tag{1}$$

where $\mathbf{H}_k$ is a $N_{R_k} \times N_T$ channel matrix corresponding to k^{th} user, whose elements are assumed to be i.i.d. complex Gaussian random variables with zero-mean and unit variance, $\mathbf{s}$ is a $N_T \times 1$ transmitted signal vector, and $\mathbf{n}_k$ is a $N_T \times 1$ additive white Gaussian noise vector with variance σ_n^2. Note that the received signal power is normalized so that $1/\sigma_n^2$ is average received SNR of k^{th} user.

Denoting the total channel matrix $\mathbf{H}$, signal vectors $\mathbf{r}$ and noise vector $\mathbf{n}$ as

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1^T & \mathbf{H}_2^T & \cdots & \mathbf{H}_K^T \end{bmatrix}^T$$

$$\mathbf{r} = \begin{bmatrix} \mathbf{r}_1^T & \mathbf{r}_2^T & \cdots & \mathbf{r}_K^T \end{bmatrix}^T$$

$$\mathbf{n} = \begin{bmatrix} \mathbf{n}_1^T & \mathbf{n}_2^T & \cdots & \mathbf{n}_K^T \end{bmatrix}^T,$$

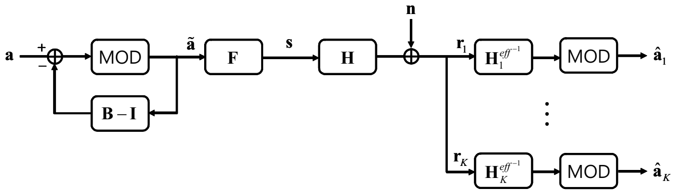

Figure 1: Block diagram of the downlink MU MIMO system in which BTHP is used.

the total received signal vector $\mathbf{r}$ can be represented by

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}. \tag{2}$$

We assume that the channel matrix $\mathbf{H}$ has full rank.

Figure 1 describes the block diagram of downlink MU MIMO system in which BTHP is used. For BTHP, the block triangular effective channel which is formed by channel decomposition is given by

$$\mathbf{H}_{eff} = \mathbf{H}\mathbf{F}. \tag{3}$$

And if we define the transmit signal vector of k th user $\tilde{\mathbf{a}}_k$ as

$$\tilde{\mathbf{a}}_k = \left(\mathbf{a}_k - \mathbf{H}_{k,k}^{eff^{-1}} \sum_{i=1}^{k-1} \mathbf{H}_{k,i}^{eff} \tilde{\mathbf{a}}_i \right)_{\mathrm{mod}} \qquad \tilde{\mathbf{a}}_1 = \mathbf{a}_1, \tag{4}$$

the received signal vector of k th user is

$$\begin{aligned} \mathbf{r}_k &= \sum_{i=1}^{k} \mathbf{H}_{k,i}^{eff} \tilde{\mathbf{a}}_i + \mathbf{n}_k \\ &= \mathbf{H}_{k,k}^{eff} \left(\mathbf{a}_k + \mathbf{d}_k \right) + \mathbf{n}_k, \end{aligned} \tag{5}$$

where $\mathbf{d}_k$ is a codevector added by the modulo device and there is no MUI in the received signal.

At the receiver side, by using zero-forcing (ZF) filter $\mathbf{H}_{k,k}^{eff^{-1}}$ and modulo device, each user can estimate the transmitted data vector. Then the modulo output vector $\hat{\mathbf{a}}_k$ is given by

$$\begin{aligned} \hat{\mathbf{a}}_k &= \left(\mathbf{H}_{k,k}^{eff^{-1}} \mathbf{r}_k \right)_{\mathrm{mod}} \\ &= \mathbf{a}_k + \mathbf{d}_k + \hat{\mathbf{d}}_k + \mathbf{H}_{k,k}^{eff^{-1}} \mathbf{n}_k, \end{aligned} \tag{6}$$

and if $\mathbf{d}_k$ is successfully canceled by $\hat{\mathbf{d}}_k$ and noise is sufficiently small, correct detection is possible.

As mentioned above, BTHP forms K SU MIMO channel, so we cannot only satisfy with the multi-stream transmission but expect the spatial diversity gain. However, because the conventional approach uses the ZF receiver, the spatial diversity gain cannot be achieved and the bit error rate (BER) performance of this system is poorer than we can obtain from the equivalent SU MIMO system. If we can use more powerful receivers like ML, better BER performance will be achieved.

3. Proposed Algorithms

3.1 Expanded ML Receiver

In BTHP, from the concept of DPC [1], a data signal has many equivalent points containing the same message, and modulo operator does the job by replicating original constellation infinitely in a two dimensional plane. This infinitely expanded constellation makes difficult to directly apply well known point-to-point MIMO receivers such as minimum mean square error (MMSE) and ML receiver. Thus, we propose an algorithm that applies ML receiver to BTHP system in this subsection.

If a user can successfully subtract a codevector $\mathbf{d}_k$ from $\mathbf{H}_{k,k}^{eff^{-1}} \mathbf{r}_k$, we can use the conventional ML receiver. However, a user cannot cancel the exact $\mathbf{d}_k$ without additional feed-forward information. Thus, we increase the candidate set for ML and refer to it as expanded ML (eML) receiver. Note that the increased number of candidates should be limited, since complexity is a critical issue of ML receiver.

First, ZF filter retransforms the received signal into an equivalent signal pair in the original plane, and then we can express the transformed signal as

$$\begin{aligned} \mathbf{r}'_k &= \mathbf{H}_{k,k}^{eff^{-1}} \mathbf{r}_k \\ &= \mathbf{a}_k + \mathbf{d}_k + \mathbf{H}_{k,k}^{eff^{-1}} \mathbf{n}_k. \end{aligned} \tag{7}$$

We set the standard constellation as a constellation that contains the ZF filter output. Next, the standard constellation and its neighbors are selected for the expanded ML. Because the probability of birth of the channel which is so distorted that the symbol with minimum distance is out of selected constellations is very small, the expanded candidate set which contains up to second or third tier from the origin is sufficient. A vector $\hat{\hat{\mathbf{a}}}_k$ is detected in the transformed plane among all candidate signal vectors in the expanded constellation as

$$\hat{\hat{\mathbf{a}}}_k = \arg \min_{\mathbf{a}_k \in A_k} \left\| \mathbf{r}_k - \mathbf{H}_{k,k}^{eff^{-1}} \mathbf{a}_k \right\|^2, \tag{8}$$

where A_k is the set of vectors in the expanded constellation of k th user. In practice, in order to reduce complexity, we can use simple version of ML receiver such as sphere decoder [6]. Finally, we can detect data signal $\hat{\mathbf{a}}_k$ as

$$\hat{\mathbf{a}}_k = \left(\hat{\hat{\mathbf{a}}}_k \right)_{\mathrm{mod}}. \tag{9}$$

3.2 Block QR Decomposition

In this subsection, we try to apply the eML algorithm as a receiver in the BTHP system as in Figure 2. We expect diversity gain and improved BER performance to be obtained. However, because of poor characteristics of the equivalent SU MIMO channels which is formed by successive optimization (SO), the conventional channel decomposition algorithm for BTHP, we can obtain little gain of the BER performance. Therefore, we propose a new block triangularization algorithm that forms the effective channel matrices which have good characteristics for the eML receiver.

As mentioned above, BTHP provides a user the SU MIMO channel but its characteristics depend on how the block triangular effective channel is formed. Because ML receiver is used in target system, our approach becomes optimal precoder problem for ML receiver and this is an open problem [8]. So, realistic goal can be to make the SU MIMO channel of which elements have the i.i.d. complex Gaussian distribution with zero mean. For this, we modify the QR algorithm to decompose a channel matrix into a unitary and a block triangular matrix.

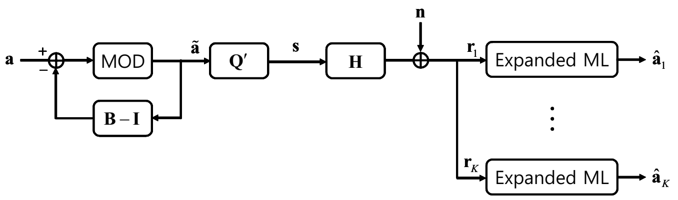

Figure 2: Block diagram of BTHP system with proposed algorithms.

Consider the QR decomposition of the conjugate transposed channel matrix

$$\mathbf{H}^{H} = \mathbf{Q}\mathbf{R}, \tag{10}$$

where $\mathbf{R}$ is an $N_R \times N_R$ upper triangular matrix and $\mathbf{Q}$ is an $N_T \times N_R$ orthogonal matrix. Note that column vectors of $\mathbf{Q}$ are basis of the channel space and elements of $\mathbf{R}$ are correlations between the corresponding channel vectors and basis. So, changing the basis can affect the elements of $\mathbf{R}$. To obtaining an orthogonal matrix and a block triangular matrix, newly define the matrix $\mathbf{Q}$ as

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_1 & \mathbf{Q}_2 & \cdots & \mathbf{Q}_K \end{bmatrix}, \tag{11}$$

where $\mathbf{Q}_k$ is a sub-matrix which contains N_{R_k} vectors that correspond to k th user. If we rotate the basis of users' sub-channel spaces by random $N_{R_k} \times N_{R_k}$ unitary matrix $\mathbf{T}_k$ as

$$\mathbf{Q}'_k = \mathbf{Q}_k \mathbf{T}_k, \tag{12}$$

we can obtain a block triangular matrix $\mathbf{R}'^{H} = \mathbf{H}\mathbf{Q}'$ as an effective channel for BTHP. Note that following theorem shows that the distribution of elements of each user's sub-channel matrix is asymptotically i.i.d. complex Gaussian.

Theorem 1: For given k th user's channel matrix $\mathbf{H}_k$ and precoding matrix $\mathbf{Q}_k$ obtained by the QR method, multiplying a random unitary matrix $\mathbf{T}_k$ makes the effective channel matrix $\mathbf{H}_k\mathbf{Q}_k\mathbf{T}_k$ to be asymptotically i.i.d. complex Gaussian.

Proof: From (12), an element in i th row and j th column of the effective channel matrix of k th user is given by

$$\mathbf{h}_{k,i}\mathbf{Q}_k\mathbf{t}_{k,j} = \sum_{l=1}^{N_T}\sum_{m=1}^{N_{R_k}} h_{k,i,l}\, q_{k,m,l}\, t_{k,j,m}. \tag{13}$$

Note that $\mathbf{h}_{k,i}$ is i th row vector of k th user's channel matrix.

1) Zero mean Gaussian distribution: To make the effective channel of k th user have stochastic properties of the Rayleigh channel, its elements need to be i.i.d. complex Gaussian random variables with zero mean and the same variance. From (13), denote a new random variable X as

$$X = \sum_{l=1}^{N_T}\sum_{m=1}^{N_{R_k}} h_{k,i,l}\, q_{k,m,l}\, t_{k,j,m} - \sum_{l=1}^{N_T}\sum_{m=1}^{N_{R_k}} E\left(h_{k,i,l}\, q_{k,m,l}\, t_{k,j,m}\right). \tag{14}$$

When $N_T N_{R_k}$ is sufficiently large, X is normally distributed with zero mean by the central limit theorem. Because the expectation of an element of a random unitary matrix is zero [10], the expectation term in (14) is zero from the independence of $t_{k,j,m}$ on $h_{k,i,l}$ and $q_{k,m,l}$. Thus, X is equal to the i th row and j th column of k th user's effective channel $\mathbf{h}_{k,i}\mathbf{q}'_{k,j}$, and is normally distributed with zero mean.

2) Independence: Consider a covariance between two different elements of the effective channel matrix of k th user. The covariance between an element in i_1 th row and j_1 th column, and that in i_2 th row and j_2 th column is given by

$$\begin{aligned} E\left\{\mathbf{q}'^{H}_{k,j_1}\mathbf{h}^{H}_{k,i_1}\mathbf{h}_{k,i_2}\mathbf{q}'_{k,j_2}\right\} &= E\left\{\mathbf{t}^{H}_{k,j_1}\mathbf{Q}^{H}_{k}\mathbf{h}^{H}_{k,i_1}\mathbf{h}_{k,i_2}\mathbf{Q}_k\mathbf{t}_{k,j_2}\right\} \\ &= \sum_{l=1}^{N_{R_k}}\sum_{m=1}^{N_{R_k}} E\left\{a_{l,m}\right\} E\left\{t^{*}_{k,j_1,l}\, t_{k,j_2,m}\right\}, \end{aligned} \tag{15}$$

where $a_{l,m}$ is the element in l th row and m th column of the matrix $\mathbf{A} = \mathbf{Q}^{H}_{k}\mathbf{h}^{H}_{k,i_1}\mathbf{h}_{k,i_2}\mathbf{Q}_k$. Because the cross correlation between two different elements of a random unitary matrix is zero [9], two elements in other columns are uncorrelated and (15) becomes

$$E\left\{\mathbf{q}'^{H}_{k,j_1}\mathbf{h}^{H}_{k,i_1}\mathbf{h}_{k,i_2}\mathbf{q}'_{k,j_2}\right\} = \sum_{l=1}^{N_{R_k}} E\left\{a_{l,l}\right\} E\left\{t^{2}_{k,j,l}\right\} \quad \text{if}\, j_1 = j_2. \tag{16}$$

Assume that i_1 is not equal to i_2. Then we can set i_1 is greater than i_2 without loss of generality. Because $\mathbf{q}_{k,l}$ is obtained by the QR method, if l is larger than i_2, $\mathbf{h}_{k,i_2}\mathbf{q}_{k,l} = 0$ and $E\left\{a_{l,l}\right\} = 0$. And if l is smaller than i_1, because of independence of $\mathbf{h}_{k,i_1}$ on $\mathbf{q}_{k,l}$, $E\left\{a_{l,l}\right\} = 0$. Accordingly, two elements in other rows is also uncorrelated.

As a result, all pair of two different elements of the effective channel of k th user are uncorrelated. Since uncorrelated two Gaussian random variables are independent, the independence property is proved.

3) Identical variance: From *2)*, variance of an element in i th row and j th column of the effective channel is given by

$$\operatorname{var}\left\{\mathbf{h}_{k,i}\mathbf{q}'_{k,j}\right\} = \sum_{l=1}^{N_{R_k}} \operatorname{var}\left\{\mathbf{h}_{k,i}\mathbf{q}_{k,l}\right\} \operatorname{var}\left\{t_{k,j,l}\right\}. \tag{17}$$

From the statistical properties of random unitary matrix and QR method in [9] and [10], (17) becomes

$$\operatorname{var}\left\{\mathbf{h}_{k,i}\mathbf{q}'_{k,j}\right\} = \frac{N_T - \sum_{l=1}^{k-1} N_{R_l}}{N_{R_k}}. \tag{18}$$

(18) means that all elements of the effective channel of k th user have identical variance. □

4. Simulation Results

In this section, we compare the BER performance of the proposed algorithms with other MU MIMO algorithms through computer simulations. For simplicity, we assume that all users have the same number of received antennas, and the number of data streams and antennas of a user are equal. The notation $\left(N_T, N_{R_k}, N_U\right)$ indexes the number of transmit antennas, receive antennas for each user, and users. The mean transmit power is set to $1/K$ for all users, and 4QAM modulation is used. Figure 3 shows the average BER performance of the proposed BTHP system applying proposed

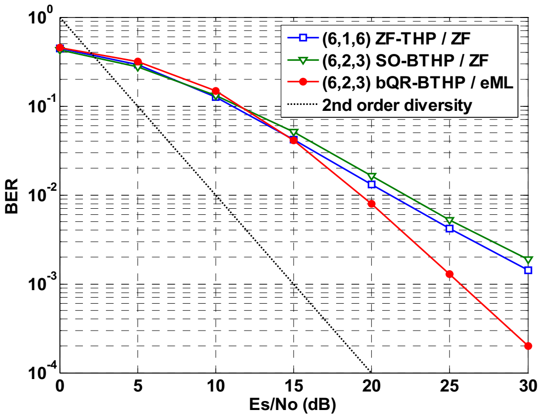

Figure 3: Average BER performance comparison between the proposed bQR-BTHP with eML and the conventional techniques.

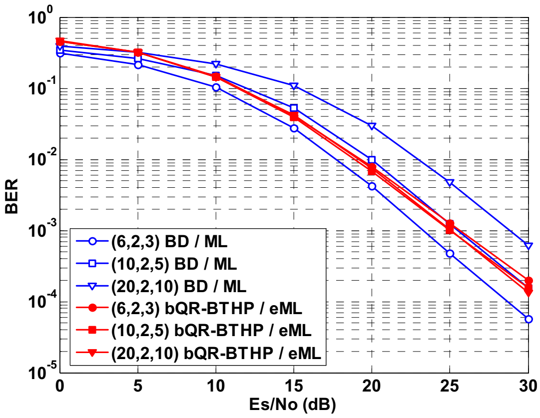

Figure 4: Average BER performance variation of the proposed bQR-BTHP/eML and BD/ML with the increase of the number of users.

bQR and the eML receiver simultaneously in the case of (6,2,3). For comparison, the BER performances of the conventional ZF-THP/ZF, SO-BTHP/ZF are described in the same case. We observe that the average BER performance of the BTHP system with proposed algorithms is better than that of the other systems. When the BER is 10^{-2}, the proposed BTHP system obtains about 3.5dB gain over the SO-BTHP/ZF. We also observe that the BTHP using the proposed algorithms obtains the full spatial diversity gain because the proposed system takes the performance of the optimal ML decoder.

Figure 8 describes the BER performance variation with the increase of the number of users and corresponding number of antennas. If there are more users, the transmit signal power for a user decreases and the average BER performance of BD/ML and that of the last user in the proposed system also becomes worse. However, in the proposed system, the average BER performance does not decrease with the increase of users. As a result, we observe that in the case of (20,2,10), bQR/eML has much better BER performance than BD/ML.

5. Conclusions

In this paper, we propose an improved BTHP system that applies a new bQR algorithm and eML receiver. Using the proposed system, we can solve the problem that the conventional THP based downlink MU MIMO systems cannot obtain the spatial diversity gain. The proposed eML algorithm provides an approach to apply the ML receiver to the THP based system which transmits multiple data streams for each user. The proposed bQR decomposition algorithm forms an effective MIMO channel that the proposed eML can obtain the full spatial diversity gain from it. And, we showed that this effective channel elements are i.i.d. complex Gaussian random variables with zero mean.

By the computer simulation, it is observed that the proposed BTHP system using the proposed bQR and eML has spatial diversity gain and considerable BER performance improvement is obtained. In the cases of (6,2,3), the proposed system provides about 3.5dB SNR gain at 10^{-2} BER compared to the conventional BTHP. For some cases, the BER performance of the proposed system is worse than BD/ML in an average view. However, the proposed system showed much better BER performance with the increase of the number of users and corresponding number of antennas.

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