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Light-Weight Performance Analysis of Wi-Fi Offload Using Mean-Field Approximation

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Abstract—**We propose a light-weight performance analysis of Wi-Fi offload using a simple Markov model. Although the proposed model does not describe the behavior of the contention window of each station in detail, it allows us to analyze the non-saturation throughput while considering interaction among interfering stations and the queueing behavior of each station. In addition to this, the proposed model does not need to use the assumption that frames arrive at relay station according to a Poisson process. In this paper, we show how to apply the mean field approximation to the model to significantly reduce the amount of computation. We show that the proposed model accurately estimates the throughput performance of the Wi-Fi offload, and that the mean-field approximation is still valid in the scenario of the Wi-Fi offload.**

I. Introduction

Due to rapid spread of mobile routers or smartphones equipped with Wi-Fi and 3G/4G interfaces, which are referred to as *client tethering devices* in this paper, Wi-Fi tethering via 3G/4G network has become a popular access technology to the Internet. Wi-Fi tethering is applicable to a variety of client terminals, such as laptops, tablets, portable gaming systems and medical devices [1]. In many countries, however, the Wi-Fi tethering via 3G/4G network often requires an expensive payment, as well as some traffic restrictions. If a client tethering device can work as a Wi-Fi relay station, Wi-Fi tethering is possible over provider's Wi-Fi access network without using cellular service. This is so called Wi-Fi offload. In fact, several mobile routers available in a market (e.g., NEC Aterm PA-W500P-B) can be used as Wi-Fi relay stations. When a tethering device is configured with the Wi-Fi hot spot, client's terminals can gain access to the Internet via the tethering device, and thereby save the trouble of connecting each device separately.

The objective of this paper is to theoretically analyze the throughput performance of the Wi-Fi tethering with Wi-Fi offload. In the common scenario of Wi-Fi offload, at least three stations – client terminal, client tethering device and provider's AP – share the same wireless medium without hidden terminals. One difficulty of the throughput analysis is the interference between these stations. Note that the interference between stations still exists in a single WLAN, and existing studies concerning a single WLAN apply the mean field approximation [2], in which the behavior of stations sharing the common wireless medium is assumed to be statistically independent. It is proved by [3] that, for a wide range of random back-off algorithms, the mean field approximation is asymptotically exact as the number of sources grows. In the case where only a small number of stations share the wireless medium like Wi-Fi offload, however, the accuracy of the mean field approach has not been well verified.

The other difficulty is how to describe the frame arrival process to the relay station. In general, the frame-arrival process at the relay station is not Poisson even if frames arrive at source stations (provider's AP and client's terminal) according to Poisson processes. This is because the frame arrival process at the relay station is the superposition of the output processes from source stations and the output process from each source station is not Poisson in general. Most of existing studies concerning the performance analysis of multi-hop WLANs assumed that the frame arrival processes at relay stations are also Poisson, but this assumption needs to be verified in Wi-Fi offload cases.

We have previously proposed a simple Markov model for the analysis of multi-hop WLANs [4], where the length of backoff period and frame-transmission period are assumed to be exponentially distributed. Although this model is far less detailed than existing analytical models for WLANs, it allows us to analyze the non-saturation throughput accurately by considering the interaction among interfering stations and queueing process of each station. It requires, however, a large amount of computation even if only a small number of stations sharing the common wireless medium. In this paper, we propose a light-weight analysis of the proposed model based on the mean-field approximation. The contributions of this paper are two folds: one is to show that the mean-field approximation is still valid in the scenario of the Wi-Fi offload. The other one is to verify that the proposed model, which is much simpler than Bianchi's model and its extensions, is useful in knowing the non-saturation throughput and queueing process when the Wi-Fi offload is used.

The paper is organized as follows. In Sec. II, we briefly

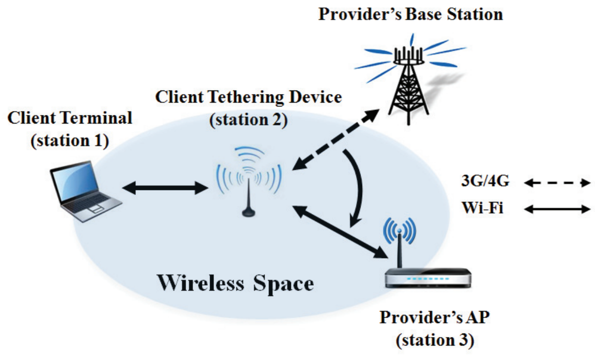

Fig. 1. Wi-Fi Offload.

summarize the related work. In Sec. III, we describe our Markov model for the analysis of Wi-Fi offload and its solution. In Sec. IV, we show how the mean field approximation is applied to the Markov model to make the analysis light-weight. Then, in Sec. V, we compare the results of the analytical models with simulation experiments. Finally, the paper ends with a concluding section.

II. Related Work

Wi-Fi term means wireless LAN (WLAN) or its product based on the 802.11 standards. In this paper, we focus on a WLAN based on the IEEE 802.11 DCF, which is a random access scheme based on CSMA/CA. Significant number of work has been done on the analysis of WLANs based on IEEE 802.11 DCF. Bianchi [5] proposed a two dimensional Markov chain model to analyze the performance under saturated conditions. Malone *et al.* [6] proposed the extension of Bianchi's model where they added post-backoff states and showed throughput results under non-saturated conditions. Liu *et al.* [7] showed an extension of Malone's model which used a three-dimensional Markov chain to integrate the contention resolution of DCF and queueing processes into one model. Their approaches focus on the single wireless LAN, and thus some extensions are required to apply them to the Wi-Fi tethering with Wi-Fi offload. Bui *et al.* [8] analyzed the throughput performance of Wi-Fi offload based on the Bianchi's approach.

III. Markov model for Wi-Fi tethering

We consider a WLAN consisting of a client terminal (station 1), client tethering device (station 2), and provider's AP (station 3) (Fig. 1). All stations work in IEEE 802.11 DCF basic mode without RTS/CTS scheme. The AP is configured with a high-speed wired access to the Internet. The tethering device is connected to the AP, and it shares a Wi-Fi as a tethering terminal. The client terminal, which needs to gain access to the Internet, is connected to the tethering device. We assume that all stations in the network share the same ideal channel without the existence of hidden-station or signal-capturing effects.

We assume that frames arrive at station i according to a Poisson process with rate λ_i. Note that $\lambda_2 = 0$ because station 2 is a relay station. Frames arriving at station 1 (from application layer) are transferred to station 3 via station 2, and frames arriving at station 3 are transferred to station 1 via station 2. Each station has a buffer with size K, which is used as a waiting room for frames to be transferred.

In the following, a station is called "BT-off" if its backoff timer is 0; otherwise it is called "BT-on." A station is also called "inactive" if it senses the frame transmission by any other stations; otherwise, it is called "active". A station is active when it senses that the wireless medium is idle or when it is transmitting a frame. In the proposed model, the backoff timer of station i turns off at random times with intensity ν_i if it is BT-on and active. This assumption means that the length of the period in which station i is BT-on and active is exponentially distributed with mean $1/\nu_i$.

When the backoff timer of a station turns off and it has at least one frame, it immediately starts the frame transmission. The frame-transmission period is defined as a time period which starts with the frame transmission and ends with the receipt of the ACK. In the proposed model, the length of the frame transmission period is assumed to be independent and exponentially distributed with mean $1/\mu$. A station is BT-off when it is transmitting a frame. When the frame-transmission period ends, the station becomes BT-on even if it leaves no frame in the buffer. This is called post-backoff which is introduced in [6]. In the proposed model, the collision of frames is ignored.

For simplifying the notation, we introduce the following variable representing the joint state of the backoff timers of all stations:

$$B \stackrel{\text{def}}{=} B_1 + 2B_2 + 4B_3.$$

where B_i denote a variable representing the state of backoff timer of station i; $B_i = 1$ ($B_i = 0$) when station i is BT-off (BT-on). For example, $B = 5$ means $(B_1, B_2, B_3) = (1, 0, 1)$. Let L_i denote the number of frames in the buffer of station i including the one being transferred by station i. The pair of the number of frames and the backoff states of the stations, (L_1, L_2, L_3, B), follows a continuous time Markov chain. Let $p(k_1, k_2, k_3, n)$ be the probability that $L_1 = k_1$, $L_2 = k_2$, $L_3 = k_3$ and $B = n$ in the stationary state. The global balance equation concerning $p(k_1, k_2, k_3, 0)$ is given as

$$\begin{aligned}
&(\mathbf{1}(k_1 < K)\lambda_1 + \mathbf{1}(k_3 < K)\lambda_3 + \nu_1 + \nu_2 + \nu_3)p(k_1, k_2, k_3, 0)\\
&= \mathbf{1}(k_1 > 0)\lambda_1 p(k_1 - 1, k_2, k_3, 0)\\
&\quad + \mathbf{1}(k_3 > 0)\lambda_3 p(k_1, k_2, k_3 - 1, 0)\\
&\quad + \mathbf{1}(k_1 < K)\{\mathbf{1}(k_2 > 0)\mu p(k_1 + 1, k_2 - 1, k_3, 1)\\
&\qquad + \mathbf{1}(k_2 = K)\mu p(k_1 + 1, k_2, k_3, 1)\}\\
&\quad + \mathbf{1}(k_3 < K)\{\mathbf{1}(k_2 > 0)\mu p(k_1, k_2 - 1, k_3 + 1, 4)\\
&\qquad + \mathbf{1}(k_2 = K)\mu p(k_1, k_2, k_3 + 1, 4)\}\\
&\quad + \mathbf{1}(k_2 < K)\mu p(k_1, k_2 + 1, k_3, 2),
\end{aligned}$$

where $\mathbf{1}(A)$ in the indicator function, which is equal to 1 if A is true and 0 if A is false. The global balance equations concerning $p(k_1, k_2, k_3, n)$ for $n = 1, \cdots, 7$ are shown in Appendix.

Let Q denote the infinitesimal generator of the continuous time Markov chain and $\boldsymbol{p}$ denote its stationary distribution. $\boldsymbol{p}$ should satisfy

$$\boldsymbol{p}Q = \mathbf{0}. \tag{1}$$

Now let $P \stackrel{\mathrm{def}}{=} I - \frac{1}{a}Q$ where $a \stackrel{\mathrm{def}}{=} \max_i\{Q_{ii}\}$ and I is the identity matrix. It follows from (1) that

$$\boldsymbol{p}P = \boldsymbol{p}.$$

Thus, we can numerically obtain the stationary distribution by recursively getting the distribution through relation $\boldsymbol{p}_{n+1} = \boldsymbol{p}_n P$ from an arbitrary initial distribution $\boldsymbol{p}_0$. This technique for numerically obtaining the stationary distribution is called uniformization method. The total throughput is obtained as

$$\text{total throughput} = \mu d \sum_{i=0}^{K}\sum_{j=1}^{K}\sum_{k=0}^{K}\sum_{B_1=0}^{1}\sum_{B_3=0}^{1} p(i,j,k,B_1+2+4B_3),$$

where d is the average length of the payload of a frame.

IV. Mean-Field Approximation

A. Description of stations

The solution of the proposed model by the uniformization requires a large amount of computation because it needs quite a large number of iteration. In this paper, we apply the mean-filed approximation to analyze the model with a much smaller amount of computation. Let A_i denote a variable representing the state of station i; $A_i = 0$ when station i is BT-on and active, $A_i = 1$ when station i is BT-on and inactive, $A_i = 1^*$ when station i is BT-off and inactive, and $A_i = 2$ when station i is BT-off and active.

A station is called in a *medium-idle* state when it senses that the wireless medium is idle. Note that station i is in a medium-idle state only when it is active. The transition of a station from a medium-idle state to an inactive state is triggered by the frame transmission of other stations, but this interdependency between stations makes the analysis complicated. To treat the state transitions of each station independently, in the mean-field approximation, we assume that station i transits from a medium-idle state to an inactive state according to a Poisson process with intensity γ_i. Parameter γ_i is first unknown and is determined in the analysis as explained later.

B. Source stations

In the Wi-Fi offload, stations 1 and 3 are source stations that generate frames, and station 2 is the relay station that only forwards frames. In this subsection, we show the global balance equations of source stations. Let $p_i(k,l)$ denote the probability that $L_i = k$ and $A_i = l$ in the stationary state. When $1 \le k \le K-1$,

$$\begin{aligned}(\lambda_i+\gamma_i+\nu_i)p_i(k,0) &= \lambda_i p_i(k-1,0) \\ &\quad + \mu(p_i(k,1)+p_i(k+1,2)),\\ (\lambda_i+\mu)p_i(k,1) &= \lambda_i p_i(k-1,1) + \gamma_i p_i(k,0) \\ &\quad + \mathbf{1}(k=1)\lambda_i p(0,1^*),\\ (\lambda_i+\mu)p_i(k,2) &= \lambda_i p_i(k-1,2) + \nu_i p_i(k,0).\end{aligned} \tag{2}$$

When $k=0$,

$$\begin{aligned}(\lambda_i+\gamma_i+\nu_i)p_i(0,0) &= \mu(p_i(0,1)+p_i(1,2)),\\ (\lambda_i+\mu)p_i(0,1) &= \gamma_i p_i(0,0),\\ (\lambda_i+\gamma_i)p_i(0,2) &= \nu_i p_i(0,0) + \mu p_i(0,1^*),\\ (\lambda_i+\mu)p_i(0,1^*) &= \gamma_i p(0,2),\end{aligned} \tag{3}$$

and when $k=K$,

$$\begin{aligned}(\gamma_i+\nu_i)p_i(K,0) &= \lambda_i p_i(K-1,0) + \mu p_i(K,1),\\ \mu p_i(K,1) &= \lambda_i p_i(K-1,1) + \gamma_i p_i(K,0),\\ \mu p_i(K,2) &= \lambda_i p_i(K-1,2) + \nu_i p_i(K,0).\end{aligned} \tag{4}$$

The stationary distribution of a source station can be numerically obtained from global balance equations (2), (3), and (4). Assume that $p_i(0,2)$ is known to be equal to c. Once $p_i(0,2)$ is known, $p_i(0,0)$, $p_i(0,1)$, and $p_i(0,1^*)$ are obtained from the second, the third and the forth equations of (3) as follows:

$$p_i(0,0) = \frac{\lambda_i(\lambda_i+\gamma_i+\mu)}{\nu_i(\lambda_i+\mu)}c, \quad p_i(0,1) = \frac{\lambda_i\gamma_i(\lambda_i+\gamma_i+\mu)}{\nu_i(\lambda_i+\mu)^2}c,$$
$$p_i(0,1^*) = \frac{\gamma_i}{\lambda_i+\mu}c.$$

The first equation of (3) gives

$$p_i(1,2) = \frac{\lambda_i+\gamma_i+\nu}{\mu}p_i(0,0) - p_i(0,1).$$

From the second and third equations of (2) with the above expression, we obtain $p_i(1,0)$ and $p_i(1,1)$ as follows:

$$\begin{aligned}p_i(1,0) &= \frac{\lambda_i+\mu}{\nu_i}p_i(1,2) - \frac{\lambda_i}{\nu_i}p_i(0,2),\\ p_i(1,1) &= \frac{\lambda_i}{\lambda_i+\mu}p_i(0,1) + \frac{\gamma_i}{\lambda_i+\mu}p_i(1,0) + \frac{\lambda_i}{\lambda_i+\mu}p_i(0,1^*)\\ &= \frac{\gamma_i}{\nu_i}p_i(1,2) + \frac{\lambda_i}{\lambda_i+\mu}p_i(0,1) - \frac{\lambda_i\gamma_i}{(\lambda_i+\mu)\nu_i}p_i(0,2)\\ &\quad + \frac{\lambda_i}{\lambda_i+\mu}p_i(0,1^*).\end{aligned}$$

Now, we show that $p(k,0)$, $p(k,1)$, and $p(k,2)$ are obtained for $k>1$ if $\{p_i(j,0),p_i(j,1),p_i(j,2)\}_{j=0}^{k-1}$ is known. First observe that $p_i(k,2)$ is obtained from the first equation of (2) as follows:

$$p_i(k,2) = \frac{\lambda_i+\gamma_i+\nu}{\mu}p_i(k-1,0) - p_i(k-1,1) - \frac{\lambda_i}{\mu}p_i(k-2,0).$$

Once $p_i(k,2)$ is obtained, $p_i(k,0)$ and $p_i(k,1)$ are also obtained from the second and the third equations of (2). For example, when $k<K$

$$\begin{aligned}p_i(k,0) &= \frac{\lambda_i+\mu}{\nu_i}p_i(k,2) - \frac{\lambda_i}{\nu_i}p_i(k-1,2),\\ p_i(k,1) &= \frac{\gamma_i}{\nu_i}p_i(k,2) + \frac{\lambda_i}{\lambda_i+\mu}p_i(k-1,1) - \frac{\lambda_i\gamma_i}{(\lambda_i+\mu)\nu_i}p_i(k-1,2),\end{aligned}$$

and when $k=K$

$$\begin{aligned}p_i(K,0) &= \frac{\mu}{\nu_i}p_i(K,2) - \frac{\lambda_i}{\nu_i}p_i(K-1,2),\\ p_i(K,1) &= \frac{\gamma_i}{\nu_i}p_i(K,2) + \frac{\lambda_i}{\mu}p_i(K-1,1) - \frac{\lambda_i\gamma_i}{\mu\nu_i}p_i(K-1,2).\end{aligned}$$

Note that we can determine the value of unknown constant $c = p_i(0,2)$ from the normalization condition that the sum of the probabilities of all states is equal to one.

Unknown parameter γ_i of station i is determined from the stationary distributions of other stations. To see this, note that the following equality derives from the global balance equations:

$$\mu p_i^{(ia)} = \gamma_i p_i^{(mi)}, \tag{5}$$

where

$$p_i^{(ia)} \stackrel{\text{def}}{=} p_i(0,1^*) + \sum_{k=0}^{K} p_i(k,1),$$
$$p_i^{(mi)} \stackrel{\text{def}}{=} p_i(0,2) + \sum_{k=0}^{K} p_i(k,0).$$

Note that $p_i^{(ia)}$ is the probability that station i is inactive, and $p_i^{(mi)}$ is the probability that station i is in a medium-idle state. Observe that $\mu p_i^{(ia)}$ is the probability flow concerning the state transition of station i from inactive states to medium-idle states. This probability flow should be equal to the probability flow of state transitions of stations other than station i triggered by the end of frame transmission, and thus

$$\mu p_i^{(ia)} = \mu \sum_{j\neq i} \sum_{k=1}^{K} p_j(k,2). \tag{6}$$

Combining (5) and (6) yields

$$\gamma_i = \frac{\mu \sum_{j\neq i} \sum_{k=1}^{K} p_j(k,2)}{p_i^{(mi)}} = \frac{\mu \sum_{j\neq i} \sum_{k=1}^{K} p_j(k,2)}{p_i(0,2) + \sum_{k=0}^{K} p_i(k,0)}. \tag{7}$$

C. Forwarding station

The global balance equations of forwarding station (station 2) are simpler than those of source stations. When $1 \leq k \leq K-1$, we have

$$(\gamma_2 + \nu_2) p_2(k,0) = \mu(p_2(k-1,1) + p_2(k+1,2)),$$
$$\mu p_2(k,1) = \gamma_2 p_2(k,0),$$
$$\mu p_2(k,2) = \nu_2 p_2(k,0) + \mathbf{1}(k=1)\mu p_2(0,1^*).$$

When $k = 0$, we have

$$(\gamma_2 + \nu_2) p_2(0,0) = \mu p_2(1,2), \quad \mu p_2(0,1) = \gamma_2 p_2(0,0),$$
$$\gamma_2 p_2(0,2) = \nu_2 p_2(0,0), \quad \mu p_2(0,1^*) = \gamma_2 p_2(0,2).$$

When $k = K$, we have

$$(\gamma_2 + \nu_2) p_2(K,0) = \mu p_2(K-1,1),$$
$$\mu p_2(K,1) = \gamma_2 p_2(K,0), \quad \mu p_2(K,2) = \nu_2 p_2(K,0).$$

Parameter γ_2 can be obtained by (7). As shown in the global balance equations in the above. the forwarding station transits from state $(k-1,1)$ to state $(k,0)$ with rate μ. This transition is triggered by the completion of frame transmission by a source station. The frame transmitted by a source station should arrive at the forwarding station, and thus the completion of the frame transmission of a source station increases the number of frames in the buffer of the forwarding station. This means that, in the proposed model, the frame arrival process at the forwarding station is exactly described by the superposition of the output processes from the source stations. The stationary distribution of forwarding station can be obtained in the same manner with source stations. The throughput of station 2 is equal to the total throughput, and thus

$$\text{total throughput} = \mu d \sum_{k=1}^{K} p_2(k,2) = \nu_2 d \sum_{k=1}^{K} p_2(k,0) + \mu d p_2(0,1*).$$

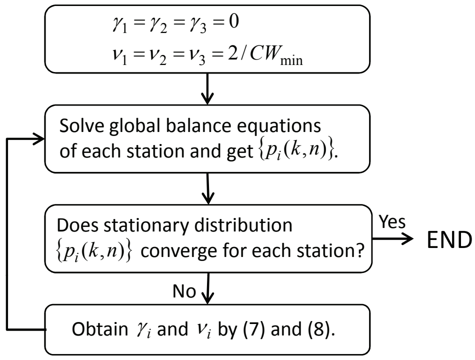

Fig. 2. Procedure for obtaining the stationary distribution.

D. Consideration of Frame Collision

Since the proposed model neglects the frame collision, the mean duration of the backoff period of each station should be equal to $T_s \times CW_{min}/2$, where T_s is the slot time. In real situations, however, the frame collision happens and it increases the mean duration of the backoff period. To approximately take into account of the influence of the frame collision, we additionally use the assumption that the frame transmission starting at time T_0 fails if one of the other stations also starts the frame transmission during $(T_0, T_0 + T_s]$. Station i starts the frame transmission only when the medium is idle, where the frame transmission by other stations occurs according to a Poisson process with intensity

$$r_i = \sum_{j\neq i} \frac{\nu_j \sum_{k=1}^{K} p_j(k,0) + \lambda_j p_j(0,2)}{\sum_{k=0}^{K} p_j(k,0) + p_j(0,2)},$$

where $\lambda_2 = 0$. Since the probability that the frame transmission by other stations does not start during $(T_0, T_0 + T_s]$ is equal to $e^{-r_1 T_s}$, the collision probability that station i experiences is $1 - e^{-r_1 T_s}$. Thus, the mean duration of the backoff period of station i, ν_i^{-1}, is given as

$$\nu_i^{-1} = e^{-r_i T_s} \sum_{n=0}^{n} (1 - e^{-r_i T_s})^n CW(n)/2,$$
$$CW(n) \stackrel{\text{def}}{=} \min\{2^n(CW_{min}+1), CW_{max}+1\} - 1. \tag{8}$$

The total throughput is given as

$$\text{total throughput} = e^{-r_i T_s} \mu d \sum_{k=1}^{K} p_2(k,2)$$
$$= e^{-r_i T_s} d \left(\nu_2 \sum_{k=1}^{K} p_2(k,0) + \mu p_2(0,1*) \right).$$

The procedure for obtaining the stationary distribution $\{p_i(k,n)\}_{k,n}$ is summarized in Fig. 2. In the next section, we show that the above approximation improves the accuracy of the analysis.

V. Numerical experiments

We compare the results obtained by the proposed analytical model with simulation results obtained by a custom-made C simulator in the scenario shown in Fig. 1. The amount of uplink traffic (from client terminal to AP) and the amount of

TABLE I. SIMULATION SETTINGS

Data rate	54 Mbps
SIFS time	16 μs
Slot time	9 μs
CW_{min}	15
Frame payload	1500 bytes
Buffer size	100 frames

downlink traffic (from AP to client terminal) are the same. All stations are within communication range of each other, sharing an IEEE802.11g-based wireless LAN. The frame transmission period is equal to $T_{DATA}+SIFS+T_{ACK}+DIFS$, where T_{DATA} and T_{ACK} represent the time length required for transmitting the data and ACK frame respectively. In the simulation, the binary exponential backoff of the DCF is precisely emulated although it is largely simplified in the proposed analysis. Other parameters are summarized in Table I.

Figure 3 shows the total throughput by increasing the offered load gradually. In the figure, the line of Analysis 1 corresponds to the result without using the mean-field approximation. Both lines of Analyses 2 and 3 show the results using the mean-field approximation; Analysis 2 does not consider the frame collision, but Analysis 3 approximately takes account of its effect by the method explained in Sec. IV-D. Note that Analysis 1 does not consider the frame collision. As shown in the simulation results, the total throughput linearly increases and reaches the highest point (about 15 Mbps) when the offered load is around 15 Mbps. Then it linearly decreases, finally stays constant when the offered load is larger than 20 Mbps. This result is consistent with our previous study [8]. Analysis 1 gives the same total throughput with Analysis 2, verifying that the mean-field approximation is still valid even in three-station cases like Wi-Fi offload. However, Analyses 1 and 2 somewhat overestimate the total throughput, which would come from the ignorance of the frame collision. Analysis 3 yields more accurate estimate of the total throughput because it takes into account of the frame collision. Note that a few days is required to get the results of the figure by Analysis 1, while only a few seconds are required to get the results by Analysis 2 or 3.

Figure 4 shows the average queue length of the relay station. It increases sharply at the point that the total throughput reaches the maximum, where the relay station becomes congested (i.e. saturated). Figure 5 shows the average queue lengths of the source stations. They become congested when the offered load reaches 20 Mbps. In the two figures, we find the discrepancy between the simulation and the analyses. The results by Analyses 1 and 2 are almost the same, and thus the discrepancy would not be caused by the mean-field approximation but by the ignorance of the frame collision. In other words, the approximate treatment of the frame collision in Sec. IV-D is not sufficient, and more consistent treatment is required to estimate the queueing behavior more accurately.

VI. Conclusion

In this paper we proposed a light-weight performance analysis of Wi-Fi offload using a simple Markov model. Although the proposed model does not describe the behavior of the contention window of each station in detail, it allows us to analyze the non-saturation throughput while considering interaction among interfering stations and the queueing behavior of each station. In addition to this, the proposed model does not need to assume that the frame-arrival process at the relay station is Poisson. The simulation experiments verify that the proposed model accurately estimates the throughput, and that the mean-field approximation is valid in the scenario of the Wi-Fi offload. More consistent treatment of the frame collision in the model remains as a future work.

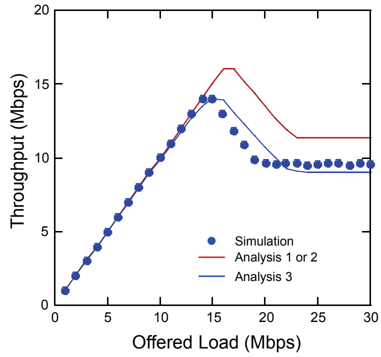

Fig. 3. Total throughput of the network

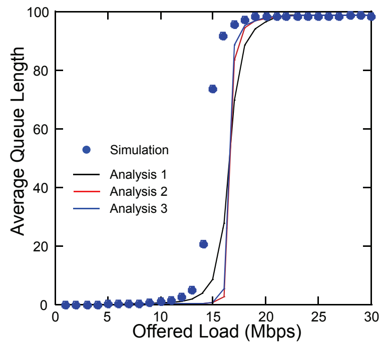

Fig. 4. Average queue length of relay station

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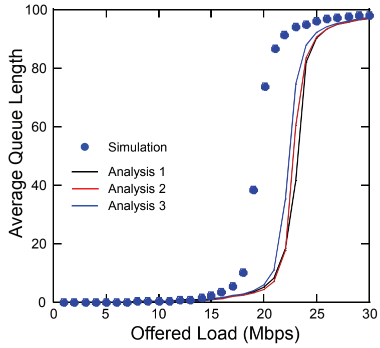

Fig. 5. Average queue length of source station

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Appendix

The global balance equation concerning $p(k_1, k_2, k_3, 1)$ is

$$\begin{aligned}
&(\mathbf{1}(k_1 < K)\lambda_1 + \mathbf{1}(k_3 < K)\lambda_3 + \mathbf{1}(k_1 > 0)\mu \\
&\quad + \mathbf{1}(k_1 = 0)(\nu_2 + \nu_3))p(k_1, k_2, k_3, 1) \\
&= \mathbf{1}(k_1 > 0)\lambda_1 p(k_1 - 1, k_2, k_3, 1) \\
&\quad + \mathbf{1}(k_3 > 0)\lambda_3 p(k_1, k_2, k_3 - 1, 1) \\
&\quad + \nu_1 p(k_1, k_2, k_3, 0) + \mathbf{1}(k_1 = 0)\mu p(0, k_2 + 1, k_3, 3) \\
&\quad + \mathbf{1}(k_1 > 0)\mathbf{1}(k_3 = 1)\lambda_3 p(k_1, k_2, k_3 - 1, 5) \\
&\quad + \mathbf{1}(k_1 = 0)\mathbf{1}(k_3 < K)\{\mathbf{1}(k_2 > 0)\mu p(0, k_2 - 1, k_3 + 1, 5) \\
&\quad\quad + \mathbf{1}(k_2 = K)\mu p(0, k_2, k_3 + 1, 5)\}.
\end{aligned}$$

The global balance equation concerning $p(k_1, k_2, k_3, 2)$ is

$$\begin{aligned}
&(\mathbf{1}(k_1 < K)\lambda_1 + \mathbf{1}(k_3 < K)\lambda_3 + \mathbf{1}(k_2 > 0)\mu \\
&\quad + \mathbf{1}(k_2 = 0)(\nu_1 + \nu_3))p(k_1, k_2, k_3, 2) \\
&= \mathbf{1}(k_1 > 0)\lambda_1 p(k_1 - 1, k_2, k_3, 2) \\
&\quad + \mathbf{1}(k_3 > 0)\lambda_3 p(k_1, k_2, k_3 - 1, 2) + \nu_2 p(k_1, k_2, k_3, 0) \\
&\quad + \mathbf{1}(k_1 = 1)\mathbf{1}(k_2 > 0)\lambda_1 p(0, k_2, k_3, 3) \\
&\quad + \mathbf{1}(k_3 = 1)\mathbf{1}(k_2 > 0)\lambda_3 p(k_1, k_2, 0, 6) \\
&\quad + \mathbf{1}(k_2 = 1)\mu(p(k_1 + 1, 0, k_3, 3) + p(k_1, 0, k_3 + 1, 6)).
\end{aligned}$$

The global balance equation concerning $p(k_1, k_2, k_3, 3)$ is

$$\begin{aligned}
&(\lambda_1 + \mathbf{1}(k_2 > 0)\mu + \mathbf{1}(k_2 = 0)\nu_3 + \mathbf{1}(k_3 < K)\lambda_3)p(0, k_2, k_3, 3) \\
&= \mathbf{1}(k_2 = 0)\nu_1 p(0, k_2, k_3, 2) + \nu_2 p(0, k_2, k_3, 1) \\
&\quad + \mathbf{1}(k_3 > 0)\lambda_3 p(0, k_2, k_3 - 1, 3) \\
&\quad + \mathbf{1}(k_2 = 1)\mathbf{1}(k_3 < K)\mu p(0, 0, k_3 + 1, 7) \\
&\quad + \mathbf{1}(k_3 = 1)\mathbf{1}(k_2 > 0)\lambda_3(0, k_2, 0, 7) \\
&(\mathbf{1}(k_1 < K)\lambda_1 + \mathbf{1}(k_1 > 0)\mu\mathbf{1}(k_1 = 0)\nu_3 \\
&\quad + \mathbf{1}(k_3 < K)\lambda_3)p(k_1, 0, k_3, 3) \\
&= \nu_1 p(k_1, 0, k_3, 2) + \mathbf{1}(k_1 > 0)\lambda_1 p(k_1 - 1, 0, k_3, 3) \\
&\quad + \mathbf{1}(k_1 = 0)\nu_2 p(k_1, 0, k_3, 1) + \mathbf{1}(k_3 > 0)\lambda_3 p(k_1, 0, k_3 - 1, 3) \\
&\quad + \mathbf{1}(k_1 > 0)\mathbf{1}(k_3 = 1)\lambda_3(k_1, 0, 0, 7).
\end{aligned}$$

The global balance equation concerning $p(k_1, k_2, k_3, 4)$ is

$$\begin{aligned}
&(\mathbf{1}(k_1 < K)\lambda_1 + \mathbf{1}(k_3 < K)\lambda_3 + \mathbf{1}(k_3 > 0)\mu \\
&\quad + \mathbf{1}(k_3 = 0)(\nu_1 + \nu_2))p(k_1, k_2, k_3, 4) \\
&= \mathbf{1}(k_1 > 0)\lambda_1 p(k_1 - 1, k_2, k_3, 4) + \mathbf{1}(k_3 > 0)\lambda_3 p(k_1, k_2, k_3 - 1, 4) \\
&\quad + \nu_3 p(k_1, k_2, k_3, 0) + \mathbf{1}(k_3 = 0)\mu p(k_1, k_2 + 1, 0, 6) \\
&\quad + \mathbf{1}(k_1 = 1)\mathbf{1}(k_3 > 0)\lambda_1 p(k_1 - 1, k_2, k_3, 5) \\
&\quad + \mathbf{1}(k_1 < K)\mathbf{1}(k_3 = 0)\{\mathbf{1}(k_2 > 0)\mu p(k_1 + 1, k_2 - 1, 0, 5) \\
&\quad\quad + \mathbf{1}(k_2 = K)\mu p(k_1 + 1, k_2, 0, 5)\}.
\end{aligned}$$

The global balance equation concerning $p(k_1, k_2, k_3, 5)$ is

$$\begin{aligned}
&(\mathbf{1}(k_1 < K)\lambda_1 + \mathbf{1}(k_1 > 0)\mu + \mathbf{1}(k_1 = 0)\nu_2 + \lambda_3)p(k_1, k_2, 0, 5) \\
&\quad = \mathbf{1}(k_1 > 0)\lambda_1 p(k_1 - 1, k_2, 0, 5) \\
&\quad + \mathbf{1}(k_1 = 0)\mathbf{1}(k_2 < K)\mu p(0, k_2 + 1, 0, 7) \\
&\quad + \mathbf{1}(k_1 = 0)\nu_3 p(k_1, k_2, 0, 1) + \nu_1 p(k_1, k_2, 0, 4) \\
&(\lambda_1 + \mathbf{1}(k_3 = 0)\nu_2 + \mathbf{1}(k_3 > 0)\mu + \mathbf{1}(k_3 < K)\lambda_3)p(0, k_2, k_3, 5) \\
&\quad = \mathbf{1}(k_3 > 0)\lambda_3 p(0, k_2, k_3 - 1, 5) \\
&\quad + \mathbf{1}(k_3 = 0)\mathbf{1}(k_2 < K)\mu p(0, k_2 + 1, 0, 7) \\
&\quad + \mathbf{1}(k_3 = 0)\nu_1 p(0, k_2, k_3, 4) + \nu_3 p(0, k_2, k_3, 1).
\end{aligned}$$

The global balance equation concerning $p(k_1, k_2, k_3, 6)$ is

$$\begin{aligned}
&(\mathbf{1}(k_1 < K)\lambda_1 + \mathbf{1}(k_2 > 0)\mu + \mathbf{1}(k_2 = 0)\nu_1 + \lambda_3)p(k_1, k_2, 0, 6) \\
&\quad = \mathbf{1}(k_2 = 0)\nu_3 p(k_1, k_2, 0, 2) + \nu_2 p(k_1, k_2, 0, 4) \\
&\quad + \mathbf{1}(k_1 > 0)\lambda_1 p(k_1 - 1, k_2, 0, 6) \\
&\quad + \mathbf{1}(k_2 = 1)\mathbf{1}(k_1 < K)\mu p(k_1 + 1, 0, 0, 7) \\
&\quad + \mathbf{1}(k_1 = 1)\mathbf{1}(k_2 > 0)\lambda_1(0, k_2, 0, 7) \\
&(\mathbf{1}(k_1 < K)\lambda_1 + \mathbf{1}(k_3 > 0)\mu + \mathbf{1}(k_3 = 0)\nu_1 \\
&\quad + \mathbf{1}(k_3 < K)\lambda_3)p(k_1, 0, k_3, 6) \\
&\quad = \nu_3 p(k_1, 0, k_3, 2) + \mathbf{1}(k_1 > 0)\lambda_1 p(k_1 - 1, 0, k_3, 6) \\
&\quad + \mathbf{1}(k_3 = 0)\nu_2 p(k_1, 0, k_3, 4) + \mathbf{1}(k_1 = 1)\mathbf{1}(k_3 > 0)\lambda_1(0, 0, k_3, 7) \\
&\quad + \mathbf{1}(k_3 > 0)\lambda_3 p(k_1, 0, k_3 - 1, 6).
\end{aligned}$$

Finally, the global balance equation concerning $p(k_1, k_2, k_3, 7)$ is

$$\begin{aligned}
&(\mathbf{1}(k_1 < K)\lambda_1 + \mathbf{1}(k_1 > 0)\mu + \lambda_3)p(k_1, 0, 0, 7) \\
&\quad = \mathbf{1}(k_1 > 0)\lambda_1 p(k_1 - 1, 0, 0, 7) + \nu_1 p(k_1, 0, 0, 6) \\
&\quad + \mathbf{1}(k_1 = 0)(\nu_2 p(k_1, 0, 0, 5) + \nu_3 p(k_1, 0, 0, 3)) \\
&(\lambda_1 + \lambda_3 + \mathbf{1}(k_2 > 0)\mu)p(0, k_2, 0, 7) \\
&\quad = \mathbf{1}(k_2 = 0)(\nu_1 p(0, k_2, 0, 6) + \nu_3 p(0, k_2, 0, 3)) + \nu_2 p(0, k_2, 0, 5) \\
&(\lambda_1 + \mathbf{1}(k_3 < K)\lambda_3 + \mathbf{1}(k_3 > 0)\mu)p(0, 0, k_3, 7) \\
&\quad = \mathbf{1}(k_3 = 0)(\nu_1 p(0, 0, k_3, 6) + \nu_2 p(0, 0, k_3, 5)) \\
&\quad + \mathbf{1}(k_3 > 0)\lambda_3 p(0, 0, k_3 - 1, 7) + \nu_3 p(0, 0, k_3, 3).
\end{aligned}$$