

# Antenna Selection for Single Carrier Cyclic Prefixed Transmission

Shuichi Ohno<sup>1</sup> and Emmanuel Manasseh<sup>2</sup>

Dept. of Artificial Complex Systems Engineering, Hiroshima University

1-4-1 Higashi-Hiroshima, Japan

E-mail: <sup>1</sup>ohno@hiroshima-u.ac.jp, <sup>2</sup>manasejc@ieee.org

**Abstract:** Single carrier cyclic prefixed transmissions between a base station having multiple antenna and a terminal having one antenna are considered. We propose two antenna selections based on the channel norm and on the worst channel gain. Our selection schemes do not always yield the optimal selection but have less computational complexities. The channel norm criterion has the least complexity, while the channel gain criterion exhibits almost the same performance with the optimal selection.

## 1. Introduction

It is well-known that multiple antennas at the transmitter and/or at the receiver can improve the performances in wireless communication systems (see e.g. [1] and references therein). This so-called Multi-Input Multi-Output (MIMO) technology has been extensively studied and adopted in the coming wireless communication networks. However, the presence of multiple radio-frequency (RF) chains which consist of amplifiers, downconverters, A/D converters and so on, makes it difficult to implement MIMO techniques especially on small devices like mobile terminals. The multiple RF chains also increase the complexity and hardware cost of the system.

To capture the benefits of multiple antennas system and to reduce the implementation costs, antenna selection, which selects some antennas from an array of multiple antennas based on a certain criterion, has been developed [2], [3]. The aim of antenna selection is thus to decrease the complexity while keeping the performance loss as small as possible. In [4], transmit antenna is selected to maximize channel capacity, while in [5], to minimize the error-rate using linear receivers. Orthogonal space-time coding (STC) scheme has been also incorporated into antenna selection. The antennas for space-time block coding (STBC) are selected based on the maximization of the Frobenius channel norm (or received signal-to-noise ratio (SNR)) [6]. An approximate [7], and comprehensive [8] theoretical analysis of the pairwise error probability (PEP) pertaining to the STBC system using antenna selection are also documented.

In high data rate transmissions, severe intersymbol interference (ISI) arises, which degrades system performance. To mitigate ISI, block transmissions, including orthogonal frequency-division multiplexing (OFDM), with redundant symbols appended to each block have been developed. In OFDM, a copy of the tail of a block, which is called cyclic prefix (CP) is appended at the top of the block. Thanks to fast Fourier transform (IFFT) at the transmitter and FFT at the receiver, OFDM renders a convolution channel into parallel flat channels, which enables very simple one-tap frequency-

domain equalization. Channel norms are used to select receive antennas in coherent STC-OFDM systems [8] and in noncoherent STC-OFDM systems [12], while in [13], to select transmit antennas in STBC-MISO-OFDM systems. On the other hand, single-carrier CP transmission discards FFT at the transmitter, which enables constant modulus transmission and exhibits better performance than OFDM [9], [10], [11], at a relatively low computational cost of the receiver. Block-wise STBC for SC-CP has been proposed to obtain transmit antenna diversity [14], [15]. However, antenna selections for SC-CP have been not well reported.

In this paper, we develop antenna selection schemes for SC-CP. We consider wireless communication between a base station having multiple antenna and a small terminal having one antenna. Our base station selects several antennas for transmission and reception. For antenna selection, we propose a channel norm criterion and a maxmin criterion that maximizes the worst channel gains. Albeit the maxmin criterion does not always yield the optimal selection, its BER performance almost matches with the optimal BER and fares better than the BER performance of the norm criterion.

## 2. Antenna Selection and CP Transmission

Let us consider a wireless communication between a small terminal and a base station with  $N_b$  multiple antennas. The uplink is modeled as a single-input and multiple-output (SIMO) system, while the downlink as a multiple-input and single-output (MISO) system, where time division duplex (TDD) is adopted. We assume that the communication resource of the base station is shared in time by multiple users so that in every time slot, we have only one point-to-point communication.

The channel is assumed to be quasi-static fading such that the channel tap coefficients are invariant over a sufficient number of symbols and independent, identically distributed (i.i.d.) of each other. Let  $\{h^{(i)}(l)\}_{l=0}^L$  be the discrete-time baseband equivalent channel impulse responses of the taps between the base station and the terminal through antenna  $i$ , where  $L$  is the maximum order of the FIR channels. Let us denote the channel frequency response at frequency  $\exp(-j2\pi n/N)$  as  $H_n^{(i)}$  defined as

$$H_n^{(i)} = \sum_{l=0}^L h^{(i)}(l)e^{-j2\pi \frac{ln}{N}}, n \in [0, N-1], \quad (1)$$

where  $N$  is our block size, which is a power of 2.

We assume that  $\bar{N}_b$  RF chains of the base station are utilized for the communication to a specific terminal. Antennas are not so costly compared to radio-frequency (RF) chains

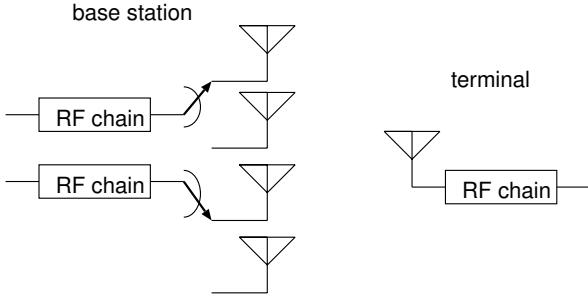


Figure 1. Antenna selection ( $N_b = 4$ ,  $\bar{N}_b = 2$ ).

which consist of amplifiers, downconverters, A/D converters and so on. Then it makes sense to select  $\bar{N}_b$  out of  $N_b$  equipped antennas and to connect  $\bar{N}_b$  antennas to the  $\bar{N}_b$  RF chains as depicted in Figure 1. This is so-called antenna selection, where a predetermined number of antennas with *good* channel condition are selected for transmission so as to enjoy antenna diversity. We assume that the knowledge of all the channel for antenna selection is acquired by the received training or pilot symbols prior to the data transmission.

We utilize the cyclic prefix (CP) to mitigate frequency-selective multipath effects. In OFDM and so-called single carrier (SC) CP or CP only block transmission [9], a CP of length  $N_{cp} \geq L$  is appended to the tail of a data sequence to avoid interference between consecutive blocks.

Thanks to inverse fast Fourier transform (IFFT) at the transmitter and FFT at the receiver, OFDM enables efficient one-tap equalizer and has been adopted in many standards. One of its major drawbacks is the high peak-to-average power ratio (PAPR), which necessitates a good power amplifier to avoid the degradation of the bit-error rate (BER) performance. On the other hand, SC-CP enables constant-modulus transmission at the expense of more involved equalization.

Let us consider SC-CP, which can be effectively implemented on small devices. For its practical importance and for the simplicity of presentation, we study two antenna selection, i.e.,  $\bar{N}_b = 2$ , as shown in Figure 1 for SC-CP. Let two transmit antennas with integer indices  $i$  and  $j$  ( $i, j \in [1, N_b]$ ) be selected and used for data transmission. (The letter  $j$  is either imaginary unit or an integer, which is clarified by the context.)

## 2.1 Uplink

At the terminal, an information sequence  $\{s(k)\}$  is stacked into information vectors of size  $N$  as  $\mathbf{s}(k)$ . If the base station connects RF chains to the selected antennas  $i$  and  $j$ , after removing the signal corresponding cyclic prefix, the received signals from antennas  $i$  and  $j$  can be expressed by

$$\mathbf{y}^{(m)}(k) = \mathbf{H}^{(m)} \mathbf{s}(k) + \mathbf{w}^{(m)}(k), \quad (2)$$

for  $m = i, j$ , where  $\mathbf{H}^{(m)}$  is a circulant matrix with  $[h^{(m)}(0), \dots, h^{(m)}(L), 0, \dots, 0]^T$  its first column and  $\mathbf{w}^{(m)}(k)$  denotes an additive white complex Gaussian noise (AWGN) vector with variance  $N_0$ .

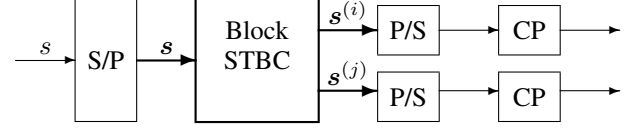


Figure 2. Transmitter of SC-CP with STBC

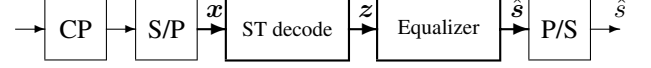


Figure 3. Receiver of SC-CP with STBC

Suppose that linear zero-forcing (ZF) equalization of the received signals. Then, the equalized output  $\hat{\mathbf{s}}(k)$  is given by

$$\hat{\mathbf{s}}(k) = \mathbf{E}^{(i,j)} \left[ \mathbf{H}^{(i)*} \mathbf{y}^{(i)}(k) + \mathbf{H}^{(j)*} \mathbf{y}^{(j)}(k) \right], \quad (3)$$

where  $*$  denotes complex conjugate transposition and

$$\mathbf{E}^{(i,j)} = \left[ \mathbf{H}^{(i)*} \mathbf{H}^{(i)} + \mathbf{H}^{(j)*} \mathbf{H}^{(j)} \right]^{-1}. \quad (4)$$

It follows from (3) and (4) that

$$\hat{\mathbf{s}}(k) = \mathbf{s}(k) + \mathbf{e}^{(i,j)}(k), \quad (5)$$

where

$$\mathbf{e}^{(i,j)}(k) = \mathbf{E}^{(i,j)} \left[ \mathbf{H}^{(i)*} \mathbf{w}^{(i)}(k) + \mathbf{H}^{(j)*} \mathbf{w}^{(j)}(k) \right]. \quad (6)$$

If  $\mathbf{w}^{(i)}(k)$  and  $\mathbf{w}^{(j)}(k)$  are independent, then the correlation of  $\mathbf{e}^{(i,j)}(k)$  is found to be

$$E\{\mathbf{e}^{(i,j)}(k) \mathbf{e}^{(i,j)*}(k)\} = N_0 \mathbf{E}^{(i,j)}, \quad (7)$$

where  $E\{\cdot\}$  stands for expectation.

With  $\gamma = E_s/N_0$  being the ratio between the transmit symbol energy  $E_s$  over noise energy  $N_0$ , one can show from (7) that SNR for each equalized symbol can be expressed as  $\gamma/\lambda^{(i,j)}$ , where

$$\lambda^{(i,j)} := \frac{1}{N} \sum_{n=0}^{N-1} \frac{1}{|H_n^{(i)}|^2 + |H_n^{(j)}|^2}. \quad (8)$$

## 2.2 Downlink

In downlink, we adopt space time block coding for SC-CP [14], [15]. It should be remarked here that even if 2 transmit antennas are available, transmission from just one antenna may outperform transmission from 2 antennas, which never happens in uplink.

Let  $\mathbf{s}(k)$  be the symbol vector to be transmitted. Schematic diagrams of the transmitter and receiver of SC-CP with STBC are depicted in Fig. 2 and in Fig. 3 respectively. Block-wise space-time block coding is then applied to  $\{\mathbf{s}(k)\}$  as

$$\mathbf{s}^{(i)}(2k) = \mathbf{s}(2k), \quad \mathbf{s}^{(i)}(2k+1) = \mathbf{P} \mathbf{s}^*(2k+1), \quad (9)$$

$$\mathbf{s}^{(j)}(2k) = \mathbf{s}(2k+1), \quad \mathbf{s}^{(j)}(2k+1) = -\mathbf{P} \mathbf{s}^*(2k), \quad (10)$$

where  $(\cdot)^*$  represents complex conjugation and  $\mathbf{P}$  is a permutation matrix. The vectors  $\mathbf{s}^{(i)}(k)$  and  $\mathbf{s}^{(j)}(k)$  undergo parallel-serial (P/S) conversion, CP addition, followed by transmission from antenna  $i$  and antenna  $j$  respectively.

At the receiver, after removal of the part which corresponds to CP, the received signals are serial-parallel (S/P) converted to obtain

$$\mathbf{x}(k) = \mathbf{H}^{(i)} \mathbf{s}^{(i)}(k) + \mathbf{H}^{(j)} \mathbf{s}^{(j)}(k) + \mathbf{w}(k), \quad (11)$$

where  $\mathbf{w}(k)$  is an AWGN vector. After block-wise space-time block decoding, we obtain a decision vector

$$\mathbf{z}(k) = \mathbf{D}^{(i,j)} \begin{bmatrix} \mathbf{F}\mathbf{s}(2k) \\ \mathbf{F}\mathbf{s}(2k+1) \end{bmatrix} + \begin{bmatrix} \mathbf{\Lambda}^{(i)} & -\mathbf{\Lambda}^{(j)} \\ \mathbf{\Lambda}^{(j)*} & \mathbf{\Lambda}^{(i)*} \end{bmatrix} \mathbf{v}(k), \quad (12)$$

where  $\mathbf{D}^{(i,j)}$  is a diagonal matrix defined as

$$\mathbf{D}^{(i,j)} = \mathbf{\Lambda}^{(i)*} \mathbf{\Lambda}^{(i)} + \mathbf{\Lambda}^{(j)*} \mathbf{\Lambda}^{(j)},$$

with diagonal matrix  $\mathbf{\Lambda}^{(i)} = \text{diag}(H_0^{(i)}, H_1^{(i)}, \dots, H_{N-1}^{(i)})$ ,  $\mathbf{F}$  is an  $N \times N$  FFT matrix, and  $\mathbf{v}(k)$  is an AWGN vector with correlation matrix  $N_0 \mathbf{I}$ . Then, ZF equalized signal can be expressed as

$$\hat{\mathbf{s}}(k) = \mathbf{s}(k) + \boldsymbol{\epsilon}^{(i,j)}(k). \quad (13)$$

The correlation of  $\boldsymbol{\epsilon}^{(i,j)}(k)$  is given by

$$E\{\boldsymbol{\epsilon}^{(i,j)}(k) \boldsymbol{\epsilon}^{(i,j)*}(k)\} = N_0 \mathbf{F} \mathbf{E}^{(i,j)} \mathbf{F}^*. \quad (14)$$

Then, the SNR of the equalized signal is found to be equal to  $\gamma/\lambda^{(i,j)}$ , which is the same with the SNR in the uplink. Thus, it suffices to develop antenna selection only for the uplink.

### 3. Criteria for Antenna Selection

For a given signal constellation, let  $\text{BER}^{(i,j)}$  be the uncoded BER of the system with active antennas  $i$  and  $j$ . Then, the BER can be minimized by selecting a pair of antennas such that

$$\arg \min_{i,j \in [1, N_b], i \neq j} \text{BER}^{(i,j)}. \quad (15)$$

If we can express  $\text{BER}^{(i,j)}$  as a function of SNR and channel state information (CSI), then the optimal two transmit antennas can at least be obtained by exhaustive numerical search through  $N_b(N_b - 1)/2$  combinations to yield the minimum.

Suppose QPSK constellation and hard-detection of linearly zero-forcing (ZF) equalized signals for example. Since the noise after equalization is still Gaussian, the BER is expressed as  $\text{BER}^{(i,j)} = Q\left(\sqrt{\gamma/\lambda^{(i,j)}}\right)$ , where  $Q(x) = (1/\sqrt{2\pi}) \int_x^\infty \exp(-t^2/2) dt$  is the Gaussian- $Q$  function. Since SNRs are all the same and  $Q(\cdot)$  is an increasing function in  $\lambda^{(i,j)}$ , (15) is equivalent to

$$\arg \min_{i,j \in [1, N_b], i \neq j} \lambda^{(i,j)}. \quad (16)$$

This criterion does not require the computation of  $Q$  function nor depends on SNR.

For devices with limited computational power, antenna selection with less computations is preferred. For example, the receive power is efficiently utilized for OFDM with receive antenna selection [8], [12] and with transmit antenna selection [13]. The same criterion can be applied to our SC-CP system. We select a pair of transmit antennas such that

$$\arg \max_{i,j \in [1, N_b], i \neq j} (\|\mathbf{h}^{(i)}\|^2 + \|\mathbf{h}^{(j)}\|^2), \quad (17)$$

where  $\|\mathbf{h}^{(i)}\|^2$  is the squared norm of channel impulse response from transmit antenna  $i$  defined as  $\|\mathbf{h}^{(i)}\|^2 = \sum_l |h^{(i)}(l)|^2$ . If there is no additive noise, then the squared norm scales in the receive power. Thus, even with additive noises, the squared norms are easily estimated by using well-designed preamble or pilot signals. Obviously, it is not assured that our antenna selection always yield the optimal selection. Its advantage lies in its reduced computations which just selects the antenna of the largest channel norm and the antenna of the second largest channel norm.

Another intuitive criterion can be developed as follows: Since the minimum of  $|H_n^{(i)}|^2 + |H_n^{(j)}|^2$  for  $n \in [0, N-1]$  affects  $\lambda^{(i,j)}$  the most, it makes sense to select a pair of antennas that maximizes the minimum such that

$$\arg \max_{i,j \in [1, N_b], i \neq j} \min_{n \in [0, N-1]} (|H_n^{(i)}|^2 + |H_n^{(j)}|^2). \quad (18)$$

For a fixed pair  $(i, j)$  of antennas,  $N$  additions and additional comparisons of  $N$  values are necessary to compute  $|H_n^{(i)}|^2 + |H_n^{(j)}|^2$  and to find their minimum. In total, excluding the comparisons, we need  $O(NN_b(N_b - 1)/2)$  additions to determine the selection.

### 4. Numerical simulations

We assess the BER performances of our antenna selection methods by numerical simulations. The symbols are drawn from a QPSK constellation and sent through an uncoded CP block transmission of block size  $N = 64$  in the uplink. The channels are i.i.d. with  $L + 1 = 7$  component channel taps of exponential power profile. The results are averaged over  $10^5$  random channel realizations.

Two antennas out of  $N_b$  antennas are chosen based on BER criterion (15), norm criterion (17), and maxmin criterion (18), which are labeled with Optimal(2), Norm(2), and maxmin(2), respectively. Similarly, one antenna out of  $N_b$  antennas is selected, whose labels are Optimal(1), Norm(1), and maxmin(1), respectively. Average( $x$ ) corresponds to BER without antenna selection with  $x$  receive antenna.

We assume the perfect knowledge, i.e., channel frequency responses and SNR for BER criterion (15), channel norms for norm criterion (17), and channel frequency responses for maxmin criterion (18), for selection.

Fig. 4 illustrates the BERs without antenna selection and the BERs with antenna selection for  $N_b = 4$ , where  $E_s$  is the energy per symbol. The advantage of systems with antenna selection over systems without antenna selection is evident. For every selection scheme, the system with two antennas has

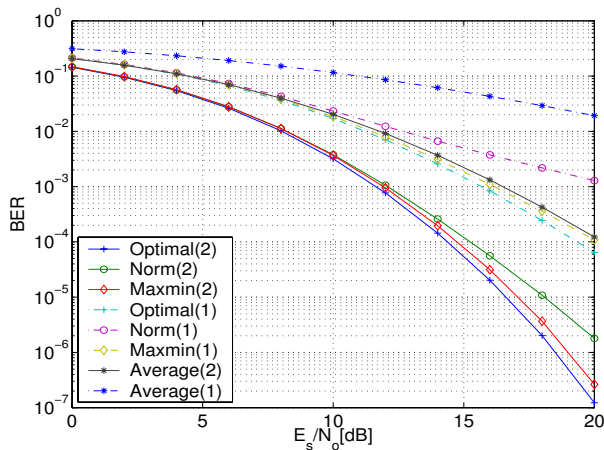


Figure 4. BER with/without antenna selection ( $N_b = 4$ ).

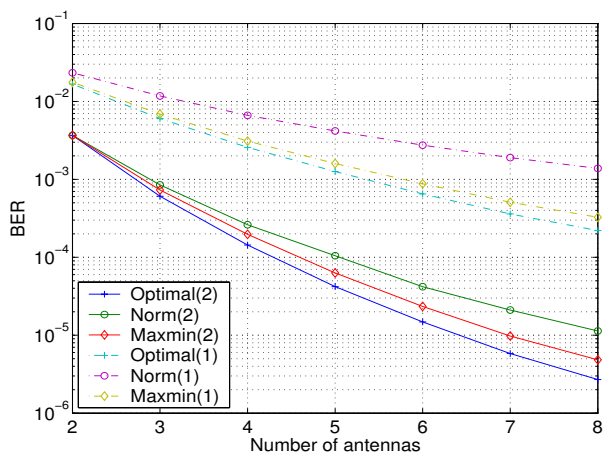


Figure 5. BER vs number of equipped antennas at 14dB.

at least about 3db antenna gain over the system with one antennas.

At low SNR, there are no significant differences among three criteria. For one antenna selection, the performance loss of the maxmin criterion is negligible, while the loss of the norm criterion is significant. For two antenna selection, BER of the maxmin criterion almost coincides with the optimal BER but the difference between the maxmin criterion and the norm criterion is not so large compared to the one selection case.

At a fix  $E_s/N_0 = 14\text{dB}$ , we evaluate the BER performances by varying the number  $N_b$  of antennas from 2 to 8. Fig. 5 depicts the results. For each scheme, the performance almost scales in the number of antennas, which clearly shows the benefits of antenna selection diversity. We also note that in two antenna selection, the maxmin criterion attains closer performance to the BER criterion than the norm criterion.

## References

[1] D. Tse and Pramod Viswanath, *Fundamentals of wireless communication*, Cambridge University Press, 2005.  
 [2] S. Sanayei and A. Nosratinia, "Antenna selection in

MIMO systems," *IEEE Communications Magazine*, vol. 42, no. 10, pp. 68–73, October 2004.

- [3] A. F. Molisch, M. Z. Win, and J. H. Winters, "Reduced-complexity transmit/receive- diversity systems," *IEEE Transactions on Signal Processing*, vol. 51, no. 11, November 2003.  
 [4] S. Sandhu, R. Nabar, D. A. Gore, and A. Paulraj, "Near optimal antenna selection of transmit antennas for a MIMO channel based on Shannon capacity," in *the Thirty-Fourth Asilomar Conference on Signals, Systems and Computers*, November 1999, pp. 567–571.  
 [5] R. Heath and A. Paulraj, "Antenna selection for spatial multiplexing systems based on minimum error rate," in *Proc. IEEE Int. Control Conf.*, 2001, pp. 2276–2280.  
 [6] D. A. Gore and A. Paulraj, "Space-time block coding with optimal antenna selection," in *Proc. of Intl. Conf. on ASSP*, May 2001, pp. 2441–2444.  
 [7] A. Ghayeb and T. M. Duman, "Performance analysis of MIMO systems with antenna selection over quasi-static fading channels," *IEEE Transactions on Vehicular Technology*, vol. 52, no. 2, pp. 281–288, March 2003.  
 [8] I. Bahceci, T. M. Duman, and Y. Altunbasak, "Antenna selection for multiple-antenna transmission systems: Performance analysis and code construction," *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp. 2669–2681, October 2003.  
 [9] B. Muquet, Z. Wang, G. B. Giannakis, M. de Courville, and P. Duhamel, "Cyclic prefixed or zero padded multi-carrier transmissions?," *IEEE Transactions on Communications*, vol. 50, pp. 2136–2148, December 2002.  
 [10] Y.-P. Lin and S.-M. Phoong, "Minimum redundancy for ISI free FIR filterbank transceivers," *IEEE Transactions on Signal Processing*, vol. 50, no. 4, pp. 842–853, April 2002.  
 [11] S. Ohno, "Performance of single-carrier block transmissions over multipath fading channels with linear equalization," *IEEE Transactions on Signal Processing*, vol. 54, no. 10, pp. 3678–3687, October 2006.  
 [12] Q. Ma and C. Tepedelenlioğlu, "Antenna selection for noncoherent space-time-frequency coded OFDM systems," in *Proc. of 40th Conf. on Information Sciences and Systems*, March 2006, pp. 1665–1669.  
 [13] K. Teo, S. Ohno, K. Yamaguchi, and T. Hinamoto, "Computationally efficient antenna selection in MISO-STBC-OFDM," in *Proc. of IEEE 2005 International Symposium on Microwave, Antenna, Propagation and EMC Technologies For Wireless Communications*, August 2005.  
 [14] N. Al-Dhahir, "Single-carrier frequency-domain equalization for space-time block-coded transmission over frequency-selective fading channels," *IEEE Communications Letters*, vol. 5, no. 7, pp. 304–306, July 2001.  
 [15] S. Zhou and G. G. Giannakis, "Single-carrier space-time block-coded transmissions over frequency-selective fading channels," *IEEE Transactions on Information Theory*, vol. 49, no. 1, pp. 164–179, January 2003.