Analysis of Noise Threshold of Regular LDPC Codes on LEO Satellite Channel

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Abstract: The purpose of this research is twofold. One is to analyze the noise threshold of the regular LDPC codes with a BPSK system on an LEO satellite channel. Through analysis of noise threshold, we can determine minimum SNR to achieve the best performance of regular LDPC codes among different code rates and degree parameters. The other is to analyze the effect of fading parameters of the LEO satellite channel on the performance of regular LDPC codes. For this, we investigate the probability density function of a fading factor depending on the parameter values of the Loo's model and analyze the performance of regular LDPC codes. Numerical analysis and simulation results are shown with discussions.

1. Introduction

In satellite mobile communications, problems such as Doppler shift, multi-path fading, etc. that give rise to a high bit error probability may be more serious than terrestrial communications, thus more powerful error correcting code techniques are required for reliable communication in the satellite mobile channel. Recently, low-density parity-check (LDPC) codes were rediscovered and have become a major research topic in error correcting codes. Due to noise threshold effect of LDPC codes, a noise threshold value of the channel usually determines the performance of LDPC codes. In this paper, therefore, we analyze the noise threshold of the regular LDPC codes with a BPSK system on an LEO satellite channel. For this, we derive the initial message probability density function required in the density evolution under the LEO satellite channel and determine the noise threshold for ensemble of regular LDPC codes having different code rates and degree parameters. The effect of fading parameters on noise threshold is also analyzed and results are shown with discussions

2. Loo's model for LEO satellite chanenl

Although there have been many trials to model a satellite fading channel, Loo's model [1] is most convenient to typify the suburban or rural tree-shadowed conditions and suits best the case of mobile omnidirectional antenna in a suburban or rural tree-shadowed environment. Furthermore, it is applicable to non-geostationary satellites and assumes flat fading which is a typical case in satellite link. [2]

In Loo's model, it is assumed that the slow amplitude fluctuations of the line-of-sight component are lognormally distributed, while the fast fading caused by the multipath propagation behaves like a Rayleigh process. Then the fading term can be expressed as a random phasor as given in eq.(1),

$$r\exp(j\theta) = z\exp(j\phi_0) + w\exp(j\phi) \quad (1)$$

where the phase ϕ_0 and ϕ are uniformly distributed random variables between 0 and 2π, z is a lognormally distributed random variable, and w is a random variable having a Rayleigh distribution. If z is temporarily kept constant, then the conditional probability density function of z can be expressed as a Rician distribution

$$p(r \mid z) = \frac{r}{b_0}\exp\left(-\frac{r^2+z^2}{2b_0}\right) I_0\left(\frac{rz}{b_0}\right) \quad (2)$$

where b_0 represents the average scattered power due to multipath in random variable w, and $I_0(\cdot)$ is the modified Bessel function of the zeroth order. Therefore, by theorem of total probability, probability density function of r is given by eq.(3).

$$p(r) = \int_0^\infty p(r,z)dz = \int_0^\infty p(r \mid z)p(z)dz \quad (3)$$

It has been assumed that p(z) has a lognormal distribution which has standard deviation $\sqrt{d_0}$ and mean μ, respectively. Finally we obtain the probability density function of a fading factor r as given in eq.(4).

$$p(r) = \frac{r}{b_0\sqrt{2\pi d_0}} \int_0^\infty \frac{1}{z}\exp\left(-\frac{r^2+z^2}{2b_0} - \frac{(\ln z - \mu)^2}{2d_0}\right) I_0\left(\frac{rz}{b_0}\right)dz \quad (4)$$

Through many experiments and calculations, Loo determined the values of model parameter, μ, b_0 , d_0 by fitting the measurements to the model. Table 1 shows the typical values of the parameters in the Loo's model. In our work, we will consider the overall case as the basic parameter values for analysis.

Table 1: Model parameters for Loo's satellite chanel model

Conditions	Standard Deviation $\sqrt{d_0}$	Mean μ	Multipath Power b_0
Infrequent light shadowing	0.115	0.115	0.158
Frequent heavy shadowing	0.806	-3.91	0.0631
Overall results	0.161	-0.115	0.126

3. Regular LDPC Codes with LEO Channel

LDPC codes exhibit an interesting noise threshold effect [3]: if the noise level of the channel is smaller than a certain noise threshold, the bit error probability goes to zero as the block size increases to infinity; if the noise level is above the noise threshold, the probability of error is always bounded away from zero. Therefore, with certain degree parameter (d_v, d_c), a noise threshold value of the channel determines the performance of the codes.

Density evolution is a numerical technique to analyze the performance of the belief propagation decoding algorithm, which enables the determination of noise thresholds at any desired degree of accuracy. Therefore we apply density evolution to the LEO satellite channel and determine the noise threshold value for each code parameter. From [3], [4], however, the basic assumption of noise threshold effect is made that the probability density of an initial message is symmetric. This will be proved in this section.

First, we will track the average fraction of incorrect messages in the l^{th} decoding round $P_{e,c}^{(l)}$ by analyzing the message-passing decoder directly. Let us consider a regular (d_v, d_c) LDPC code with a modulation scheme of BPSK. It is possible to assume without loss of generality that all-zero codeword is sent. Then error is occurred when 1 is in the estimated codeword. This means that error is occurred when the average of messages has negative sign. Therefore, the average fraction of incorrect message is expressed by

$$P_{e,c}^{(l)} = \int_{-\infty}^{0} p(\alpha^{(l)})d\alpha^{(l)} \quad (5)$$

where $p(\alpha^{(l)})$ denotes the probability density function of the average variable node message against all variable nodes at the l th iteration. Furthermore, $p(\alpha^{(l)})$ could be calculated from the two recursive formula and this density function evolves to make the fraction of incorrect messages zero as iteration goes on. Here we will only show the result of two recursive formula rather than deriving all the processes.

$$p(\alpha^{(l)}) = F^{-1}\left(F\left(p(\alpha^{(l)})\right)F\left(p(\beta^{(l)})\right)^{d_v-1}\right) \quad (6)$$

$$p(\gamma(\beta^{(l)})) = F^{-1}\left(F\left(p(\gamma\left(\alpha^{(l-1)}\right))\right)^{d_c-1}\right) \quad (7)$$

where $F(\cdot)$ denotes the Fourier transform and $\gamma(\cdot)$ is a map form the real numbers $[-\infty,\infty]$ to $GF(2)\times[0,\infty]$ defined by $\gamma(x) := (\operatorname{sgn}(x), -\ln(\tanh(|x|/2)))$.

From now on, we will derive the probability density function of an initial message $\alpha^{(0)}$ in LEO satellite channel. If the code symbol x is mapped into the signal point $w = 1 - 2x$ (BPSK modulation) and transmitted across the LEO satellite channel with additive white Gaussian noise n, the sampled matched filter output $y = w \cdot r + n$ will have the following conditional probability density function.

$$p(y \mid w, r) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y - w\cdot r)^2}{2\sigma^2}\right) \quad (8)$$

where $\sigma^2 = \frac{1}{2R}\left(\frac{E_b}{N_0}\right)^{-1}$ is the variance of the noise n, and R is the code rate and r is the fading factor that follows Loo's model distribution. Assuming probability of code symbols are equal, i.e. $\Pr\{x=0\} = \Pr\{x=1\} = 1/2$, initial message observed from the channel can be expressed as

$$\alpha^{(0)} = \frac{2}{\sigma^2} y \cdot r \quad (9)$$

Since we assume the all-zero codeword is sent, i.e. $w = 1$, a change of variable in eq.(8) yields the conditional probability density function of $\alpha^{(0)}$:

$$p(\alpha^{(0)} \mid r) = \frac{\sigma}{2r\sqrt{2\pi}} \exp\left(-\frac{\left(\alpha^{(0)} - \frac{2r^2}{\sigma^2}\right)^2}{2\left(\frac{2r}{\sigma}\right)^2}\right) \quad (10)$$

Then we can get the probability density function of $\alpha^{(0)}$ using the theorem of total probability.

$$p(\alpha^{(0)}) = \frac{\sigma}{4\pi b_0\sqrt{d_0}} \int_0^\infty \exp\left(-\sigma^2 \frac{\left(\alpha^{(0)} - \frac{2r^2}{\sigma^2}\right)^2}{8r^2}\right) \times \quad (11)$$
$$\int_0^\infty \frac{1}{z} \exp\left(-\frac{r^2+z^2}{2b_0} - \frac{(\ln z - \mu)^2}{2d_0}\right) I_0\left(\frac{rz}{b_0}\right) dz dr$$

From this initial message distribution, we can easily see the symmetric property.

$$p(\alpha^{(0)}) = p(-\alpha^{(0)})\exp(\alpha^{(0)}) \quad (12)$$

To determine noise threshold value σ^*, calculate $P_{e,c}^{(l)}$ based on density evolution with certain noise parameter σ. If $P_{e,c}^{(l)}$ tends to zero then slightly increase the noise parameter and calculate $P_{e,c}^{(l)}$ again until $P_{e,c}^{(l)}$ does not converges to zero. Then we can say the last value of σ is the noise threshold value of (d_v, d_c) codes on a given channel.

4. Numerical and Simulation Results

4. 1 Noise threshold calculation

Using the density evolution technique discussed in section 3, we can obtain the noise threshold values of regular LDPC codes for the LEO satellite channel. Table 2 shows the noise threshold values for various code parameters in LEO satellite channel with $\mu = -0.115$, $d_0 = 0.161^2$,

$b_0 = 0.126$, which is the general case of the LEO satellite channel. In the numerical analysis, we set the maximum iteration number as 2000, and obtain the noise threshold value with the accuracy of 10^{-5}. In the table we express the noise threshold value by both σ^* and its corresponding $(E_b / N_0)^*$ (dB). Since $\sigma^* = \frac{1}{2R} \cdot \left(\frac{E_b}{N_0}\right)^{-1}$ the threshold can be defined as the smallest E_b / N_0 such that $\lim_{l \to \infty} P_{e,c}^{(l)} = 0$.

From the result shown in the table, we can conclude that the lowest number of (d_v, d_c) has the best performance for the same code rate R. This is because the lower density (lower degree parameters) is more look alike cycle-free case which is the basic assumption in the LDPC decoder.

To validate the numerical analysis results, we carry out some simulations of detection of LDPC codes via LEO satellite channel with Loo's model. The BPSK modulation is assumed, and each LDPC code used in the simulation has the block size of $10^3, 10^4, 10^5$. Figure 1 compares the noise threshold values obtained from the numerical analysis and the simulation results for (3,6) regular LDPC code. From the figure, we can conjecture that as the block size goes to infinity the threshold obtained from simulation results will converge to the threshold obtained from numerical analysis.

Table 2: Noise threshold value σ^* for various code parameter (d_v, d_c) in LEO satellite fading channel with $\mu = -0.115$, $d_0 = 0.161^2$, $b_0 = 0.126$.

Parameter (d_v, d_c)	Code Rate R	Threshold value σ^*	$(E_b / N_0)^*$ (dB)
(3,6)	1/2	0.82930	1.62577
(4,8)	1/2	0.77214	2.24597
(4,6)	1/3	0.96639	2.05786
(3,4)	1/4	1.25148	1.03182
(6,8)	1/4	1.11242	1.93014
(4,5)	1/5	1.16413	2.65937

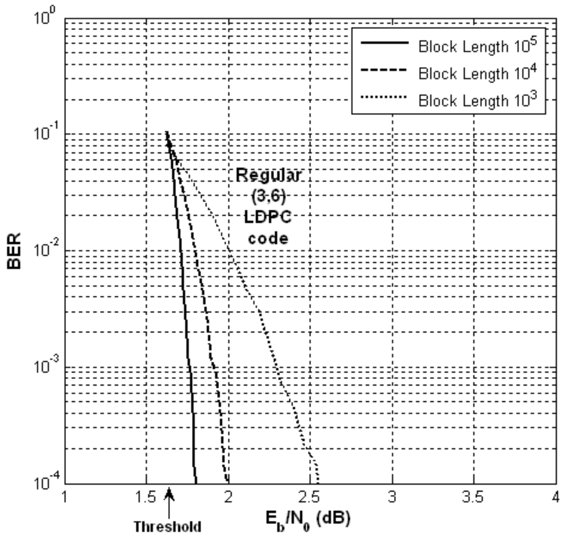


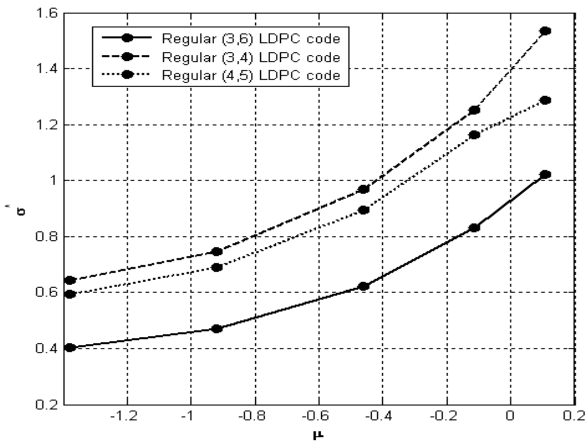
Figure 1: Comparison of thresholds and simulation results for regular LDPC codes on the LEO satellite fading channel. The maximum iteration number in decoder is 2000.

4. 2 Effect of fading parameters in the Loo's model

To analyze the effect of fading parameters on the performance of regular LDPC codes, we change the value of one parameter in density evolution while other parameters are fixed to a value of the overall case in Loo's model. Table 3 to Table 5 show the noise threshold values obtained from numerical analysis for various code parameters depending on a specific fading parameter while others are fixed. Figure 2 is corresponding graph to Table 3.

From this, we can conclude that parameter μ may affect the performance most dominantly with exponential order. Therefore we can mostly consider on parameter μ during the satellite communication and can apply required SNR values from our results for power control to achieve the best performance

Figure 2: The relationship between μ and noise threshold value σ^* for various regular LDPC codes.



5. Conclusion

In this paper, we analyze the performance of regular LDPC codes for fading characteristics of the satellite channel using Loo's model, considering the mobile communications via an LEO satellite channel. The main work carried out in this thesis is twofold. First, we derive the initial message probability density function using the density evolution under the LEO satellite channel and determine the noise threshold for ensemble of regular LDPC codes having different code rates and degree parameters numerically. We verify the numerical analysis results through the simulations. From comparison of the analytical and the simulation results, we confirm that as the block size increases, the performance converges into the analytical required E_b / N_0 for error-free communications. Second, we investigate the probability density function of a fading factor depending on the parameter values of the Loo's model and analyze their effect on the performance of regular LDPC codes. From analytical and numerical analysis, we have shown that the mean of lognormal process in Loo's model, μ , is the dominant parameter to determine the noise threshold value in the fading channel. More specifically, it has been shown that when the channel experiences increasing of μ the noise threshold value σ^* increases with the order of $\exp(\mu)$.

Table 3: Noise threshold value σ^* for various code parameter (d_v, d_c) in LEO satellite fading channel with μ .

Parameter (d_v, d_c)	Code Rate R	μ	Threshold value σ^*	(E_b / N_0) (dB)
(3,6)	1/2	-1.380	0.40228	7.90727
		-0.920	0.47084	6.54253
		-0.460	0.62419	4.09366
		-0.115	0.82930	1.62577
		0.115	1.02306	-0.19802
(3,4)	1/4	-1.380	0.64479	6.82193
		-0.920	0.74757	5.53726
		-0.460	0.96868	3.28669
		-0.115	1.25148	1.06182
		0.115	1.53492	-0.71141
(4,5)	1/5	-1.380	0.59314	8.51626
		-0.920	0.68880	7.21753
		-0.460	0.89607	4.93256
		-0.115	1.16413	2.65937
		0.115	1.28605	1.79424

Table 4: Noise threshold value σ^* for various code parameter (d_v, d_c) in LEO satellite fading channel with b_0 .

Parameter (d_v, d_c)	Code Rate R	b_0	Threshold value σ^*	(E_b / N_0) (dB)
(3,6)	1/2	0.0630	0.80770	1.85500
		0.1260	0.82930	1.62577
		0.1575	0.84155	1.49840
(3,4)	1/4	0.0630	1.20540	1.38767
		0.1260	1.25148	1.06182
		0.1575	1.28695	0.81907
(4,5)	1/5	0.0630	1.12284	2.97304
		0.1260	1.16413	2.65937
		0.1575	1.19355	2.44259

Table 5: Noise threshold value σ^* for various code parameter (d_v, d_c) in LEO satellite fading channel with d_0 .

Parameter (d_v, d_c)	Code Rate R	$\sqrt{d_0}$	Threshold value σ^*	(E_b / N_0) (dB)
(3,6)	1/2	0.161	0.82930	1.62577
		0.483	0.86117	1.29822
		0.805	0.84368	1.47644
(3,4)	1/4	0.161	1.25148	1.06182
		0.483	1.28184	0.85362
		0.805	1.24043	1.13885
(4,5)	1/5	0.161	1.16413	2.65937
		0.483	1.21050	2.32010
		0.805	1.18354	2.51574

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