A zero-forcing Tomlinson-Harashima precoding with stream permutation based on the bit rate maximization for multi-user MIMO systems

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Abstract: In this paper, we propose a zero-forcing (ZF) Tomlinson-Harashima precoding (THP) scheme for multi-user (MU) MIMO systems. The ZF-THP scheme is computationally efficient because we do not need to calculate the inverse of a MIMO channel covariance matrix that is required to construct an MMSE-THP system. We propose to construct a permutation matrix, which determines the order of the pre-interference cancellation, so that the bit rate is maximized. In our past work, we have proposed a modified sorted QR decomposition to construct the ZF-THP, whose permutation matrix was generated so as to maximize the bit rate, for single-user (SU) scenario. We have shown that the BER performance was superior to the MMSE-THP scheme when we applied variable bit allocation to each substream. In this paper, we propose to apply the methodology to the MU-MIMO system. Finally, we execute the feasibility study of the proposed method.

Keywords—**Zero-forcing, THP, MU-MIMO, QRD**

1. Introduction

Broadband wireless mobile systems such as the Long Term Evolution (LTE) and the mobile WiMAX adopt orthogonal frequency division multiple access (OFDMA), which realizes high frequency efficiency, as a multiple access scheme. The OFDMA allocates each user block, which is referred to as a resource block, into a two-dimensional space defined by the frequency and the time. Recently, the space has been incorporated into the third dimension of the multiple access. The OFDMA with space division multiple access (SDM) is the one to realize the requirement, and it has been studied aggressively. The SDM becomes possible by using multi-user multiple input multiple output (MU-MIMO) methodology, which is a key technology for the next generation wireless systems. Actually, the LTE-advanced standard has already been designed so that we can implement the MU-MIMO into base station transmitters [1].

The MU-MIMO scheme can be realized by a precoder installed in a base station transmitter. The precoder carries out interchannel interference cancellation beforehand the transmission. Many studies have been done on the precoder design. Most of them are concerning a block diagonalization (BD) method for multiple receive antennas per user. Implementing the BD method is very challenging because it requires high computation power to carry out the algorithm [2]. In this paper, the number of antenna per user is restricted to one because we assume the case of using very small devices. In this case, a minimum mean square error (MMSE) Tomlinson-Harashima precoding (THP) with symmetric permutation method proposed in [3] has shown the best BER performance by optimizing the order of the pre-interference cancellation in the transmitter. The MMSE-THP can be realized by reasonable computational complexity since it uses the Cholesky decomposition algorithm; however, we have to calculate the inverse of a MIMO channel covariance matrix.

In order to develop high performance and low complexity MU-MIMO algorithm, we focus our target on a zero-forcing (ZF) THP method [4]. In general, the QR decomposition (QRD) of the MIMO channel matrix is carried out to obtain a precoder for the ZF-THP. We do not need to generate the covariance matrix of the MIMO channel and calculate its inverse matrix; therefore, the ZF-THP is more computationally efficient. We can permute the order of the pre-interference cancellation based on the mean square error (MSE); however, the BER performance is to be inferior to the MMSE-THP scheme [4]. On the other hand, we have proposed a modified sorted QRD (MSQRD) to construct the ZF-THP, whose permutation matrix was generated so as to maximize the bit rate, for single-user (SU) scenario[5]. We have shown that the BER performance was superior to the MMSE-THP scheme when we applied variable bit allocation to each substream.

The purpose of this paper is to develop a new ZF-THP scheme for the MU-MIMO scenario. We modify the transmitter and receiver structure, algorithm and methodology proposed in [5] concerning the SU-MIMO system so that we can treat the multiple users. In more details, we propose a transmitter and receiver structure of an MU-MIMO system based on the ZF-THP with permutation matrix, and an algorithm based on the MSQRD. Finally, we execute the feasibility study of the proposed method.

Notation: Throughout this paper, $\mathbb{C}$ denote a set of complex numbers. $\mathbb{C}^{N\times 1}$ and $\mathbb{C}^{N\times M}$ stand for a set of $N \times 1$ complex vectors and $N \times M$ complex matrices respectively. $[\cdot]^T$, $[\cdot]^H$, and $det(\cdot)$ indicate the transpose of a matrix, the Hermitian transpose of a complex matrix, and the determinant of a matrix. $diag\{\boldsymbol{x}\}$ represents a diagonal matrix whose diagonal elements are equal to the vector $\boldsymbol{x}$.

2. MMSE-THP for MU-MIMO systems

We assume a narrow band MIMO channel, which represents a subcarrier of OFDM systems. N_{tx} and N_{rx} are respectively the number of transmit antennas and the number of receive antennas. Each user has one receive antenna; therefore, N_{rx}

equals to the number of users. For simplicity, we assume that $N_{tx} = N_{rx} = N$.

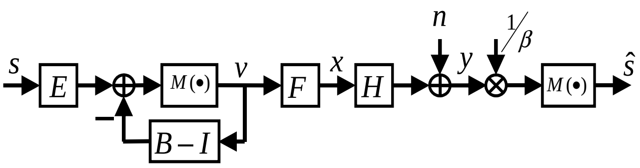

Figure 1. MU-MIMO system with MMSE-THP.

Figure 1 illustrates an MU-MIMO system proposed in [3]. We define the transmitted signal vector $\boldsymbol{x} \in \mathbb{C}^{N\times 1}$ and the received signal vector $\boldsymbol{y} \in \mathbb{C}^{N\times 1}$. Then, the system in Fig. 1 can be expressed as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{n} \in \mathbb{C}^{N\times 1}$ is the noise vector whose elements are independent additive white Gaussian noises (AWGN) with zero mean and variance σ_n^2. $\boldsymbol{H} \in \mathbb{C}^{N\times N}$ denotes the MIMO channel matrix. In Fig. 1, $\boldsymbol{E}$ is a permutation matrix defined as

$$\boldsymbol{E} = \begin{bmatrix} \boldsymbol{e}_{k_1} & \boldsymbol{e}_{k_2} & \cdots & \boldsymbol{e}_{k_N} \end{bmatrix}, \tag{2}$$

where $\boldsymbol{e}_i$ denotes an $N \times 1$ vector whose i-th element is one and the others are zeros.

The MMSE-THP is constructed by using the equations below[3].

$$\boldsymbol{\Phi}^{-1} = \left(\boldsymbol{H}\boldsymbol{H}^H + \gamma^{-1}\boldsymbol{I}\right)^{-1}. \tag{3}$$

$$\boldsymbol{E}\boldsymbol{\Phi}^{-1}\boldsymbol{E}^T = \boldsymbol{L}^H\boldsymbol{D}\boldsymbol{L}. \tag{4}$$

$$\boldsymbol{F}^H = \beta\boldsymbol{H}^H\boldsymbol{E}^T\boldsymbol{L}^H\boldsymbol{D}. \tag{5}$$

$$\boldsymbol{B}^H = \boldsymbol{L}^{-1}. \tag{6}$$

γ is the signal-to-noise (SNR) ratio and $\beta = 1/\sqrt{\chi}$ where

$$\chi = \| \boldsymbol{F}^H(:,1) \|_2^2 + \sigma_v^2 \| \boldsymbol{F}^H(:,2:N) \|_F^2 . \tag{7}$$

"$M(\bullet)$" in Fig. 1 denotes the modulo operation, which is installed in the precoder to suppress the power enhancement of the precoded signals. The modulo operation is defined as

$$M(c) = c - \lfloor \frac{Re(c)}{\tau} + \frac{1}{2} \rfloor \tau - j \lfloor \frac{Im(c)}{\tau} + \frac{1}{2} \rfloor \tau \tag{8}$$

with $\tau = 2\sqrt{M}$, where the modulation scheme is M-QAM, and $Re(c)$ and $Im(c)$ denote a real and an imaginary component of a complex variable c, and $\lfloor \bullet \rfloor$ stands for the floor operator which gives the largest integer smaller than or equal to the argument.

In Fig. 1, each receiver just multiplies a scaling factor, β, to decode the signals. This means that all equivalent channels connected to users have the same channel gain β, which leads to the same receiving SNRs in each receiver, and the system results in minimizing the total MSE[5].

3. Proposed ZF-THP for MU-MIMO systems

Figure 2 illustrates the proposed MU-MIMO system. The difference between Fig. 1 and 2 can be seen in the receiver. While the receiver of the MMSE-THP system multiplies the same scaling factors, $1/\beta$, to the receiving signals of each user, the proposed system does different scaling factors. That is, $\boldsymbol{G}_p$ is a diagonal matrix with different diagonal elements. Figure 3 shows an alternative linear representation, where $\boldsymbol{a}$ is a perturbation vector produced by the modulo operation.

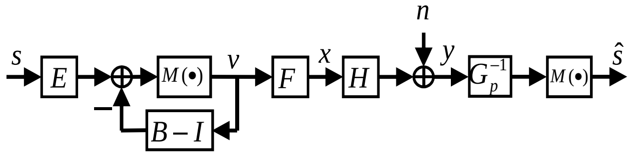

Figure 2. Proposed MU-MIMO system.

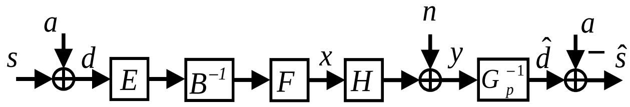

Figure 3. Linear model.

To construct a ZF-THP system, we apply the QRD to $(\boldsymbol{E}\boldsymbol{H})^H$. Then, we get

$$\boldsymbol{E}\boldsymbol{H} = \boldsymbol{S}\boldsymbol{F}^H \tag{9}$$

where $\boldsymbol{F} \in \mathbb{C}^{N\times N}$ is a unitary matrix and $\boldsymbol{S} \in \mathbb{C}^{N\times N}$ is a lower triangular matrix. We define a diagonal matrix $\boldsymbol{G}$ so that $\boldsymbol{G} = diag\{\boldsymbol{s}_d\}$, where the elements of the vector $\boldsymbol{s}_d$ equal to the diagonal elements of $\boldsymbol{S}$. Then, a precoding matrix, $\boldsymbol{B}$, is given by

$$\boldsymbol{B} = \boldsymbol{G}^{-1}\boldsymbol{S}. \tag{10}$$

It is easy to confirm that when

$$\boldsymbol{G}_p^{-1} = \boldsymbol{E}^T\boldsymbol{G}^{-1}\boldsymbol{E}, \tag{11}$$

a decoded signal vector $\hat{\boldsymbol{s}}$ approximates a transmit signal vector $\boldsymbol{s}$. From Eq. (11), $\boldsymbol{G}_p = \boldsymbol{E}^T\boldsymbol{G}\boldsymbol{E}$. This relation suggests that $\boldsymbol{G}_p$ is the symmetric permutation of $\boldsymbol{G}$[3].

From Fig. 3,

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{F}\boldsymbol{B}^{-1}\boldsymbol{E}\boldsymbol{d} + \boldsymbol{n}. \tag{12}$$

Since

$$\boldsymbol{H} = \boldsymbol{E}^T\boldsymbol{S}\boldsymbol{F}^H = \boldsymbol{E}^T\boldsymbol{G}\boldsymbol{B}\boldsymbol{F}^H \tag{13}$$

from Eqs. (9) and (10), we have

$$\boldsymbol{y} = \boldsymbol{E}^T\boldsymbol{G}\boldsymbol{E}\boldsymbol{d} + \boldsymbol{n} = \boldsymbol{G}_p\boldsymbol{d} + \boldsymbol{n}. \tag{14}$$

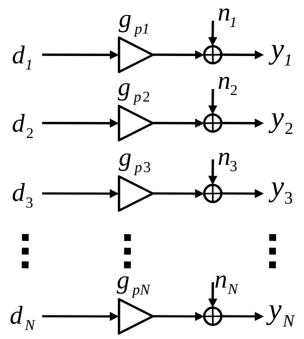

Figure 4. Equivalent channel model.

Equation (14) suggests that the equivalent channel of the proposed system can be represented by a parallel MIMO system as illustrated in Fig.4. In Fig.4,

$$\boldsymbol{G}_p = diag\{ \ [g_{p1} \quad g_{p2} \quad \cdots \quad g_{pN}] \ \}. \tag{15}$$

The bit rate of the system can be written by[5]

$$R = log_2 det\big(\frac{\gamma}{N}\boldsymbol{G}_p^2 + \boldsymbol{I}\big). \tag{16}$$

We propose to use this bit rate as our cost function to determine an optimum permutation matrix $\boldsymbol{E}$.

In order to construct the precoder, the modified sorted QRD (MSQRD) algorithm proposed in [5] is useful for the MU-MIMO scenario. The MSQRD, which is reconstructed for the MU-MIMO structure, is shown in Table 1.

Table 1. Modified sorted QRD for MU-MIMO scenario.

$\boldsymbol{R} = \boldsymbol{0}, \boldsymbol{Q} = \boldsymbol{H}^H, \boldsymbol{E} = \boldsymbol{I}.$
for $i = 1, 2, \cdots, N$
  $l_i = \arg \max_{j=i,\cdots,N} \parallel \boldsymbol{q}_j \parallel^2$
  exchange columns, i and l_i, in $\boldsymbol{Q}$,$\boldsymbol{R}$ and $\boldsymbol{E}$
  $r_{ii} = \parallel \boldsymbol{q}_i \parallel$
  $\boldsymbol{q}_i = \frac{\boldsymbol{q}_i}{r_{ii}}$
  for $k = i+1, \cdots, N$
    $r_{ik} = \boldsymbol{q}_i^H \boldsymbol{q}_k$
    $\boldsymbol{q}_k = \boldsymbol{q}_k - r_{ik} \cdot \boldsymbol{q}_i$
  end
  (We can skip this loop when $i = N$.)
end
  $\boldsymbol{S} = \boldsymbol{R}^H, \boldsymbol{G} = diag\{\boldsymbol{s}_d\}, \boldsymbol{F} = \boldsymbol{Q}, \boldsymbol{B} = \boldsymbol{G}^{-1}\boldsymbol{S}.$
  $Output \boldsymbol{E}, \boldsymbol{B}, \boldsymbol{S}.$

When $s_d(i)$ is the i-th diagonal element of $\boldsymbol{G}$ the MSDRD algorithm results in the relation:

$$s_d(1) \geq s_d(2) \geq \cdots \geq s_d(N). \tag{17}$$

The diagonal matrix, $\boldsymbol{G}$, connects with the equivalent channel matrix, $\boldsymbol{G}_p$, by Eq. (11). This means that each user is to receive signals with different SNRs. In this case, it is reasonable to allocate the number of bits flexibly according to the amount of gain to improve the receiver performance. That is, we assign many bits to users with large channel gain and do a few or zero bits to users with small channel gain as illustrated in Fig.5. This variable bit allocation is an important point of our proposal.

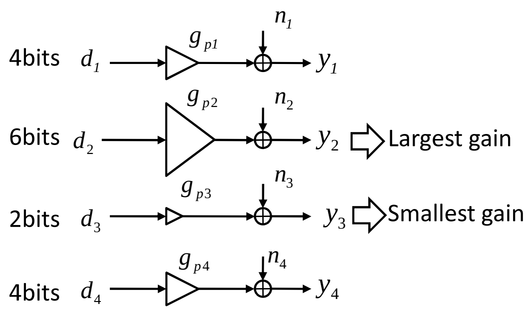

Figure 5. Example of bit allocation.

4. Numerical results

In this section, we demonstrate some computer simulations to confirm the effectiveness of the proposed method. Simulation parameters of the physical layer are listed in Table 2.

Table 2. Simulation parameters

PHY	Single carrier MIMO model
Antennas	$N = 4$
Modulation	QPSK, 16-QAM, 64-QAM
Bit rate	16 bits/s
FEC	No coding

bits/s : bits per symbol vector.

In Fig. 2, the elements of the MIMO channel matrix, $\boldsymbol{H}$, are independent and identically distributed (i.i.d) with the complex Gaussian.

To confirm the feasibility of the MSQRD in Table 1, we demonstrate achievable bit rate of the proposed ZF-THP system. We calculate Eq. (16) with respect to the SNRs. A permutation matrix is selected by using two cases, the MSQRD and the exhaustive search, so as to maximize the bit rate. No permutation case is also demonstrated for the reference. The results are shown in Fig.6.

From Fig.6, we can see that the permutation matrices constructed by the exhaustive search and MSQRD give the same bit rate. This means that the MSQRD is a proper algorithm to get a permutation matrix which maximizes the bit rate of the ZF-THP system.

Figure 7 shows the average bit error rate (BER) of the system. Bit allocation patterns, (4444), (6442), (6622) and (6640), are applied to the proposed ZF-THP. The average BER of the MMSE-THP, which uses the bit allocation pattern of (4444), is also shown for the comparison. We can

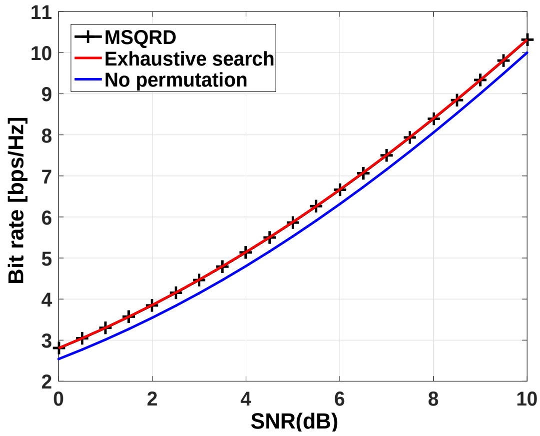

Figure 6. Achievable bit rate.

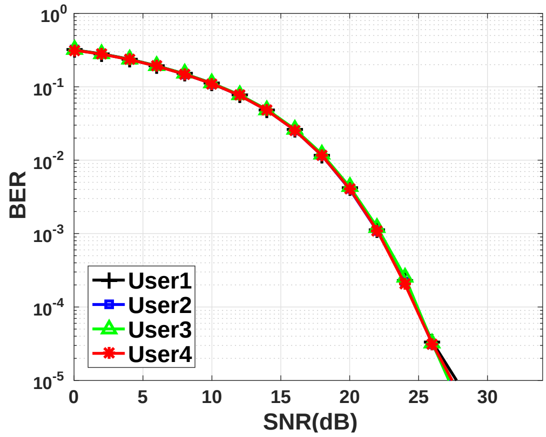

Figure 8. BER of each user with (6640).

see that the proposed scheme outperforms the MMSE-THP in high SNR region when (6640) is selected as the bit allocation pattern.

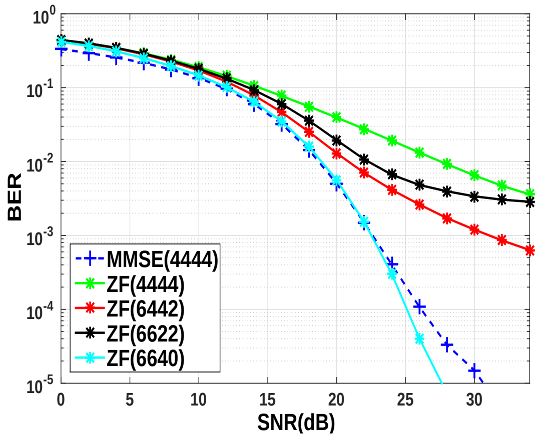

Figure 7. Bit error rate.

Figure 8 shows the BER of each user when (6640) is adopted as the bit allocation pattern. Because the elements of the MIMO matrix are i.i.d., all users demonstrate the same BER regardless of enforcing the zero transmission on a user.

5. Conclusions

In this study, we proposed a ZF-THP scheme for the MU-MIMO scenario when each user had one receive antenna. The transmitter and receiver structure were shown, and a computationally efficient algorithm, the MSQRD, in order to generate a permutation matrix so as to maximize the bit rate of the ZF-THP system was proposed. The numerical results demonstrated that the BER was superior to the MMSE-THP based MU-MIMO system when an appropriate bit allocation was selected.

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