

Design of Framed and Slotted Time Structure for ALOHA-based Collision Arbitration Scheme in RFID Networks

Jin Kyung Park¹, Woo Cheol Shin¹, Jun Ha¹, Chulhun Seo², Joonmo Kim¹, and Cheon Won Choi¹

¹ School of Electronics and Computer Engineering, Dankook University
126 Jukjeon Dong, Suji Gu, Yongin Si, Gyeonggi Do, 448-701, Korea

² School of Electronic Engineering, Soongsil University
511 Sangdo Dong, Dongjak Gu, Seoul, 156-743, Korea

E-mail: ¹cchoi@dku.edu, ²chulhun@e.ssu.ac.kr

Abstract: Consider an RFID network which consists of a reader and a crowd of tags sojourning in the vicinity of the reader. In the RFID network, a response of a tag may collide with responses of other tags. However, the time for the reader's cognizing the tag is limited in the network. Assuming a framed and slotted ALOHA for arbitrating a collision, we choose the cognizance ratio for given cognizance time as a key performance measure. Then, we formulate a problem to find an optimal structure of framed and slotted time which maximizes the cognizance ratio under a constraint on the cognizance time. In some simple cases, we present an optimal time structure analytically. Also, we confirm that there exists a non-trivial number of frames which maximizes the cognizance ratio for given constraint on the cognizance time.

1. Introduction

Radio frequency identification (RFID) is a system which, in a contactless fashion, attains the information stored at an electronic tag by using a radio wave. We consider a usual radio frequency identification network of star topology where a single reader is located in the middle of the crowd of tags. In a typical application of such an RFID network, the reader is only allowed to spend a limited time for cognizing tags. On the other hand, two or more tags in the network may attempt to respond at the same time, which results in a collision among the tags' responses. For arbitrating a collision among tag's responses, framed and slotted ALOHA schemes were proposed and adopted in some standards [1][2]. In a framed and slotted ALOHA, time is divided into frames and a number of slots are provided in each frame. Then, a tag randomly selects a slot in the response part of a frame and attempts to respond using the selected slot. Previous research works on framed and slotted ALOHA tried to optimize the time structure with aiming to maximize the long-term cognizance rate [3][4][5][6]. However, such an optimal structure does not necessarily maximize the cognizance ratio (defined as the ratio of the number of cognized tags against the number of all tags in the vicinity of the reader) while the time for cognizing tags is limited.

In this paper, we choose the cognizance ratio for given cognizance time as a key performance measure. Then, we formulate a problem to find an optimal structure of framed and slotted time, (i.e. optimal number of frames and

optimal number of slots in each frame) which maximizes the cognizance ratio under a constraint on the cognizance time. In simple cases, we obtain these optimal numbers analytically.

In section 2, we describe a version of framed and slotted ALOHA. In section 3, we calculate the cognizance ratio. In section 4, we formulate a problem to find an optimal structure of framed and slotted time and solve the problem analytically in some simple cases. Section 5 is devoted to numerical examples.

2. Framed and Slotted ALOHA

In this section, we present a version of framed and slotted ALOHA for arbitrating a collision among tags' responses. Figure 1 shows an exemplary frame structure in the proposed version of framed and slotted ALOHA. A frame is divided into a part for the inquiry of the reader and a part for the responses of tags. The part for tags' responses is divided again into a number of slots.

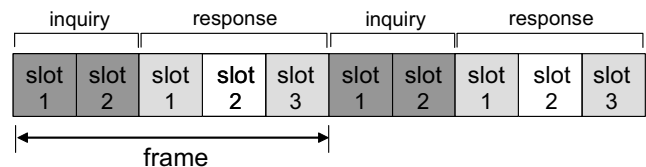


Figure 1. Exemplary frame structure in framed and slotted ALOHA.

At the start of each frame, the reader inquires the identities of tag and announces the number of slots in the frame by using the inquiry part of the frame. Then, each tag equally likely selects a slot among the slots involving in response part of the frame and responds to the reader's inquiry using the selected slot.

3. Cognizance Ratio

In this section, we consider a framed and slotted ALOHA employing the frame structure illustrated in figure 1. In the structure, we set that the inquiry part of the n th frame consists of μ_n slots while the response part is comprised of ν_n slots for $n \in \{1, 2, \dots\}$.

Let Y_n denote the number of tags which succeed in responding (without collision) during the n th frame. Suppose that M tags sojourns in the neighborhood of the

This work was supported by 2008 Dankook University Project of funding Internal Strategic Research Project Team.

reader. Then, Y_n has the same distribution as the number of boxes with only one ball when M indistinguishable balls are equally likely put into v_n boxes [7]. Let f_n denote the mass for Y_n . Then,

$$f_n(y) = \frac{(-1)^y v_n! M!}{y! v_n^M} \times \sum_{j=y}^{\min\{v_n, M\}} \frac{(-1)^j (v_n - j)^{M-j}}{(j-y)! (v_n - j)! (M-j)!} \quad (1)$$

for $y \in \{0, \dots, \min\{v_n, M\}\}$. Note that the Y_n tags may contain some tags which already succeeded in responding during a previous frame. For $n \in \{1, 2, \dots\}$, let U_n denote the number of tags which for the first time succeed in responding, (i.e., which are newly cognized) during the n th frame. Set

$$V_n = \sum_{j=1}^n U_j \quad (2)$$

for $n \in \{1, 2, \dots\}$. Then V_n represents the number of cognized tags until the n th frame. Since every tag independently attempts to respond in each frame, the sequence $\{V_n, n=0, 1, \dots\}$ is a discrete-time Markov chain on the finite state space $S = \{0, \dots, M\}$. (We set $V_0 = 0$ almost surely.) Let $g_n : S^2 \rightarrow [0, 1]$ be the n th transition probability function of $\{V_n, n=0, 1, \dots\}$. Then,

$$g_n(p, q) = \sum_{y=q-p}^q [p(U_n = q - p | V_{n-1} = p, Y_n = y) \times P(Y_n = y)] \quad (3)$$

for $p \in S$ and $q \in \{p, \dots, M\}$. Given V_{n-1} and Y_n , U_n has a hypergeometric distribution since every tag independently and equally likely chooses a slot in each frame. Thus, we have

$$g_n(p, q) = \sum_{y=q-p}^q \frac{\binom{M-p}{q-p} \binom{p}{y-q+p}}{\binom{M}{y}} f_n(y) \quad (4)$$

for $p \in S$ and $q \in \{p, \dots, M\}$.

Suppose that the lengths of inquiry and response parts of each frame are fixed, i.e., $\mu_j = \mu$ and $\nu_j = \nu$ for positive integers μ and ν . Then, $\{V_n, n=0, 1, \dots\}$ is a homogeneous Markov chain. Let $g : S^2 \rightarrow [0, 1]$ be the transition probability function of $\{V_n, n=0, 1, \dots\}$. Then, we have

$$g(p, q) = \sum_{y=q-p}^q \frac{\binom{M-p}{q-p} \binom{p}{y-q+p}}{\binom{M}{y}} f(y) \quad (5)$$

for $p \in S$ and $q \in \{p, \dots, M\}$, where

$$f(y) = \frac{(-1)^y v! M!}{y! v^M} \times \sum_{j=y}^{\min\{v, M\}} \frac{(-1)^j (v-j)^{M-j}}{(j-y)! (v-j)! (M-j)!} \quad (6)$$

for $y \in \{0, \dots, \min\{v, M\}\}$. Hereafter, we only consider the homogeneous Markov chain $\{V_n, n=0, 1, \dots\}$.

Recall that the cognizance ratio is the ratio of the average number of cognized tags against the total number of tags. Let $\rho_{\alpha v}$ denote the cognizance ratio until the α th frame when the response part of each frame consists of v slots. Then, we have

$$\rho_{\alpha v} = \frac{1}{M} E(V_\alpha) = \sum_{q=0}^M q \sum_{p_1 \in S} \dots \sum_{p_{n-1} \in S} g_0(0) g(0, p_1) \dots g(p_{n-1}, q) \quad (7)$$

where g_0 is the initial mass for $\{V_n, n=0, 1, \dots\}$ such that $g_0(0) = 1$. In general, the cognizance ratio is not easily represented in a handy form. In simple cases, however, the cognizance ratio is obtained in a tractable form as shown in the following theorem:

Theorem 1: For $M \in \{1, \dots, 5\}$, the cognizance ratio is expressed as follows:

$$\begin{aligned} \rho_{\alpha v} &= 1 \text{ when } M = 1 \\ \rho_{\alpha v} &= 1 - \frac{1}{v^\alpha} \text{ when } M = 2 \\ \rho_{\alpha v} &= 1 - \frac{(2v-1)^\alpha}{v^{2\alpha}} \text{ when } M = 3 \\ \rho_{\alpha v} &= 1 - \frac{(3v^2 - 3v + 1)^\alpha}{v^{3\alpha}} \text{ when } M = 4 \\ \rho_{\alpha v} &= 1 - \frac{(4v^3 - 6v^2 + 4v - 1)^\alpha}{v^{4\alpha}} \text{ when } M = 5. \end{aligned} \quad (8)$$

Proof: A straightforward calculation yields $\rho_{\alpha v}$ in the theorem.

4. Optimal Framed and Slotted Structure

In this section, we find an optimal structure of framed and slotted time which maximizes the cognizance ratio under a constraint on the cognizance time. Let γ be the constraint on the cognizance time. Then, the problem to find such an optimal structure is formulated as follows:

$$\begin{aligned} &\text{Maximize } \rho_{\alpha v} \\ &\text{with respect to } \alpha \in \{1, 2, \dots\} \text{ and } v \in \{1, 2, \dots\} \\ &\text{subject to } \alpha(\mu + \nu) \leq \gamma \end{aligned} \quad (9)$$

for $\gamma \in \{2, 3, \dots\}$. It is obvious that the maximum cognizance ratio is attained when $\alpha(\mu + \nu) = \gamma$. Thus, we modify the problem as follows:

$$\begin{aligned}
& \text{Maximize } \rho_{\alpha v} \\
& \text{with respect to } \alpha \in \{2,3,\dots\} \text{ and } v \in \{1,2,\dots\} \\
& \text{subject to } \alpha(\mu + v) = \gamma
\end{aligned} \tag{10}$$

for $\gamma \in \{2,3,\dots\}$. As stated in section 3, the cognizance ratio is not easily expressed in a tractable form so that a numerical method is often required to solve the problem. In simple cases, however, we can analytically obtain optimal numbers of frames and slots. In the problem given in (10), let A_γ be the set of all feasible values for α for given γ . Then, A_γ is the set of all divisors of γ which make $\mu + v$ be a positive integer. Define $\lceil x \rceil_{A_\gamma}$ and $\lfloor x \rfloor_{A_\gamma}$ to be the greatest value less than or equal to x and the smallest value greater than or equal to x in A_γ . Then, the optimal numbers of frames and slots, denoted by α^* and v^* , are presented in the following theorems:

Theorem 2: Suppose that $M = 2$. Let x^* be the root of the following equation:

$$\log(x) + \mu x + 1 = 0 \tag{11}$$

for $x \in (0,1)$. Set $\beta = (\gamma x^*) / (\mu x^* + 1)$. Then, α^* is either $\lceil \beta \rceil_{A_\gamma}$ or $\lfloor \beta \rfloor_{A_\gamma}$. Also, $v^* = (\gamma - \mu \alpha^*) / \alpha^*$.

Proof: Set $x = \alpha / (\gamma - \mu \alpha)$. Then, α is a monotone increasing function of $x \in (0,1)$. Set

$$h(x) = \rho_{\alpha v} = 1 - x^{\frac{\gamma x}{\mu x + 1}} \tag{12}$$

for $x \in (0,1)$. Then, h has a unique local maximum and is differentiable. Differentiating $h(x)$ and equating it to 0, we have (11). Let x^* be the critical point. Then, x^* is also the extreme point.

Theorem 3: Suppose that $M = 3$. Let x^* be the root of the following equation:

$$(2 - x) \log(x(2 - x)) + 2(1 - x)(1 + \mu x) = 0 \tag{13}$$

for $x \in (0,1)$. Set $\beta = (\gamma x^*) / (\mu x^* + 1)$. Then, α^* is either $\lceil \beta \rceil_{A_\gamma}$ or $\lfloor \beta \rfloor_{A_\gamma}$. Also, $v^* = (\gamma - \mu \alpha^*) / \alpha^*$.

Proof: Set $x = \alpha / (\gamma - \mu \alpha)$. Then, α is a monotone increasing function of $x \in (0,1)$. Set

$$h(x) = \rho_{\alpha v} = 1 - [x(2 - x)]^{\frac{\gamma x}{\mu x + 1}} \tag{14}$$

for $x \in (0,1)$. Then, h has a unique local maximum and is differentiable. Differentiating $h(x)$ and equating it to 0, we have (14). Let x^* be the critical point. Then, x^* is also the extreme point.

5. Numerical Examples

In this section, we present some numerical examples which illustrate the effect of the number of tags, the number of frames, the number of slots in the response part, and the constraint on the cognizance time on the cognizance ratio.

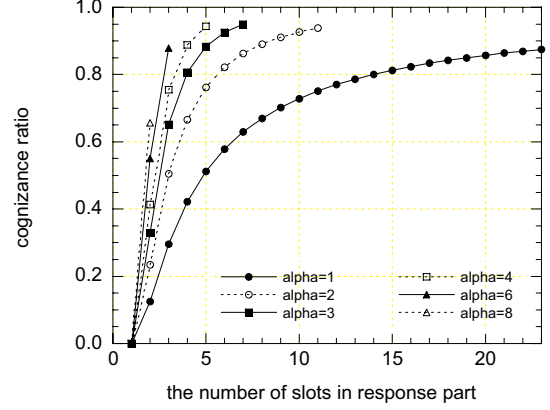


Figure 2. Cognizance ratio with respect to the number of slots in the response part.

Figure 2 shows the cognizance rate with respect to the number of slots in the response part. In this figure, we set that the number of tags $M = 4$, the number of frames $\alpha \in \{1,2,3,4,6,8\}$, the number of slots in the inquiry part $\mu = 1$, the number of slots in the response part $v \in \{1, \dots, \gamma/\alpha - \mu\}$, and the constraint on the cognizance time $\gamma = 24$. We observe that the cognizance ratio is maximized when the number of slots in the response part is maximized for given number of frames, i.e., $v = \gamma/\alpha - \mu$.

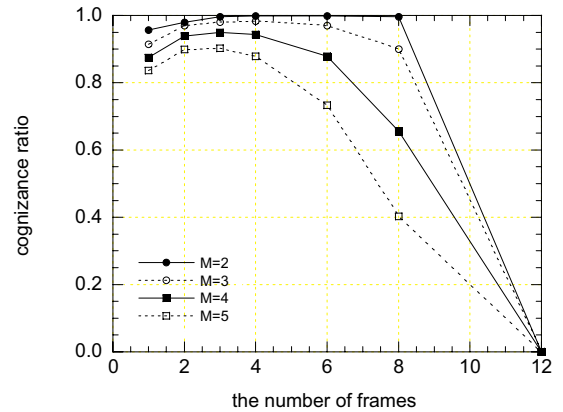


Figure 3. Cognizance ratio with respect to the number of frames.

Figure 3 shows the cognizance rate with respect to the number of frames. In this figure, we set that the number of tags $M \in \{2,3,4,5\}$, the number of frames $\alpha \in \{1,2,3,4,6,8,12\}$, the number of slots in the inquiry part $\mu = 1$, the number of slots in the response part $v = \gamma/\alpha - \mu$, and the constraint on the cognizance time

$\gamma = 24$. We observe that for a larger number of tags, the cognizance ratio is maximized by a smaller number of frames.

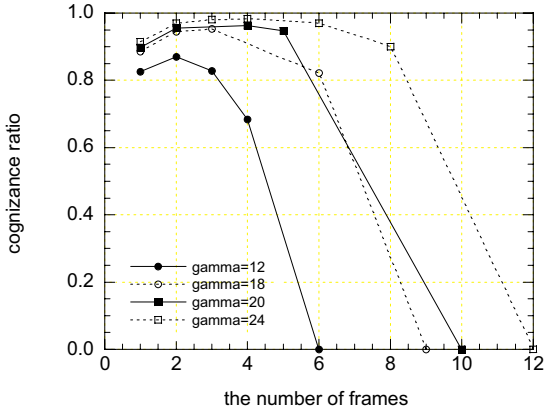


Figure 4. Cognizance ratio with respect to the number of frames.

Figure 4 shows the cognizance rate with respect to the number of frames. In this figure, we set that the number of tags $M = 3$, the number of frames $\alpha \in A_\gamma$, the number of slots in the inquiry part $\mu = 1$, the number of slots in the response part $\nu = \gamma/\alpha - \mu$, and the constraint on the cognizance time $\gamma \in \{12,18,20,24\}$. We observe that there exists a non-trivial number of frames which maximizes the cognizance ratio for given constraint on the cognizance time. Given the constant $\gamma = 18$, for example, the cognizance ratio is maximized when the number of frames $\alpha = 3$ and the number of slots in the response part $\nu = 5$. We also notice that a smaller number of frames maximizes the cognizance ratio when a tighter constraint is forced on the cognizance time.

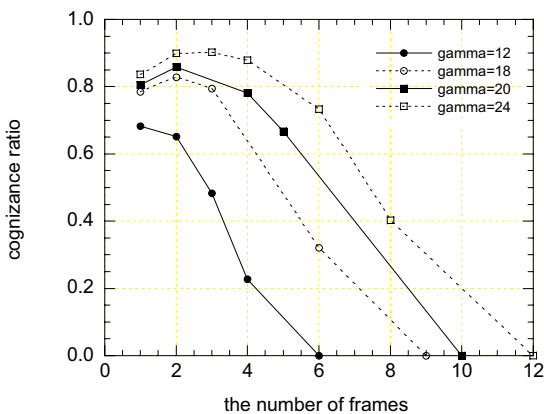


Figure 5. Cognizance ratio with respect to the number of frames.

Figure 5 shows the cognizance rate with respect to the number of frames. In this figure, we set that the number of

tags $M = 5$, the number of frames $\alpha \in A_\gamma$, the number of slots in the inquiry part $\mu = 1$, the number of slots in the response part $\nu = \gamma/\alpha - \mu$, and the constraint on the cognizance time $\gamma \in \{12,18,20,24\}$. As in figure 4, we observe that there exists a non-trivial number of frames which maximizes the cognizance ratio for given constraint on the cognizance time. We also notice that a smaller number of frames maximizes the cognizance ratio when a tighter constraint is forced on the cognizance time. Comparing the cognizance ratios in figures 4 and 5, we observe that a smaller number of frames maximizes the cognizance ratio when a larger number of tags sojourn in the vicinity of the reader. Given the constant $\gamma = 18$, for example, the cognizance ratio is maximized when the number of frames $\alpha = 2$ and the number of slots in the response part $\nu = 8$. On the other hand, these optimal numbers are 3 and 5, respectively, in figure 4.

5. Conclusions

In this paper, we considered an RFID network which consists of a reader and a crowd of tags sojourning in the vicinity of the reader. In the RFID network, a response of a tag may collide with responses of other tags. However, the time for the reader's cognizing the tag is limited in the network.. Assuming a framed and slotted ALOHA for arbitrating a collision, we chose the cognizance ratio as a key performance measure and formulated a problem to find an optimal structure of framed and slotted time which maximizes the cognizance ratio under a constraint on the cognizance time. In some simple cases, we presented an optimal time structure analytically.

References

- [1] B. Glover and H. Bhatt, RFID Essentials. O'Reilly, 2006.
- [2] K. Finkenzeller, RFID Handbook - Fundamentals and Applications in Contactless Smart Cards and Identification. John Wiley & Son, 2006.
- [3] H. Vogt, "Efficient Object Identification with Passive RFID Tags," Proceedings of International Conference on Pervasive Computing 2002, pp. 98-113, 2002.
- [4] J. Cha and J. Kim, "ALOHA-type Anti-collision Algorithms Using Tag Estimation Method in RFID system," Journal of KICS, vol. 30, no. 9A, pp. 814-821, September 2005.
- [5] J. Park, W. Shin, J. Ha, and C. Choi, "Bayes Action for Tag Cognizance in RFID Networks," Proceedings of IEEK/IEICE ITC-CSCC 2008, 2008.
- [6] J. Park, W. Shin, J. Ha, and C. Choi, "Decision of the Number of Slots for Framed Slotted ALOHA in RFID Networks: Bayesian Approach," Proceedings of IEEE APWCS 2007, 2007.
- [7] W. Feller, An Introduction to Probability Theory and Its Applications. 2nd edition, John Wiley & Sons, 1968.