The Optimum power allocation method in the uplink of OFDMA system

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Abstract: Orthogonal Frequency Division Multiple Access (OFDMA) is an optimized technology established based on the traditional Orthogonal Frequency Division Multiplexing (OFDM) method that allows to allocated the subcarriers to different users in a single frame. Note that in OFDMA, each user can occupy certain amount of power to achieve its target signal to noise ratio (SNR) level. This paper proposes a novel transmission power and subcarrier allocation strategy that results in minimum power level transmission subject to target SNR constraints defined for each allocated subcarrier. It is shown (via Matlab-based simulations) that the proposed allocation resource allocation algorithm outperforms the conventional waterfilling method (in terms of power savings).

1. Introduction

The optimized OFDM (or OFDMA) which is generally employed in 802.11ac, 802.16, WiMAX and LTE, is a standard wireless system being widely used nowadays [1]. Each OFDMA symbol is divided into several subcarriers to carry the users' data stream. In general, each subcarrier is allocated to the user with the highest channel gain to maximize the total network throughput. Note that in wireless network, the scarcest resource is the transmission power of user terminals. Hence, the problem of achieving the minimum power level while preserving the high transmission quality (i.e., SNR). In this paper, we come up with a power allocation model for OFDMA system where the transmission power p is allocated subject to the target SINR constraints γ using game-theoretic approach. In our model, the players are the subcarriers which aim to increase their transmission power (to achieve higher SNR) but are limited by the transmission cost.

2. System Model

Let us define the $N\times M$-dimensional matrix

$$\mathbf{p} = \begin{bmatrix} p_{11} & \cdots & p_{1M} \\ \vdots & \ddots & \vdots \\ p_{N1} & \cdots & p_{NM} \end{bmatrix}$$

with the non-negative real components p_{ij} being the transmission power allocated to i^{th} subcarrier and j^{th} user at the current frame.

It is well-known that the system capacity C is defined by Shannon expression as

$$C = max_{p_{ij}}\left(B \cdot log_2(1 + \frac{g_{ij}p_{ij}}{\sigma^2})\right) \quad (1)$$

where B stands for bandwidth, g_{ij} is the channel gain of subcarrier i allocated to user j, p_{ij} is the power occupied by subcarrier I, σ^2 is the noise power density.

$$\gamma_{ij} = \frac{B}{R} * \frac{g_{ij}p_{ij}}{\sigma^2} \quad (2)$$

$\boldsymbol{\gamma_{ij}}$ stands for this subcarrier's SNR and $\boldsymbol{R}$ is the bit's speed.

Obviously, when we calculate system's capacity, or system's throughput, if all subcarriers can provide with higher signal to noise ratio (SNR), system's throughput will increase as well.

So here is the problem, we can find that, for condition (1), the user with poor channel gain means he needs to occupy more power from whole system, which may cause interference to other user's subcarrier, and also it's unfair to other users as well since allocating more power to this subcarrier can only contribute less to system's throughput than normal users. As to condition (2), lower target SNR means this subcarrier needs less power than usual, which means lower ratio of system's total power is taken away from whole system and cause less interference other users.

3. Conventional Power allocation Techniques

In traditional Waterfilling algorithm [2], the standard of filling power into each subcarrier is defined as a constant value called power baseline, which is consist of two parts:

$$power\ baseline = X_{ij} + P_{ij} \quad (3)$$

$$X_{ij} = 1/SNR \quad (4)$$

Where X_{ij} and P_{ij} stands for the noise to signal ratio and power belonging to user j's subcarrier i.

Power baseline is defined as a constant value. In the process of allocation, according to each subcarrier's SNR, system will fill power to them until an equal value (power baseline).

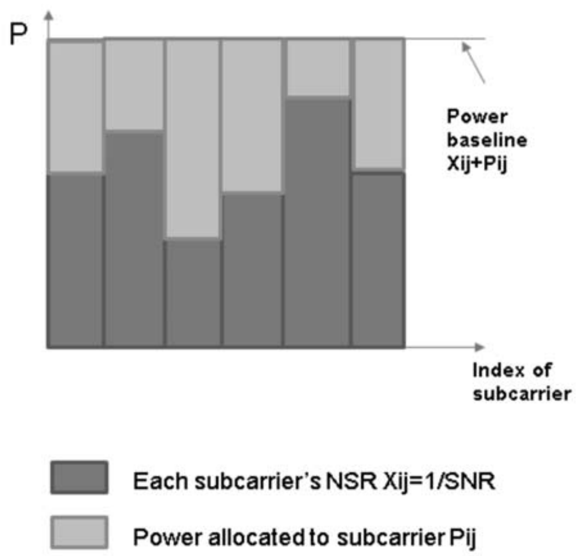

Fig 2. Power baseline's definition in Waterfilling algorithm

Note that, in order to properly implement waterfilling algorithm, the power baseline should be calculated at first, which means both of the subcarrier ***i***'s power and its SNR should be know before calculation, and it's proved to be impossible.

So that's why in most of experiments, they choose to calculate an average power for each subcarrier by ***Ptot/N*** to get a first power baseline (its result shown in the graph down there).

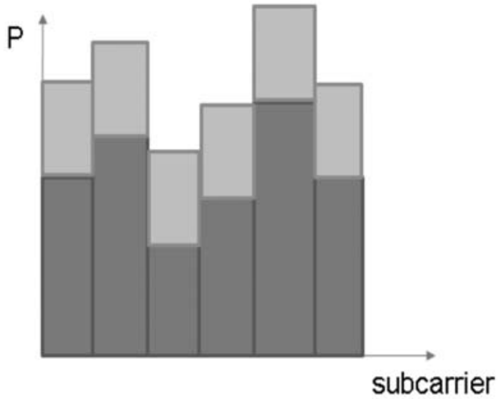

Fig 3. Result of Waterfilling algorithm with power being allocated averagely.

Obviously the accuracy of system's capacity would decrease and the power allocated to each user can't be customized according to their required SNR or channel situation. And Waterfilling method only simply offer enough power to each subcarrier, ignoring what kind of channel gain this subcarrier will suffer and how much SNR its owner requires to achieve his transmission. Thus here I come up with a new optimized power allocation algorithm without needing to calculate power baseline at first.

4. Proposed algorithm

We implement game theory to simulate the competition for power by users' subcarriers.

Defining system utilization as a sum of extra SNR over each subcarrier's target SNR minus the interference caused by over occupying power from system's total power [3].

System utility function should satisfy the following points:
(1) $\boldsymbol{U}$ is a strictly increasing function of SNR;
(2) $\boldsymbol{U}$ is a strictly increasing function of transmitted power, but the increment should be gradually reduced;
(3) When the transmission power reaches infinity, the limit of the utility function should be 0.

According to the system's condition:
(1) Each user wants to obtain larger power to improve its SNR, and its utility to the system;
(2) Increment of a user's power will increase the interference on other users;
(3) Pricing mechanism is used to find a balance between the utility provided by each subcarrier's SNR and the expenditure caused by the interference over other users.

Defines $\boldsymbol{p_{ij}} = \begin{bmatrix} \boldsymbol{p_{11}} & \cdots & \boldsymbol{p_{1M}} \\ \vdots & \ddots & \vdots \\ \boldsymbol{p_{N1}} & \cdots & \boldsymbol{p_{NM}} \end{bmatrix}$ as the power allocation's vector, $\boldsymbol{c_{ij}} = \begin{bmatrix} \boldsymbol{c_{11}} & \cdots & \boldsymbol{c_{1M}} \\ \vdots & \ddots & \vdots \\ \boldsymbol{c_{N1}} & \cdots & \boldsymbol{C_{NM}} \end{bmatrix}$ as the subcarrier allocation's matrix, $\boldsymbol{i}$ is subcarrier's index number and $\boldsymbol{j}$ is user's index number, $\boldsymbol{N}$ and $\boldsymbol{M}$ stands for subcarrier and user's total amount respectively.

Under these restriction and avoiding the demerit of Waterfilling method (needs each subcarrier's power and SNR before allocation), I define the utility provided by each subcarrier to whole system's function as:

$$U_{ij} = argmax_{p_{ij},c_{ij}}\{log_2\,(c_{ij}\alpha(\gamma_{ij} - \gamma_{ij}^{tar})) - \lambda p_{ij}\} \quad (5)$$

S.T.

$$\begin{cases} p_{ij} \neq 0, & while\ c_{ij} = 1 \\ p_{ij} = 0, & while\ c_{ij} = 0 \end{cases} \quad (6)$$

Defining $\boldsymbol{U}$ as a parameter standing for the utility made by each subcarrier to the whole system. If a subcarrier only request for quite low target SNR and the channel condition it suffers is better than any other, according to Shannon theory, obviously this subcarrier is easier to reach high SNR and higher system throughput with certain amount of power than other subcarrier. Thus defining this subcarrier can provide system with more utility.

Subcarriers can only be allocated with power only if it's been successfully allocated to proper user.

$$\gamma_{ij} \geq \gamma_{ij}^{tar}, i \in \{1, \dots, N\}\ j \in \{1, \dots, M\} \quad (7)$$

$$\sum p_{ij} \leq p_{tot} \quad (8)$$

$$p_{ij} \geq 0, i \in N\ j \in M \quad (9)$$

Where

$$\gamma_{ij} = \frac{B}{R} * \frac{g_{ij}p_{ij}}{\sigma^2} \quad (10)$$

In equation (5), $\boldsymbol{U_{ij}}$ is the utility provided by user ***j***'s subcarrier ***i*** to system, $\boldsymbol{\alpha}$ is positive biased parameter and λ is punishment parameter, $\boldsymbol{\gamma_{ij}}$ is subcarrier i's SNR and $\boldsymbol{\gamma_{ij}^{tar}}$ is this subcarrier's target SNR, p_{tot} means the total power system has and $\boldsymbol{\sigma^2}$ is the power of noise, $\boldsymbol{B}$ is bandwidth, $\boldsymbol{R}$ is bit speed and $\boldsymbol{g_{ij}}$ stands for subcarrier ***i***'s channel gain.

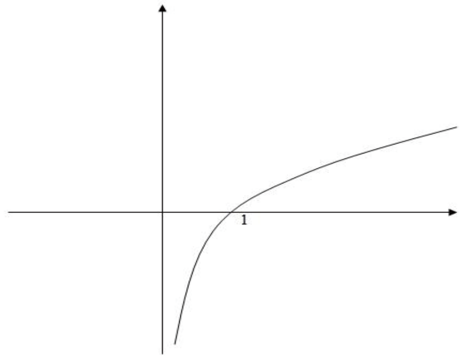

Fig 4. A typical log2 function's graph.

The graph of an log2 function is given in the figure above. When the power of ***i***th subcarrier increases, it's SNR increases at well, and the increment of utility provided by subcarrier ***i*** decreases when its SNR keeps growing up (especially after its target SNR is reached), and finally converges to a constant value. This is the same to actual condition.

The second part of the equation is the product of the punishment parameter and subcarrier ***i***'s power, which stands for the interference caused by subcarrier ***i*** and ensuring that this function converges to some equilibrium point. For example, in a wireless network system, one of the user needs quite high target SNR, and result in very high power being required, it will cause serious interference to other user. So when we build allocating strategy, we can give this user's subcarrier a low biased parameter $\boldsymbol{\alpha}$ and high punishment parameter $\boldsymbol{\lambda}$. On one hand, the increment of utility will become more flat when this user's subcarrier ask for more power, on the other hand, this user need to pay more for increasing power. By these two way can we limit this user's power and reduce the interference in system, thus achieving optimization in resourcing allocation.

By valuing biased parameter α and punishment parameter λ can we calculate a power p for subcarrier ***i*** which can either guarantee its target SNR being reached and cause as little interference as subcarrier ***i*** can while providing system with the largest utility. And one more, this method doesn't need to know each subcarrier's power and its SNR before allocation starts.

In order to get biggest utility ***U***, calculate ***U*** derivative of $\boldsymbol{p_{ij}}$:

$$\frac{dU_{ij}}{dP_{ij}} = \frac{1}{\alpha(\gamma_{ij}-\gamma_{ij}^{tar})ln2}\frac{\frac{B}{R}g_{ij}}{\sigma^2} - \lambda = 0 \tag{11}$$

The only variable value in this partial derivative is the denominator $\alpha(\gamma_{ij}-\gamma_{ij}^{tar})ln2$, input γ 's definition, $\frac{1}{\alpha(\gamma_{ij}-\gamma_{ij}^{tar})ln2}\frac{\frac{B}{R}g_{ij}}{\sigma^2}$ (which is positive) will decreases as power Pi increases and finally $\frac{dUij}{dPij}$ equals zero and ***U*** gets its maximum point.

5. Simulation results

In this paper, we implement a Greedy algorithm in subcarrier allocation field, that is, the user who can get higher channel gain, offer him with higher privilege to get more subcarrier first. Through theoretical analysis and actual modeling, we present the result of power allocation result and how does system's throughput performs in my power allocation strategy. And, we provide the comparison with the result from a traditional Waterfilling method.

Valuing simulation environment value as follows:

Table 1. Simulation settings.

User's amount	4
Subcarrier's amount	20
Bandwidth	$10*10^6$Hz
Each user's max power	10w
Start of noise's power	$1*10^{-15}$W
Goal of noise's power	$10*10^{-15}$W
Interleave of noise's power	$1*10^{-15}$W

Results are shown in the figures below:

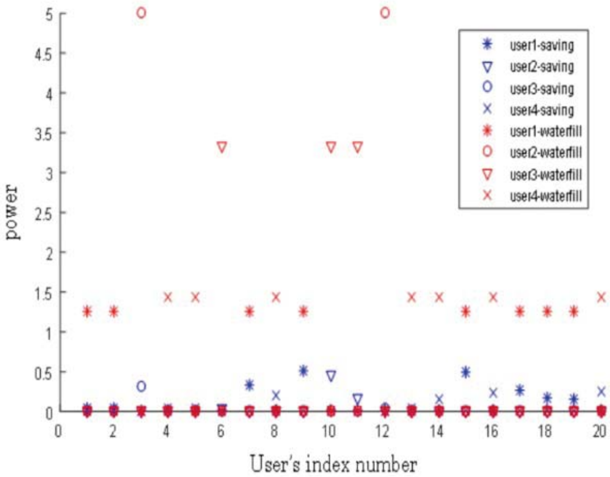

Fig 5. Result of the power being allocated to each user's subcarrier, since only one subcarrier will be provided with power once, every point stands for the total power occupied by each user.

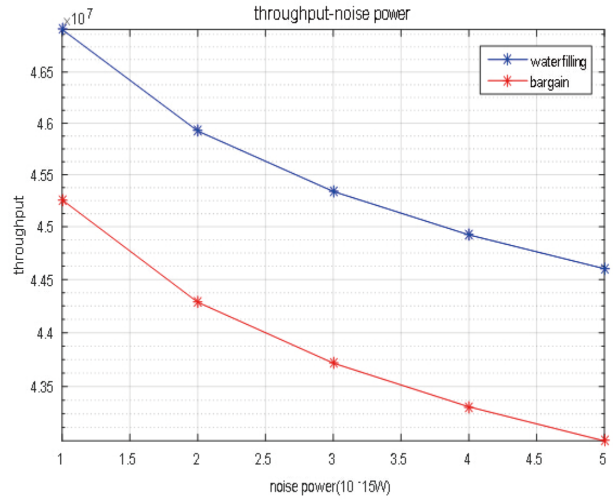

Fig 6. Whole system's throughput calculated according to the power allocation result in Fig 5.

In Fig 5, each kind of point mark stands for a user's subcarrier, and each point represents the power being allocated to this subcarrier. Red point is the result from tradition Waterfilling method and blue point is from my algorithm. User's index number ranges alone the X label and Y label is power's intensity.

Fig 6 shows the whole system's throughput when the power is allocated in Fig 5's strategy. Red line and blue line monitors the changing of system's throughput under my gaming allocation algorithm and Waterfilling method respectively when channel's noise power increases

Once the allocation occurs, only one subcarrier will be allocated with power, so each point in figure stands for the total power gotten by this user. We can see from the figure showed above, the power used by proposed gaming based power allocation is about 80% less than Waterfilling method while the sacrifice of whole system's throughput is only 3-5% comparing to Waterfilling. Especially in uplink where transmitting power is quite limited, this gaming based power allocation algorithm can be more useful than traditional Waterfilling method.

6. Conclusion

This paper presents a new algorithm to increase the throughput of OFDMA system while limiting power used. Previous allocation methods mostly consider only transmitting power as part of subcarrier, power has been considering as a component of subcarrier and in most of the algorithm, the main purpose of algorithm 3 is to reach the pinnacle of throughput rate of system or most of users when allocating subcarrier without caring about the cost of whole system.

The result performs well. When all of the user's target SNR is reached, the power costed is quite lower than Waterfilling method and whole system's throughput will just sacrifice a little of itself, which is suitable for the environment of uplink.

References

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