Boundary estimation with a modified multi-layer neural network

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Abstract: This work presents a boundary estimation approach for binary mixture fields based on a modified multi-layered neural network and front point approach. The boundary shape of anomaly is expressed with front points and the unknown front points are estimated with the proposed multi-layered neural network. Numerical experiments show that the proposed approach has a good possibility for the application in the visualization of a binary mixture boundary for real-time monitoring and has enhanced performances.

1. INTRODUCTION

Boundary estimation method is relatively new technique to visualize the boundary shape of object in the phantom [1].

Regarding boundary estimation, when the shape of anomalies is near circular or elliptic a small number of Fourier coefficients would be enough to describe the interface and the Newton-raphson algorithm could show good performance. If the shape of anomalies are complex, however, higher order Fourier coefficients will be required. As the Fourier order increases the sensitivity to the change of the measured voltage tends to decreases. Furthermore, even if the boundary is deformed, partly, the Fourier coefficients should be altered totally.

This work is considering boundary estimation. The interface is represented as the discrete front points located along the boundary rather than Fourier coefficients [4, 5]. In doing so complex boundary can be described easily and partly deformation of the boundary will result in a partly change in the front points. The front points are tracked with the aid of proper inverse problem algorithm.

The neural network (NN) has been introduced to the ERT image reconstruction because it has advantages in fast calculation, ease of implementation, and the ability to control the compromise between the noise treatment and the spatial resolution. Adler and Guardo [2] and Ratajewicz-Mikolajczak [3] applied the NN algorithm to estimate the conductivity distribution, while Jeon [4] and Kim [5] reconstructed the interfacial boundary described in terms of Fourier coefficients with the multi-layer neural network (MNN) and the weighted multi-layer neural network (WMNN), respectively.

In this paper, to estimate the unknown front points, we propose the multilayer neural network whose bipolar sigmoid functions change each training step. Because, If the sigmoid function of multilayer neural network is fixed, the output of the sigmoid function can be saturated. It mean the sigmoid function of multilayer neural network define its estimation performance.

In the numerical experiments, comparison with Weighted multi layer neural network is successfully carried out to show the performance of the proposed approach.

2. MATHEMATICAL MODEL

2.1 Boundary expression and FEM discretization

The boundary shape of anomaly is expressed by front points [6]. Firstly, we chose M points which is spacing equivalently along the polar angle in the frame of polar coordinate. The angle of *k-th* position will be written as:

$$\theta_k = 2\pi(k-1)/M, \qquad k = 1, 2, \cdots, M \tag{1}$$

The set of front points R like as:

$$\begin{aligned} R &= \{r_k \mid k = 1, 2, \cdots, M \} \\ &= R^{ref} + \Delta R \end{aligned} \tag{2}$$

Where, r_k is the distance of the *k-th* front point from the center of the reference coordinate. R^{ref} is reference points and ΔR means the change from the reference points. If we assume the region of interest (ROI) set Q whose elements are changed into front point, the change will be expressed as:

$$\Delta R = \begin{cases} r_j^{change} & , j \in Q \\ 0 & , otherwise \end{cases} \tag{3}$$

To express the boundary smoothly, we use Fourier interpolation. In the Fig. 1, we show an example of boundary expression by front point method. To express the boundary shape of object, we use 20 front points and the ROI set is defined as $Q = \{6, 7, 8, 9\}$.

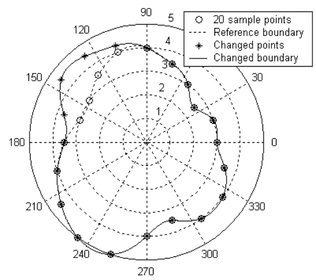


Fig. 1. Reference boundary and changed boundary.

The relationship between the measured voltages on the electrodes and the front points set R are very nonlinear. However, it should be clarified and then incorporated into the forward solver. In this part, we will express the discretization of the forward model as a mapping from the front point set R to the boundary data of the injected currents and the measured potential. The spatial discretization of the forward problem is made with FEM, so the relation will be described in the sense of finite element method. In boundary estimation problems, the object boundary may not fit to the mesh boundary. Hence, the mesh should be reformed to adapt to the object boundary or the elements crossing the object boundary should be treated in a proper manner. This paper include the treatment of boundary-crossing elements, the FEM implementation of this mapping can be accomplished in next 4 states [7] :

Step1: Classify mesh nodes into inside or outside for a given boundary $C_\ell(s)$.

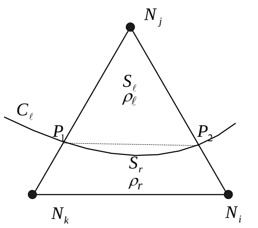


Fig. 2. Schematic of the finite element mesh crossing the object boundary $C_l(s)$.

Step 2: Classify the finite elements as inside, outside, or intercepted by the given boundary $C_\ell(s)$.

Step 3: Determine intersections of element edges, $C_\ell(P_1)$ and $C_\ell(P_2)$, with the given boundary $C_\ell(s)$.

Step 4: Compute the system matrices. In this step, we approximate the boundary C_ℓ with a straight line from the intersection point $C_\ell(P_1)$ to the point $C_\ell(P_2)$, and then split the intercepted element into two parts. We approximate the effective conductivity of the intercepted element, ρ_{cross} , as an area-averaged value

$$\rho_{cross} = \frac{\rho_\ell S_\ell + \rho_r S_r}{S_\ell + S_r} \tag{4}$$

Where, ρ_ℓ and ρ_r are the conductivity of the object and the background, respectively. S_ℓ and S_r are the area belonging to the object and the background, respectively.

Now, the front point set R is implicitly engaged in the finite element formulation through the area terms in (4).

2.2 Multi-layer neural network with changed bipolar sigmoid function

We consider the multi-layer neural network approach to reconstruct the boundary shape of the anomaly. In this work, multi-layer neural network with changed bipolar sigmoid function is introduced due to its enhanced feature compared with WMNN [6]. During training, the output of hidden layer can unexpectedly be saturated without further improvement. In this approach, to avoid this, the parameter value of unipolar sigmoid function depend on root mean squar of error has been devised. The schematic diagram of MNN is shown in Fig. 3.

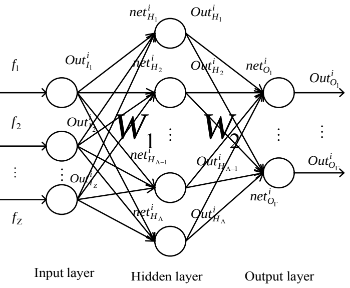


Fig. 3. Multi-layer neural network.

Here, The W_1 , W_2 are weighting matrix of multi-layer neural network, which is used to estimate the change of front points ΔR . To train the weighting matrices W_1 , W_2 of the MNN, we use front point change ΔR and normalized boundary voltages f . The bipolar sigmoid function g_k in hidden layer shows the non-linearity of system. In the bipolar sigmoid function, parameter α is slope parameter and it is generally 1. If α is fixed, however, the output of bipolar sigmoid function can be saturated [8]. So, we use changed bipolar sigmoid function whose parameter α depend on error. The bipolar sigmoid function can be written as:

$$g_k(x) = \frac{1}{1 + e^{-\alpha_k \cdot (W_1 \cdot x)}} \tag{5}$$

$$\alpha_k = \begin{cases} \dfrac{M}{e_k} & if\ \alpha_k < 1.0 \\ 1 & else \end{cases} \tag{6}$$

$$e = \mathrm{mean}(\max(\left|\Delta R - \Delta\hat{R}\right|)) \tag{7}$$

$$M = \mathrm{mean}(\mathrm{mean}(\Delta R)) \tag{8}$$

Where, e is the mean of maximum error value between known changed front point and estimated changed front point. M is the mean of known front point. The α_k is changed each training step, if the α_k is lower than 1.

The normalized boundary voltage f_n is expressed as:

$$f_n = \frac{v_n^i - v_n^{ref}}{v_n^{ref}} \quad i = 1, 2, \cdots, s_j \ , \quad n = 1, 2, \cdots, N \tag{9}$$

Where, v_n is the *k-th* voltage from measured voltage on each frame, s_j is a number of sampling data and v^{ref} is the reference voltage. The changed front point ΔR is expressed as:

$$\Delta R^k = R^k - R^{ref} \quad where \quad k = 1, \cdots, K \tag{10}$$

Where, K is the total number of sampling data.

To train multi-layer neural network, we use error back propagation algorithm. The cost function of it is defined as

$$E = \frac{1}{2}\left\| \Delta R - Out_O \right\| \tag{11}$$

Where, ΔR set is known value, Out_O set is estimated value.

The updating procedure for the weighting matrices W_1, W_2 can be summarized by the following algorithm:

Step 1: Initialize the weights W_1, W_2

Step 2: Calculate the outputs of multi-layer neural network

$$Out_O^k = W_2 \cdot g_k(W_1 \cdot f_n^k) \tag{12}$$

Step 3: Calculate the error $e = \Delta R - Out_O$

Step 4: Update bipolar sigmoid function g_k

Step 5: Update Weightings W_1, W_2

$$W_2(n+1) = W_2(n) + \Delta W_2 \tag{13}$$

$$\Delta W_2 = -\eta \frac{\partial E}{\partial W_2} = -\eta \frac{\partial E}{\partial Out_O}\frac{\partial \Delta \hat{R}}{\partial W_2} \tag{14}$$

$$W_1(n+1) = W_1(n) + \Delta W_1 \tag{15}$$

$$\Delta W_1 = -\eta \frac{\partial E}{\partial W_1} = -\eta \frac{\partial E}{\partial Out_O}\frac{\partial Out_O}{\partial g_k}\frac{\partial g_k}{\partial W_1} \tag{16}$$

Where, η is learning rate, g_k is the output of bipolar sigmoid function.

Step 6: Repeat from Step 2 to Step 5 until designed iteration number is reached

The schematic diagram of training procedure is summarized in Fig. 4.

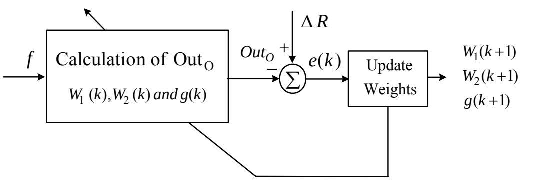


Fig. 4. Schematic diagram of training procedure.

After training, we can estimate the changed front point by using the trained weights W_1, W_2 and bipolar sigmoid function g_k and normalized boundary voltage f :

$$\Delta \hat{R} = W_2 \cdot g_k(W_1 \cdot f) \tag{17}$$

3. NUMERICAL SIMULATION

To verify the performance of the proposed method, numerical experiments have been conducted. In the simulation, we estimate the changed front points ΔR, which have not been used for teaching during training process. We consider a circular object whose radius is 14cm and it has 16 electrodes along the boundary. To calculate boundary voltage, we discretize the circular object into 1968 triangular elements. We assume the resistivity values of the object and the background are 600Ωcm and 300Ωcm respectively. The adjacent current injection method is adopted and 256 boundary voltages are used for a single frame of image.

The boundary shape of object is expressed with 30 front points. To verify the performance of the proposed method, we compare WMNN [3] with proposed method. The reference front points R^{ref}

$$R^{ref} = [2\ \ 3\ \ 5\ \ 7\ \ 8\ \ 7.5\ \ 7\ \ 6.5\ \ 6\ \ 6\ \ 5\ \ 5\ \ 5\ \ 5\ \ 5\ \ 5\ \ 5\ \ 5.5\ \ 6\ \ 7\ \ 8.5\ \ 9\ \ 9\ \ 8\ \ 7\ \ 4\ \ 2.5\ \ 2\ \ 1.5\ \ 1.5]$$

It is assumed that only seven points are in ROI set Q

$$Q = [1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 30]$$

The proposed approach and WMNN are applied to the synthesized voltage data :

Example 1: $\Delta R = [0.5\ \ 0.3\ \ 2.0\ \ 1.2\ \ 0.4\ \ 0.5\ \ 0.4]$

Example 2: $\Delta R = [0.3\ \ 0.4\ \ 1.5\ \ 2.5\ \ 1.5\ \ 0.5\ \ 0.4]$

The results of examples will give an indication of the performance of the proposed approach. It is assumed that the measured boundary voltages are contaminated with 0%, 1%, 2% and 3% random noises, respectively, in which the distribution of noise is Gaussian with the corresponding percentage of standard deviation.

To evaluate the estimation quality of ΔR, the root mean square error (RMSE) based on front point is defined:

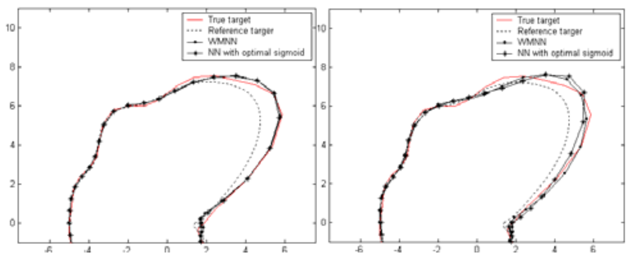
$$RMSE_{Front} = \frac{\left\| \Delta R - \Delta \hat{R} \right\|}{\left\| \Delta R \right\|} \tag{18}$$

Where, ΔR is known changed front points and and $\Delta \hat{R}$ is estimated front points.

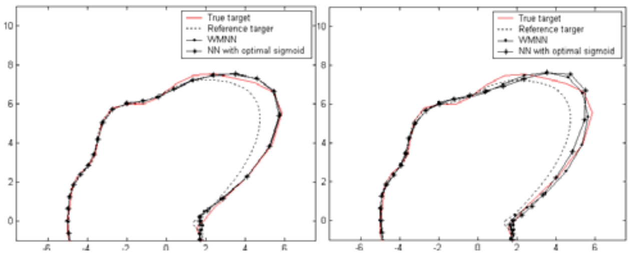
The root mean square error (RMSE) based on the boundary voltages U obtained from the estimated interfacial boundary is defined:

$$RMSE_U = \frac{\left\| U - V(\hat{R}) \right\|}{U U^T} \tag{19}$$

Where, U is calculated boundary voltage by known front point and $V(\hat{R})$ is calculated boundary voltage by estimated front point.



(a) 0% noise (b) 1% noise



(c) 0% noise (d) 1% noise

Fig. 5. Estimated boundary shape of object (example 1).

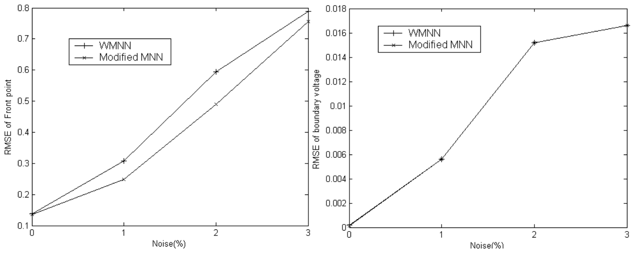
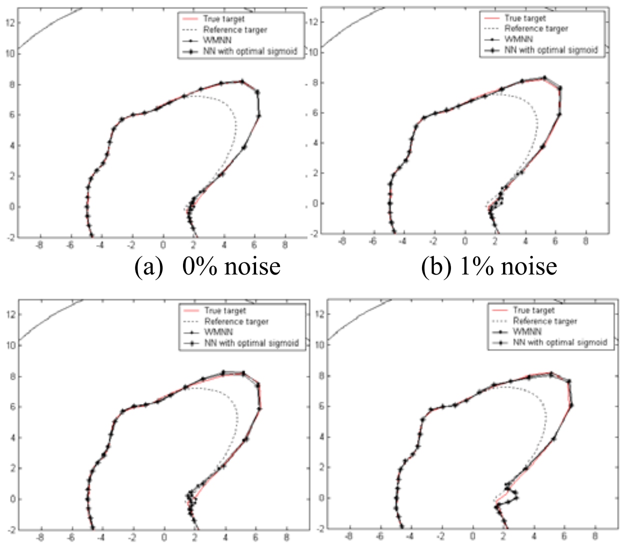


Fig. 6. RMSE of Front point and boundary voltage (example 1).



(a) 0% noise (b) 1% noise

(b) 2% noise (d) 3% noise

Fig. 7. Estimated boundary shape of object (example 2).

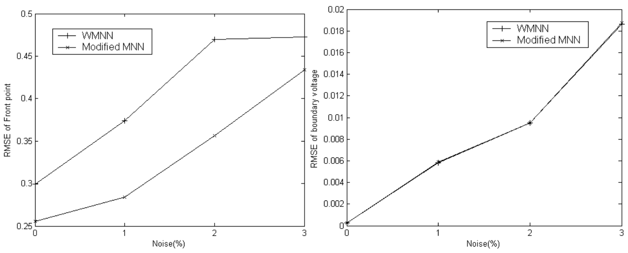


Fig. 8. RMSE of Front point and boundary voltage (example 2).

As can be seen in Fig. 5 to Fig. 8. , the proposed approach improved performance in sense of boundary estimation accuracy.

4. CONCLUSION

In this paper, we have presented a multi-layer neural network with changed bipolar sigmoid function to estimate the complex boundary shape of anomaly. The boundary shape of anomaly is expressed in terms of front points discretely placed on the interfacial boundary. The unknown front points are estimated with the boundary voltages and proposed approach. The advantages of the proposed approach include negligibly small reconstruction time and enhanced estimation performance.

To verify the performance of the proposed multi-layer neural network, we have conducted numerical simulation with and without measurement noise. Numberical experiemnts show that the proposed approach has enhanced performance in terms of boundary estimation accuracy.

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