On Each Condition of Soundness for Acyclic Free Choice Workflow Nets

Shingo Yamaguchi and Naoki Nakahara
Graduate School of Sciences and Technology for Innovation, Yamaguchi University
2-16-1 Tokiwadai, Ube, Yamaguchi 755-8611, Japan
Email: shingo@yamaguchi-u.ac.jp

Abstract: Workflow nets (WF-nets for short) are Petri nets which represent workflows. Soundness is a criterion of logical correctness defined for WF-nets. A WF-net is said to be sound if it satisfies three conditions: (i) option to complete, (ii) proper completion, and (iii) no dead tasks. Our result shows that for an acyclic free choice WF-net, (1) Conditions (i) and (ii) of soundness are respectively equivalent to liveness and boundedness of the short-circuited net; (2) Checking of Conditions (i) and (ii) are respectively NP-complete; and (3) If the short-circuited net has no disjoint paths from a transition to a place (or no disjoint paths from a place to a transition), Conditions (i) and (ii) can be checked in polynomial time.

Keywords—**Petri net, workflow net, soundness, liveness, boundedness**

1. Introduction

Workflow nets [1] (WF-nets for short) are Petri nets [2] which represent workflows. A WF-net is said to be sound if it satisfies three conditions: (i) option to complete, (ii) proper completion, and (iii) no dead tasks. Soundness is a criterion of logical correctness for workflows. Most actual workflows can be modeled as a subclass of WF-nets called as free-choice (FC for short) WF-nets. Van der Aalst [1] proposed a polynomial time procedure to decide soundness for FC WF-nets.

If a given WF-net is non-sound, we should modify it. We may find a clue to the modification by analyzing which conditions of soundness are not satisfied. Nevertheless, little is known about properties and computation complexity for checking each condition.

In this paper, focusing our analysis on acyclic FC WF-nets, we reveal properties and computation complexity for checking each condition of soundness. After the introduction in Sect. 1, Sect. 2 gives the definition and properties of WF-nets. In Sect. 3, we show the correspondence between each condition and Petri nets' basic properties such as liveness and boundedness. Then we prove the NP-completeness for checking each condition. In Sect. 4, by imposing structural restrictions on acyclic FC WF-nets, we show that each condition of soundness can be checked in polynomial time. Section 5 gives the conclusion and future work.

2. WF-Nets and Properties

2.1 WF-Nets

A Petri net [2] is a three tuple $N{=}(P,T,A)$, where P, T, and A are respectively finite set of places, transitions, and arcs. For a place or transition x, $\overset{N}{\bullet}x$ and $x\overset{N}{\bullet}$ respectively denote $\{y|(y,x){\in}A\}$ and $\{y|(x,y){\in}A\}$. N is said to be free choice (FC for short) if $\forall p_1,p_2{\in}P$: $p_1\overset{N}{\bullet}{\cap}p_2\overset{N}{\bullet}{\neq}\emptyset{\Rightarrow}|p_1\overset{N}{\bullet}|{=}|p_2\overset{N}{\bullet}|{=}1$. A marking of N is a mapping $M{:}\,P{\rightarrow}\mathbb{N}$. We represent M as a bag over P, i.e. $M{=}[p^{M(p)}|p{\in}P,M(p){>}0]$. A transition t is said to be firable in a marking M if $M{\geq}\overset{N}{\bullet}t$. This is denoted by $M[N,t\rangle$. Firing t in M results in a new marking M' $({=}M{\cup}t\overset{N}{\bullet}{\setminus}\overset{N}{\bullet}t)$. This is denoted by $M[N,t\rangle M'$. A marking M_n is said to be reachable from a marking M_0 if there exists a transition sequence σ $({=}t_1t_2\cdots t_n)$ such that $M_0[N,t_1\rangle M_1[N,t_2\rangle M_2\cdots[N,t_n\rangle M_n$. This is denoted by $M_0[N,\sigma\rangle M_n$. σ is called firing sequence. The set of all possible firing sequences from M_0 is denoted by $L(N,M_0)$. A transition t in (N,M_0) is said to be dead if t does not appear in any firing sequence in $L(N,M_0)$. A marking is said to be dead if no transition is firable in the marking. The set of all possible markings reachable from M_0 is denoted by $R(N,M_0)$.

N is said to be a WF-net if (i) N has a single source place p_I and a single sink place p_O and (ii) every node is on a path from p_I to p_O. Each action of a workflow is modeled as a transition. Causalities between actions are modeled as places and arcs. The initial marking of any WF-net is $[p_I]$. If we add a transition t^* to a WF-net N which connects p_O with p_I, then we obtain a strongly-connected net. The net is called the short-circuited net of N and is denoted by $\overline{N}$.

We say a path from a node x to a node y as an XY-path, where if $x{\in}P$ then X is P, otherwise X is T; if $y{\in}P$ then Y is P, otherwise Y is T. Two paths are said to be (node) disjoint if they have the same start node and the same end node but do not have any internal node in common. Let c be a cycle in $\overline{N}$. A path is called a handle of c if h and a part of c are disjoint. A path $b = y_1y_2\cdots y_m$ $(m{\geq}2)$ is called a bridge between c and h if b shares exactly one node, y_1 or y_m, with each of c and h.

2.2 Soundness

A Petri net N with an initial marking M_0, i.e. (N,M_0), is said to be live if, for every marking $M{\in}R(N,M_0)$ and every transition $t{\in}T$, there exists a marking $M'{\in}R(N,M)$ such that $M'[N,t\rangle$. (N,M_0) is said to be k-bounded or simply bounded if, for every marking $M{\in}R(N,M_0)$ and every place p, $M(p){\leq}k$. Soundness is a criterion of logical correctness defined for WF-nets. A WF-net N is said to be sound if

(i) Option to complete:
$\forall M{\in}R(N,[p_I])$: $\exists M'{\in}R(N,M)$: $M'{\geq}[p_O]$;
(ii) Proper completion:
$\forall M{\in}R(N,[p_I])$: $M{\geq}[p_O]\Rightarrow M{=}[p_O]$; and
(iii) No dead tasks:
There is no dead transition in $(N,[p_I])$.

A WF-net N is sound iff $(\overline{N},[p_I])$ is live and bounded.

3. Properties and Computation Complexity

In this section, we reveal that for acyclic FC WF-nets, (1) Conditions (i) and (ii) of soundness are respectively equivalent to liveness and boundedness of the short-circuited nets; and (2) Checking of Conditions (i) and (ii) are respectively NP-complete.

3.1 Condition (i) of Soundness

Theorem 1: An acyclic FC WF-net N satisfies Condition (i) of soundness, i.e. $\forall M \in R(N,[p_I])$: $\exists M' \in R(N,M)$: $M' \geq [p_O]$, iff $(\overline{N},[p_I])$ is live. ■

Lemma 1: An acyclic FC WF-net N satisfies Condition (i) of soundness, i.e. $\forall M \in R(N,[p_I])$: $\exists M' \in R(N,M)$: $M' \geq [p_O]$, iff $(\overline{N},[p_I])$ has no dead marking. ■

Proof: The proof of "if" part: Since $\overline{N}$ is strongly-connected, transition t^* repeatedly fires in $(\overline{N},[p_I])$. This means that in $(N,[p_I])$, $[p_I]$ is necessarily transformed to a marking which has a token at p_O. Thus N satisfies Condition (i) of soundness.

The proof of "only if" part: Condition (i) of soundness can be rewritten as the following property: A token necessarily arrives at p_O in $(N,[p_I])$. Let us consider the monotonicity of this property: Let M be any marking of N such that $M > [p_I]$, a token necessarily arrives at p_O in (N,M). The structure of the place having two or more output transitions is called a conflict. For each conflict structure, whichever transition we choose, a token arrives at p_O in $(N,[p_I])$. Since N is FC, there is no marking such that some transitions of a conflict structure are firable and the others are not firable. Therefore, we can choose which transition to fire in (N,M) as well as $(N,[p_I])$. Whichever transition we choose, a token in (N,M) arrives at p_O. Thus Condition (i) of soundness has monotonicity. Since we have

- $[p_I]$ is necessarily transformed to a marking M ($>[p_O]$) in N (Condition (i) of soundness)
- $M = [p_O] \cup M' [\overline{N}, t^*\rangle [p_I] \cup M'$
- $[p_I] \cup M'$ is necessarily transformed to a marking L ($>[p_O]$) in N (Monotonicity of Condition (i) of soundness)
- $L = [p_O] \cup L' [\overline{N}, t^*\rangle [p_I] \cup L'$

$$\vdots$$

Thus $(\overline{N},[p_I])$ has no dead marking. Q.E.D.

Lemma 2: Let N be an acyclic FC WF-net. $(\overline{N},[p_I])$ has no dead marking iff $(\overline{N},[p_I])$ is live. ■

Proof: The proof of "if" part: Immediate from the definition.

The proof of "only if" part: Since $(\overline{N},[p_I])$ has no dead marking, p_I is necessarily marked repeatedly. Assume that there are dead transitions in $(\overline{N},[p_I])$. Let t is a dead transition which is a nearest from p_I. Since there is no other dead transition between p_I and t, a token of p_I arrives at an input place of t. Since p_I is marked repeatedly, the other input places also can be marked. This means that t is not dead. In the same way, we can make sure that all the transitions are not dead. Since p_I is marked infinitely, every transition can always fire again. Thus $(\overline{N},[p_I])$ is live. Q.E.D.

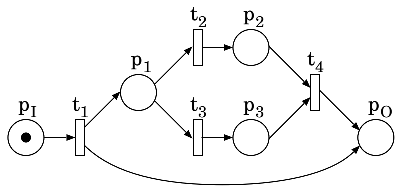

Figure 1. A non-sound acyclic FC WF-net N_1. N_1 satisfies Condition (i) of soundness.

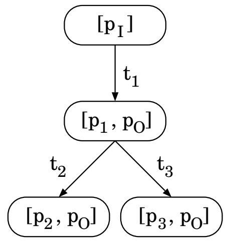

Figure 2. The reachability tree of $(N_1,[p_I])$.

Proof of Theorem 1: Immediate from Lemmas 1 and 2. Q.E.D.

Theorem 1 means that for an acyclic FC WF-net N, Condition (i) of soundness is equivalent to liveness of the short-circuited net $(\overline{N},[p_I])$.

Let us consider an acyclic FC WF-net N_1 shown in Fig. 1. $(N_1,[p_I])$ has the reachability tree shown in Fig. 2. $(N_1,[p_I])$ satisfies Condition (i) of soundness, because every marking in $R(N_1,[p_I])$ is reachable to $[p_2,p_O]$ or $[p_3,p_O]$. By Theorem 1, we can say that $(\overline{N_1},[p_I])$ is live. Let us make sure of that. $\overline{N_1}$ has two minimal siphons $\{p_I,p_1,p_2,p_O\}$ and $\{p_I,p_1,p_3,p_O\}$. Every siphon in $\overline{N_1}$ contains a marked trap $\{p_I,p_O\}$. This means that $(\overline{N_1},[p_I])$ is live by Commoner's theorem (Theorem 12 of Ref. [2]). On the other hand, $(N_1,[p_I])$ does not satisfy Condition (ii) of soundness, because $[p_1,p_O]$ ($\in R(N_1,[p_I])$) is greater than $[p_O]$.

Theorem 2: The following problem is NP-complete: Given an acyclic FC WF-net N, to decide whether N violates Condition (i) of soundness. ■

Proof: By Theorem 1, the condition is equivalent to the liveness of $(\overline{N},[p_I])$. The non-liveness problem for $(\overline{N},[p_I])$ is NP-complete (Theorem 1 of Ref. [3]). Q.E.D.

This theorem means that for acyclic FC WF-nets, Condition (i) of soundness cannot be decided in polynomial time if $P \neq NP$.

3.2 Condition (ii) of Soundness

Theorem 3: An acyclic FC WF-net N satisfies Condition (ii) of soundness, i.e. $\forall M \in R(N,[p_I])$: $M \geq [p_O] \Rightarrow M = [p_O]$, iff $(\overline{N},[p_I])$ is bounded. ■

Proof: Be similar to the proof of Property 2 of Ref. [4].

The proof of "if" part: Let $(\overline{N},[p_I])$ be bounded. Assume that $(N,[p_I])$ has a marking M in $R(N,[p_I])$ such that $M > [p_O]$. M can be divided into two sub-markings $[p_O]$ and M'. Note that $M' > \emptyset$. We have

$$M = [p_O] \cup M' [\overline{N}, t^*\rangle [p_I] \cup M'$$
$$[\overline{N}, *\rangle\ M \cup M' (\geq M).$$

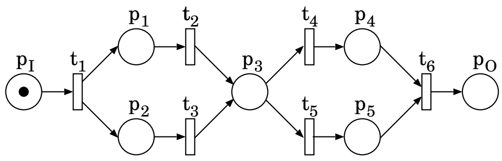

Figure 3. A non-sound acyclic FC WF-net N_2. N_2 satisfies Condition (ii) of soundness.

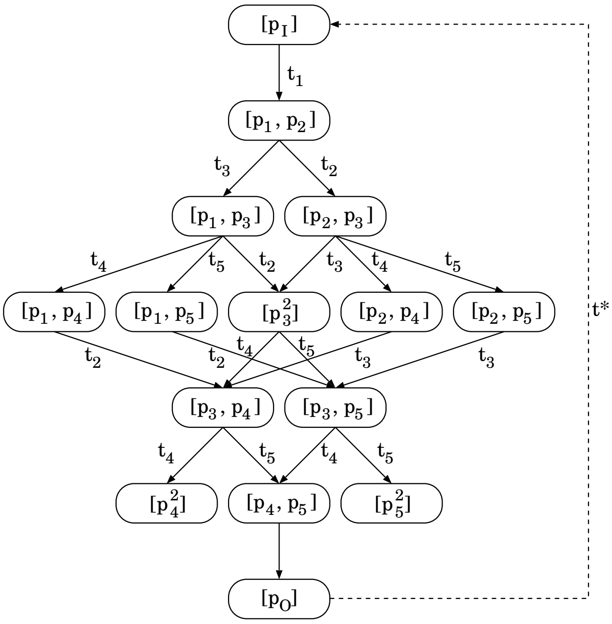

Figure 4. The reachability tree of $(N_2, [p_I])$ drawn by the solid lines. The dashed lines and solid lines represent the reachability tree of $(\overline{N_2}, [p_I])$.

This means that $(\overline{N}, [p_I])$ is unbounded. Therefore, the assumption must be wrong. Thus N satisfies Condition (ii) of soundness.

The proof of "only if" part: Let us consider the contraposition. If every marking M in $R(N, [p_I])$ has no token at p_O, i.e. $\forall M{\in}R(N, [p_I]) : M(p_O){=}0$, then transition t^* never fires in $(\overline{N}, [p_I])$. Therefore, $R(\overline{N}, [p_I]){=}R(N, [p_I])$ holds. Since N is acyclic, $(N, [p_I])$ is bounded. This means that $(\overline{N}, [p_I])$ is bounded.

If there exists a marking M in $R(N, [p_I])$ such that M has a token at p_O, then $M{=}[p_O]$. Since $[p_O][\overline{N}, t^*\rangle[p_I]$, $R(\overline{N}, [p_I]){=}R(N, [p_I])$ holds. Since N is acyclic, $(N, [p_I])$ is bounded. This means that $(\overline{N}, [p_I])$ is bounded. Q.E.D.

This theorem means that for an acyclic FC WF-net N, Condition (ii) of soundness is equivalent to boundedness of the short-circuited net $(\overline{N}, [p_I])$.

Let us consider an acyclic FC WF-net N_2 shown in Fig. 3. $(N_2, [p_I])$ has the reachability tree shown in Fig. 4. $(N_2, [p_I])$ satisfies Condition (ii) of soundness, because it has only $[p_O]$ as a marking with a token in p_O. By Theorem 3, we can say that $(\overline{N_2}, [p_I])$ is bounded. Let us make sure of that. Figure 4 shows that every marking in $R(\overline{N_2}, [p_I])$ has at most two tokens in each place. This means that $(\overline{N_2}, [p_I])$ is 2-bounded or simply bounded. On the other hand, $(N_2, [p_I])$ does not

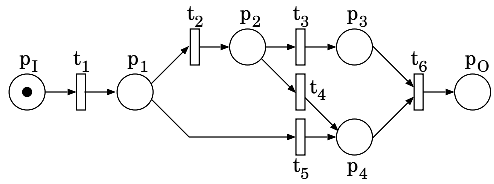

Figure 5. A non-sound acyclic FC WF-net N_3. $\overline{N_3}$ has no disjoint TP-paths but has disjoint PT-paths: $p_1t_2p_2t_3p_3t_6$ and $p_1t_5p_4t_6$. N_3 does not satisfy Condition (i) of soundness but satisfies Condition (ii).

satisfy Condition (i) of soundness, because $[{p_4}^2]$ and $[{p_5}^2]$ are dead.

Theorem 4: The following problem is NP-complete: Given an acyclic FC WF-net N, to decide whether N violates Condition (ii) of soundness. ■

Proof: By Theorem 3, the condition is equivalent to the boundedness of $(\overline{N}, [p_I])$. The unboundedness problem for $(\overline{N}, [p_I])$ is NP-complete (Theorem 3 of Ref. [4]). Q.E.D.

This theorem means that for acyclic FC WF-nets, Condition (ii) of soundness cannot be decided in polynomial time if $P{\neq}NP$.

3.3 Condition (iii) of Soundness

The computation complexity to check Condition (iii) of soundness is an open question.

4. Structural Restricted Nets Checkable in Polynomial Time

In this section, by imposing structural restrictions on acyclic FC WF-nets, we show that Conditions (i) and (ii) of soundness can be checked in polynomial time

Property 1: Let N be an acyclic FC WF-net such that $\overline{N}$ has no disjoint TP-paths.

(a) N satisfies Condition (i) of soundness iff $\overline{N}$ has no disjoin PT-paths.

(b) N always satisfies Condition (ii) of soundness. ■

Proof: (a) The proof of "if" part: Since $\overline{N}$ has neither disjoint TP-paths nor disjoint PT-paths, N is acyclic well-structured. Since N is sound, N satisfies Condition (i) of soundness.

The proof of "only if" part: Let us consider the contraposition. Since $\overline{N}$ has disjoint PT-paths, there exists a circuit c having a PT-handle ρ in $\overline{N}$. Assume that ρ has a TP-bridge. This TP-bridge forms disjoint TP-paths with c and ρ. Therefore, the assumption must be wrong. Since ρ has no TP-bridge, $(\overline{N}, [p_I])$ is non-live and/or unbounded. Since $\overline{N}$ has no disjoint TP-paths, $(\overline{N}, [p_I])$ is bounded (as explained later). Thus $(\overline{N}, [p_I])$ is non-live. This means by Theorem 1 that N violates Condition (i) of soundness.

(b) Since $\overline{N}$ has no disjoint TP-paths, no circuit of $\overline{N}$ has a TP-handle. This means by Theorem 3.1 of Ref. [5] that $\overline{N}$ is structurally bounded. $(\overline{N}, [p_I])$ is bounded. By Theorem 3, we show that N satisfies Condition (ii) of soundness. Q.E.D.

Property 1 can be decided in polynomial time by using the max-flow min-cut algorithm.

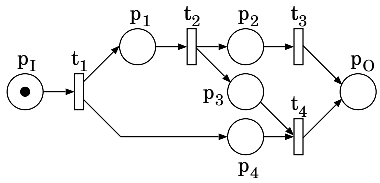

Figure 6. A non-sound acyclic FC WF-net N_4. $\overline{N_4}$ has no disjoint PT-paths but has disjoint TP-paths: $t_1p_1t_2p_2t_3p_O$ and $t_1p_4t_4p_O$. N_4 satisfies Condition (i) of soundness but does not satisfy Condition (ii).

As an example, let us consider an acyclic FC WF-net N_3 shown in Fig. 5. $\overline{N_3}$ has no disjoint TP-paths but has disjoint PT-paths: $p_1t_2p_2t_3p_3t_6$ and $p_1t_5p_4t_6$. Property 1 (a) shows that N_3 does not satisfy Condition (i) of soundness. We have $[p_I][N_3, t_1t_5\rangle[p_4]$. $[p_4]$ is, however, dead, so it is not reachable to a marking with a token in p_O. Thus N_3 does not satisfy Condition (i) of soundness. On the other hand, Property 1 (b) shows that N_3 satisfies Condition (ii) of soundness. There is no marking in $R(\overline{N_3}, [p_I])$ with a token in p_O. This means that N_3 satisfies Condition (ii) of soundness.

Property 2: Let N be an acyclic FC WF-net such that $\overline{N}$ has no disjoint PT-paths.

(a) N always satisfies Condition (i) of soundness.

(b) N satisfies Condition (ii) of soundness iff $\overline{N}$ has no disjoin TP-paths. ■

Proof: (a) Since $\overline{N}$ has no disjoint PT-paths, no circuit of $\overline{N}$ has a PT-handle. This means by Theorem 3.1 of Ref. [5] that $\overline{N}$ is repetitive. In the same way as Theorem 1, we show that $(\overline{N}, [p_I])$ is live and satisfies Condition (i) of soundness.

(b) The proof of "if" part: Since $\overline{N}$ has neither disjoint PT-paths nor disjoint TP-paths, N is acyclic well-structured. Since N is sound, N satisfies Condition (ii) of soundness.

The proof of "only if" part: Let us consider the contraposition. Since $\overline{N}$ has disjoint TP-paths, there exists a circuit having a TP-handle in $\overline{N}$. $(\overline{N}, [p_I])$ is non-live and/or unbounded. Since $(\overline{N}, [p_I])$ is live, $(\overline{N}, [p_I])$ is unbounded. This means by Theorem 3 that N violates Condition (ii) of soundness. Q.E.D.

Property 2 can be decided in polynomial time by using the max-flow min-cut algorithm.

As an example, let us consider an acyclic FC WF-net N_4 shown in Fig. 6. $\overline{N_4}$ has no disjoint PT-paths but has disjoint TP-paths: $t_1p_1t_2p_2t_3p_O$ and $t_1p_4t_4p_O$. Property 2 (a) shows that N_4 satisfies Condition (i) of soundness. $\overline{N_4}$ has two minimal siphons: $\{p_I, p_1, p_2, p_4, p_O\}$ and $\{p_I, p_1, p_2, p_3, p_O\}$. Every siphon in $\overline{N_4}$ contains a marked trap $\{p_I, p_1, p_2, p_O\}$. This means that $(\overline{N_4}, [p_I])$ is live by Commoner's theorem. Thus N_3 satisfies Condition (i) of soundness. On the other hand, Property 2 (b) shows that N_4 does not satisfy Condition (ii) of soundness. We have $[p_I][N_4, t_1t_2t_3\rangle[p_3, p_4, p_O]$ $(>[p_O])$. Thus N_4 does not satisfy Condition (ii) of soundness.

5. Conclusion

In this paper, we revealed that (1) Conditions (i) and (ii) of soundness are respectively equivalent to liveness and boundedness of the short-circuited net; (2) Checking of Conditions (i) and (ii) are respectively NP-complete; and (3) If the short-circuited net has no disjoint PT-paths (or no disjoint TP-paths), Conditions (i) and (ii) can be checked in polynomial time. These results would help us to find a clue to modifying non-sound WF-nets. As an example, let us consider the non-sound WF-net shown in Fig. 3. This WF-net does not satisfy Condition (i). In fact, it has two dead markings: $[{p_4}^2]$ and $[{p_5}^2]$. We can apply the supervisory control method proposed in Ref. [6] to prevent those dead markings. Figure 7 shows the modified WF-net. We would like to consider this in the future research.

Acknowledgment

This work was partially supported by Interface Corporation.

References

[1] W.M.P. van der Aalst, "The application of Petri nets to workflow management," J. Circuits, Systems, Computers, vol.8, no.1, pp.21–65, 1998.

[2] T. Murata, "Petri nets: properties, analysis and applications," Proc. of IEEE, vol.77, no.4, pp.541–580, 1989.

[3] N. Nakahara, S. Yamaguchi, "On liveness of non-sound acyclic free choice workflow nets," Proc. of CANDAR 2014, pp.336–341, 2014.

[4] S. Yamaguchi, "The state number calculation problem for free-choice workflow nets is intractable," Proc. of ITC-CSCC 2014, pp.838–841, 2014.

[5] J. Esparza, M. Silva, "Circuits, handles, bridges and nets," Proc. on APN 1990, pp.210–242, 1991.

[6] S. Yamaguchi, S. Nagano, "A place-invariant based method for supervisory control of workflow nets," Proc. of ITC-CSCC 2015, pp.268–271, 2015.

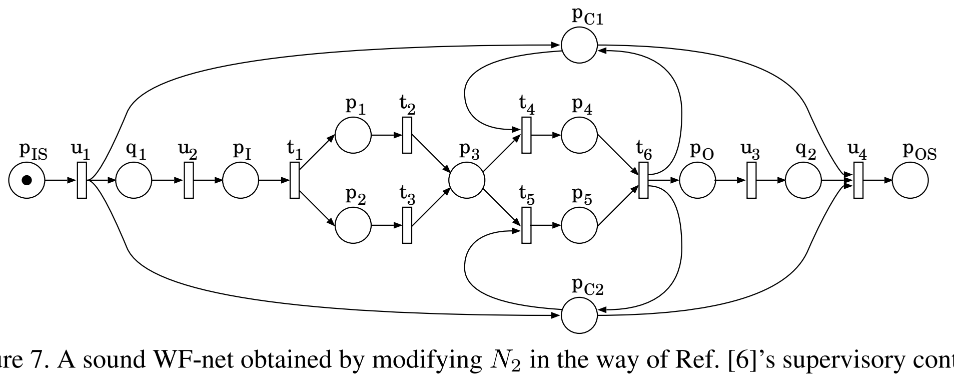

Figure 7. A sound WF-net obtained by modifying N_2 in the way of Ref. [6]'s supervisory control.