An MILP Approach to Optimal Surveillance over Graphs

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Abstract: The surveillance problem over graphs is to find an trajectory of an agent that travels each node as evenly as possible. This problem has several application such as city safety management and disaster rescue. In this paper, the finite-time optimal surveillance problem is formulated, and is reduced to a mixed integer linear programming (MILP) problem. Based on the policy of model predictive control, an optimal trajectory is generated by solving the MILP problem at each discrete time, and persistent surveillance can be realized.

1. Introduction

The surveillance problem over graphs is to find an trajectory of agents that travels each node as evenly as possible [1], [4], [5]. This problem has several application such as city safety management and disaster rescue. Furthermore, in the case of centralized control, a controller computes the next location of agents, and sends to each agent through communication networks. In this sense, the surveillance problem can be regarded as a synthesis problem of networked control systems.

On the other hand, to overcome the hardness of control of complex systems, it is natural to approximately solve complex problems using simplification/abstraction techniques (see, e.g., [11]). In the surveillance problem, it is appropriate that a surveillance area is given by a graph. The surveillance problem over graphs has been studied in e.g., [1]. However, to the best of our knowledge, the method based on model predictive control (MPC) has not been studied so far (see, e.g., [3], [10] for details of MPC).

In this paper, we consider the surveillance problem over graphs for a single agent. In order to express behavior of agents in the surveillance problem, we adopt a mixed logical dynamical (MLD) system model [2]. The MLD (mixed logical dynamical) system model is well known as a powerful model of hybrid systems. In this paper, the optimal surveillance problem over a given graph is formulated. This problem is a kind of the finite-time optimal control problem. Behavior of an agent is modeled by the MLD system model. Then, the optimal surveillance problem is reduced to a mixed integer linear programming problem, which can be solved by a suitable commercial/free solver. In order to generate the next location, an on-line procedure based on model predictive control [3] is also presented.

Notation: Let $\mathcal{R}$ denote the set of real numbers. Let $\{0,1\}^n$ denote the set of n-dimensional vectors, which consists of elements 0 and 1. Let 1_n denote the n-dimensional vector whose elements are all one. Let I_n and $0_{m\times n}$ denote the $n\times n$ identity matrix and the $m\times n$ zero matrix, respectively. For simplicity of notation, we sometimes use the symbol 0 instead of $0_{m\times n}$, and the symbol I instead of I_n. For the vector v, let $v^\top$ denote the transpose of v.

2. Problem Formulation

A surveillance area is given by an undirected connected graph $G=(V,E)$, where $V=\{v_1,v_2,\dots,v_n\}$ is the set of nodes, and $E\subseteq V\times V$ is the set of undirected edges. We assume that an agent can move according to a given graph, and behavior of an agent is expressed by a discrete-time system. The number of agents is given by m.

As an example, consider the undirected connected graph in Fig. 1. Suppose that the initial location of an agent is given by v_4. Then, the candidates of the location at the next time is constrained to the set $\{v_2,v_4,v_5,v_6,v_7\}$. Thus, a complicated surveillance area can be modeled by an undirected graph.

For each vertex, we define a penalty $x_i(k)\in\mathcal{R}$, $k\in\{0,1,2,\dots\}$ as follows:

$$x_i(k+1)=\begin{cases}0 \text{ if the agent is located on } v_i \text{ at time } k,\\ x_i(k)+1 \text{ otherwise.}\end{cases} \tag{1}$$

Then, the optimal surveillance problem is formulated as follows.

Problem 1: For the undirected connected graph $G=(V,E)$ and time evolution (1) of the penalty, suppose that the initial locations of an agent, the initial penalty $x_i(0)$, and the prediction horizon N are given. Then, find trajectories of m agents minimizing that following cost function

$$J=\sum_{k=0}^{N}\sum_{i=1}^{n}q_i x_i(k) \tag{2}$$

where $q_i\geq 0$ is a given weight.

We may impose a constraint such as $x_i(k)\leq\alpha$, where $\alpha>0$ is a given scalar. Temporal logic constraints can also be imposed for Problem 1 (see e.g., [6], [7], [9]). In this paper, for simplicity of discussion, we consider the case where no constraints are imposed.

As an example, consider the undirected connected graph in Fig. 1 again. Suppose that the initial location of an agent and the initial penalty $x_i(0)$ are given by v_4 and $x_i(0)=0$, respectively. Then, $x_i(1)$ can be obtained as $x_4(1)=0$ and $x_i(1)=1$, $i=1,2,3,5,6,\dots,14$. Next, suppose that the the location at time 1 is given by v_7. Then, $x_i(2)$ can be obtained as $x_7(2)=0$, $x_4(2)=1$, and $x_i(2)=2$, $i=1,2,3,5,6,8,9,\dots,14$. From this example, we see that an trajectory of an agent that travels each node as evenly as possible can be obtained by using the cost function (2).

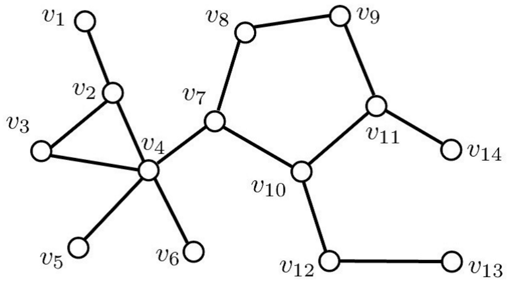

Figure 1. Example of undirected connected graphs.

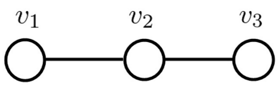

Figure 2. Simple example of undirected connected graphs.

3. Modeling of an Agent and Penalties Using the MLD System

In this section, we consider modeling behavior of an agent and time evolution of penalties by using a mixed logical dynamical (MLD) system, which is one of the powerful model of hybrid systems (see [2]).

First, we define a binary variable $\delta_i(k)$ as follows:

$$\delta_i(k) = \begin{cases} 1 \text{ if the agent is located on } v_i \text{ at time } k, \\ 0 \text{ otherwise.} \end{cases}$$

From this definition, we impose the following equality constraint:

$$\delta_1(k) + \delta_2(k) + \delta_3(k) = 1, \; k \in \{0, 1, 2 \dots\}. \tag{3}$$

Using a binary variable, consider modeling behavior of an agent. We present a simple example.

Example 1: Consider the undirected graph shown in Fig. 2. This graph implies that

(i) If the agent is located on v_1 at time k, the position of the agent at time $k+1$ is either v_1 or v_2,

(ii) If the agent is located on v_2 at time k, the position of the agent at time $k+1$ is at least any one of v_1, v_2, and v_3,

(iii) If the agent is located on v_3 at time k, the position of the agent at time $k+1$ is either v_2 or v_3.

Using a binary variable, these conditions can be expressed by

$$\begin{cases} \delta_1(k) \le \delta_1(k+1) + \delta_2(k+1), \\ \delta_2(k) \le \delta_1(k+1) + \delta_2(k+1) + \delta_3(k+1), \\ \delta_3(k) \le \delta_2(k+1) + \delta_3(k+1). \end{cases} \tag{4}$$

For example, if $\delta_1(k) = 1$ (i.e., $\delta_2(k) = \delta_3(k) = 0$ holds from the equality constraint (3)), then either $\delta_1(k+1) = 1$ or $\delta_2(k+1) = 1$ must hold, and $\delta_3(k+1) = 0$ must hold. Thus, three linear inequalities (4) and the equality constraint (3) can express behavior of an agent moving along a given undirected graph.

We consider a general case. Define

$$\delta(k) = [\; \delta_1(k) \;\; \delta_2(k) \;\; \cdots \;\; \delta_n(k) \;]^\top .$$

Then, behavior of an agent can be modeled by

$$\delta(k) - \Phi\delta(k+1) \le 0, \tag{5}$$

where Φ is an adjacency matrix of a given graph. See [2], [8] for further details.

Next, consider modeling time evolution of the penalty $x_i(k)$. Using $\delta_i(k)$, time evolution of $x_i(k)$ can be expressed by

$$x_i(k+1) = (1 - \delta_i(k))(x_i(k) + 1). \tag{6}$$

Defining

$$z_i(k) := \delta_i(k)x_i(k) - 1, \tag{7}$$

(6) can be rewritten as the following linear system:

$$x_i(k+1) = x_i(k) - \delta_i(k) - z_i(k). \tag{8}$$

Without loss of generality, $x_i(k)$ is constrained by $x_i(k) \in [0, x_{\max}] \subset \mathcal{R}$, where $x_{\max} < \infty$ can be determined from a given undirected connected graph. Then, (7) can be equivalently transformed into

$$\begin{cases} -1 \le z_i(k) \le x_{\max}\delta_i(k) - 1, \\ x_i(k) - x_{\max}(1 - \delta_i(k)) - 1 \le z_i(k) \le x_i(k) - 1, \end{cases} \tag{9}$$

See [2] for further details.

From (3), (5), (8), and (9), behavior of an agent and time evolution of the penalty can be expressed by the following MLD system:

$$\begin{cases} x(k+1) = Ax(k) + B_1u(k) + B_2z(k), \\ Cx(k) + D_1u(k) + D_2z(k) \le E, \end{cases} \tag{10}$$

where

$$\begin{aligned} x(k) &= \left[\; x_1(k) \; x_2(k) \; \cdots \; x_n(k) \; \delta^\top(k) \;\right]^\top \\ &\in \mathcal{R}^n \times \{0,1\}^n, \\ u(k) &= \delta(k+1) \in \{0,1\}^n, \\ z(k) &= [\; z_1(k) \; z_2(k) \; \cdots \; z_n(k) \;]^\top \in \mathcal{R}^n, \end{aligned}$$

and

$$A = \begin{bmatrix} I_n & -I_n \\ 0 & 0 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0 \\ I_n \end{bmatrix}, \; B_2 = \begin{bmatrix} I_n \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & I_n \\ 0 & 0 \\ 0 & -x_{\max}I_n \\ I_n & x_{\max}I_n \\ -I_n & 0 \\ 0 & 1_n^\top \\ 0 & -1_n^\top \end{bmatrix}, \; D_1 = \begin{bmatrix} \Phi \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \; D_2 = \begin{bmatrix} 0 \\ -I_n \\ I_n \\ -I_n \\ I_n \\ 0 \\ 0 \end{bmatrix},$$

$$E = \begin{bmatrix} 0 \\ 1_n \\ -1_n \\ (x_{\max}+1)1_n \\ -1_n \\ 1 \\ -1 \end{bmatrix}.$$

4. Reduction of the Optimal Surveillance Problem to an MILP problem

Using the MLD system (10), consider reducing Problem 1 to an MILP (mixed integer linear programming) problem. Hereafter, for simplicity of notation, the MLD system (10) is denoted by

$$\begin{cases} x(k+1) = Ax(k) + Bv(k), \\ Cx(k) + Dv(k) \leq E, \end{cases} \tag{11}$$

where $v(k) := [\ u^\top(k) \quad z^\top(k)\ ]^\top$, $B := [\ \overline{B}_1 \quad \overline{B}_2\ ]$, and $D := [\ \overline{D}_1 \ \overline{D}_2\ ]$.

First, using

$$x(k) = A^k x_0 + \sum_{i=1}^{k} A^{i-1} Bu(k-i)$$

obtained from the state equation in (11), we can obtain

$$\overline{x} = \overline{A}x(0) + \overline{B}\overline{v} \tag{12}$$

where

$$\begin{aligned} \overline{x} &:= [\ x^T(0)\ x^T(1)\ \cdots\ x^T(N)\ ]^\top, \\ \overline{v} &:= [\ v^T(0)\ v^T(1)\ \cdots\ v^T(N-1)\ ]^\top, \end{aligned}$$

and

$$\overline{A} = \begin{bmatrix} I \\ A \\ A^2 \\ \vdots \\ A^N \end{bmatrix}, \ \overline{B} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ B & 0 & \cdots & 0 \\ AB & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ A^{N-1}B & \cdots & AB & B \end{bmatrix}.$$

From the linear inequality in (11), we obtain

$$\overline{C}\overline{x} + \overline{D}\overline{v} \leq \overline{E} \tag{13}$$

where

$$\overline{C} = \begin{bmatrix} C & & 0 & 0 \\ & \ddots & & \vdots \\ 0 & & C & 0 \end{bmatrix}, \ \overline{D} = \begin{bmatrix} D & & 0 \\ & \ddots & \\ 0 & & D \end{bmatrix},$$

$$\overline{E} = \begin{bmatrix} E \\ \vdots \\ E \end{bmatrix}.$$

Next, defining $Q = [\ q_1\ \cdots\ q_n\ ][\ I_n\ 0\ ]$, the cost function (2) can also be rewritten as

$$J = \overline{Q}\overline{x} \tag{14}$$

where $\bar{Q} = [\ Q\ \cdots\ Q\ ]$. By substituting (12) into (13) and (14), Problem 1 can be equivalently rewritten as the following problem.

Problem 2: Suppose that the initial state $x(0)$ is given. Then, find $\overline{v} \in (\{0,1\}^{nm+n} \times \mathcal{R}^n)^N$ minimizing the following linear cost function

$$J = \overline{QB}\overline{v} + \overline{QA}x(0)$$

subject to the following linear constraint

$$(\overline{CB} + \overline{D})\overline{v} \leq \overline{E} - \overline{CA}x(0).$$

This problem is the form of an MILP problem, which can be solved by using a free/commercial solver.

Finally, we present a procedure of model predictive control (MPC) using Problem 2.

Procedure of MPC-Based Optimal Surveillance:

Step 1: Set $t = 0$, and give $x_i(0)$ (the initial penalty for each node) and $\delta(0)$ (the initial location for each agent).

Step 2: Solve Problem 2.

Step 3: Move an agent based on $\delta_{i,j}(1)$ obtained.

Step 4: Set $t + 1 \to t$, and return to Step 2.

5. Conclusion

In this paper, we propose an optimal surveillance method for a given graph. Overall model including time evolution of the penalty and behavior of an agent is given by a mixed logical dynamical system, and the optimal surveillance problem is reduced to a mixed integer linear programming (MILP) problem. According to the receding horizon policy, the MILP problem is solved at each discrete time. Therefore, an appropriate surveillance can be achieved. The proposed method can be extended to the case of multiple agents.

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