Design of Cascade Observers for Robot Angular Link Velocity

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Abstract: A new observer, namely a cascade observer, for a general class of nonlinear SISO systems was presented by the authors [1]. Inspired by but different from a high-gain observer, the cascade observer features a cascade structure and adaptive observer gains. In doing so the cascade observer attempts to overcome some of the typical problems that may pose to a high-gain observer. Based on the cascade observer, the estimation problem of the angular link velocity is presented in this paper. As in the case of a high-gain observer, the cascade observer structure is simple and universal in the sense that it is independent of the system dynamics and parameters. It is shown that the proposed cascade observer is applicable to robotic state estimation using only joint position measurements effectively.

1. Introduction

Most popular model-free observers are high-gain observers that can estimate the derivatives of the output [2],[3]. High-gain observers to estimate the joint velocities of robot have been used in a number of studies [4]. The difficulty arising in practical applications, however, is the determination of an appropriate value for the observer gain. By choosing the observer gain large enough (therefore the term "high-gain"), the observer error can be made arbitrarily small. But for high values, initial peaking (called 'peaking phenomenon') is generated, i.e. large mismatched values between real and estimated values for the short initial period [3]. For values too low, desired bounds on the observer error can not be achieved. The high-gain observer induces a time scale separation between the considered system and the estimate error dynamics; hence, the singular perturbation techniques [5] have been used in the stability analysis of the error dynamics.

One way to remedy the velocity feedback problem is to determine a velocity signal by first-order numerical differentiation of the accurate position signal. The simplicity of this technique makes it particularly useful from an implementation point of view. However, for low and high velocities especially, such a simple approximation of the velocity signal may be inadequate [7]. Moreover, the quantization effect that inherently goes along with this approach may produce undesired oscillations in the robot joint response, or even cause it to become unstable. In addition, there is no theoretical justification for this *ad hoc* solution. To solve this problem, the authors in [1] have proposed a new cascade observers for output feedback control and presented that the estimation errors converge to zero and all the internal variables are globally uniformly bounded. Based on the cascade observer, an adaptive output feedback controller is designed for a class of SISO nonlinear systems represented globally by a nth-order differential equation.

In this paper, we propose how the proposed observer with cascade structure is applied to nonlinear systems. Then by using the cascade observer, velocity estimation for a robot can be implemented. In addition to the cascade structure, the gains of a cascade observer consist of two type: fixed and adaptive ones. In doing so the cascade observer attempts to overcome some of the typical problems outlined above for a high-gain observer. The cascade observer structure is simple and universal in the sense that it is independent of the system dynamics and parameters. It is shown that the proposed cascade observer is applicable to robotic state estimation using only joint position measurements effectively.

2. Cascade Observer

2.1 Observer Design

A basic design objective of the proposed cascade observer is to guarantee the convergence of observer errors in a 'cascade' fashion, that is, $\hat{x}_1 = x_1 \rightarrow \hat{x}_2 = \dot{\hat{x}}_1 \rightarrow \hat{x}_2 = \dot{\hat{x}}_1 = \dot{x}_1 \rightarrow \hat{x}_3 = \dot{\hat{x}}_2 \rightarrow \hat{x}_3 = \dot{\hat{x}}_2 = \ddot{x}_1, \cdots, \hat{x}_{n+1} = \dot{\hat{x}}_n \rightarrow$

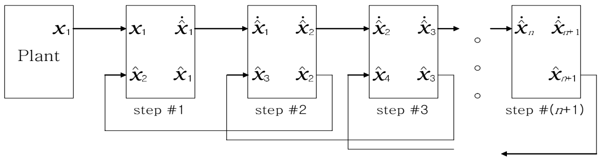

Figure 1. The structure of cascade observer.
figure

$\hat{x}_{n+1} = \dot{\hat{x}}_n = x_1^{(n)}$. Based on this concept, a cascade observer is designed as

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + l_1(y - \hat{x}_1) + \hat{\rho}_1 sgn(y - \hat{x}_1) \\ \dot{\hat{x}}_i &= \hat{x}_{i+1} + l_i(\dot{\hat{x}}_{i-1} - \hat{x}_i) + \hat{\rho}_i sgn(\dot{\hat{x}}_{i-1} - \hat{x}_i)\ n \\ \dot{\hat{x}}_{n+1} &= l_{n+1}(\dot{\hat{x}}_n - \hat{x}_{n+1}) + \hat{\rho}_{n+1} sgn(\dot{\hat{x}}_n - \hat{x}_{n+1}) \\ \hat{y} &= \hat{x}_1 \end{aligned} \tag{1}$$

where we utilize the condition such that $|\hat{x}_{i+1} - \ddot{\hat{x}}_{i-1}| \leq \rho_i$, $i = 1, \cdots, n$. i is a step number.

$$sgn(a) := \begin{pmatrix} 1 & \text{if} & a > 0 \\ 0 & \text{if} & a = 0 \\ -1 & \text{if} & a < 0 \end{pmatrix}.$$

Note that ρ_i, $i = 1, \cdots, n$ and ρ_{n+1} are upper bound values and constants. And this will be verified through Theorem 1. It is also shown in Theorem 1 that the ρ_i's exist and are bounded. $\hat{\rho}_i$ denotes the estimate of ρ_i, and l_i is a positive constant. When l_is are chosen as commonly used, l_i will be larger than l_{i+1} because the estimation of the former step contributes more than the estimation of the latter one in terms of the cascade construction. Such a gain selection requires a smaller gain value as the state order increases, unlike the high-gain observer demands a higher gain value as the state order increases. Fig.1 shows the structure of the proposed observer. Each step is connected in a cascade way.

Notation. For convenience, $\dot{\hat{x}}_0$, in this paper, refers to y.

Adaptive law of $\hat{\rho}_i$ in (1) is designed as

$$\begin{aligned} \dot{\hat{\rho}}_i &= \gamma_i|\dot{\hat{x}}_{i-1} - \hat{x}_i| \quad if \quad \hat{\rho}_i \leq \bar{\rho}_i \quad i = 1, \cdots, n+1 \\ &= \gamma_i[1 + \frac{\bar{\rho}_i - \hat{\rho}_i}{\delta_i}]|\dot{\hat{x}}_{i-1} - \hat{x}_i| \ \ otherwise \end{aligned} \tag{2}$$

where δ_i is a positive constant that plays the role to reduce the adaptation gain in the case of $\hat{\rho}_i > \bar{\rho}_i$ and prevent $\hat{\rho}_i$ from being divergent. $\bar{\rho}_i$ can be set to any positive value but should be chosen judiciously as they affect the transient response of the cascade observer. Equation (2) is the updating law to ensure that $\hat{\rho}_i$ is bounded.

Remark 1: The upper bounds of $|\hat{x}_{i+1} - \ddot{\hat{x}}_{i-1}|$ and $|\ddot{\hat{x}}_n|$, i.e. the ρ_i's, can be obtained under certain circumstances (please note that the ρ_i's exist and are bounded as shown in Theorem 1). For example, in the case of vehicle dynamics, the maximum velocity and acceleration values are known from performance specifications. However, to determine the values of these upper bounds a priori can be troublesome or may be difficult under certain circumstances. Therefore it is often more desirable to estimate them using adaptation laws. As in typical adaptation schemes, it does not matter that the estimated values do not converge to the real ones. The estimated values serve to organize the cascade observer to guarantee stability in the sense of Theorem 1.

Theorem 1: For system (Σ'), the proposed cascade observer guarantees asymptotic stability of the estimation error.

Proof of Theorem 1 To examine the stability analysis of the proposed observer, we consider the Lyapunov function candidate

$$V = \frac{1}{2}\sum_{i=1}^{n+1}(\dot{\hat{x}}_{i-1} - \hat{x}_i)^2 + \frac{1}{2}\sum_{i=1}^{n+1}\frac{1}{\gamma_i}\tilde{\rho}_i^2 \tag{3}$$

where $\tilde{\rho}_i = \rho_i - \hat{\rho}_i$.

Differentiating V along the system trajectory gives

$$\dot{V} \leq -\sum_{i=1}^{n+1} l_i(\dot{\hat{x}}_{i-1} - \hat{x}_i)^2. \tag{4}$$

From (4), we can recognize that $(\dot{\hat{x}}_{i-1} - \hat{x}_i)$ and $\tilde{\rho}_i$ are bounded. Also, $\hat{\rho}_i$ is bounded from (2). Thus, ρ_i and $|\hat{x}_{i+1} - \ddot{\hat{x}}_{i-1}|$ are bounded. Integrating (4) gives

$$\begin{aligned} \int_0^t \sum_{i=1}^{n+1} l_i(\dot{\hat{x}}_{i-1}(\tau) - \hat{x}_i(\tau))^2 d\tau &\leq -V(t) + V(0) \leq V(0) \\ &= \frac{1}{2}\sum_{i=1}^{n+1}(\dot{\hat{x}}_{i-1}(0) - \hat{x}_i(0))^2 + \sum_{i=1}^{n+1}\frac{1}{2\gamma_i}\tilde{\rho}_i^2(0). \end{aligned} \tag{5}$$

Thus, because $V(0)$ and l_i are constants, $\sum_{i=1}^{n+1}(\dot{\hat{x}}_{i-1} - \hat{x}_i) \in L_2$. Since we have proven that $\rho_i, \hat{\rho}_i, (\dot{\hat{x}}_{i-1} - \hat{x}_i)$ and $|\hat{x}_{i+1} - \ddot{\hat{x}}_{i-1}|$ are bounded, we have $\sum_{i=1}^{n+1}\frac{d}{dt}(\dot{\hat{x}}_{i-1} - \hat{x}_i) \in L_\infty$. Using the Barbalat's Lemma [6], $\lim_{t\to\infty}\sum_{i=1}^{n+1}(\dot{\hat{x}}_{i-1}(t) - \hat{x}_i(t)) = 0$.

Remark 2: From Theorem 1, it follows that state estimation errors converge to 0 in a 'cascade' fashion, i.e., first $\hat{x}_1$ converges to x_1, $\hat{x}_2$ then converges to $\dot{\hat{x}}_1 = \dot{x}_1$. It follows that $\hat{x}_3$ converges to $\dot{\hat{x}}_2 = \ddot{x}_1$. Therefore $\hat{x}_i$ converges to $\dot{\hat{x}}_{i-1} = x_1^{(i-1)}$. Moreover, the succession of convergence implies that $\hat{x}_i$ are bounded. Therefore all the internal variables are globally uniformly bounded.

3. Application to the Robot Angular Link Velocity

3.1 Manipulator Model

Consider the dynamic model for a robot manipulator with two degrees of freedom [6]. The corresponding Lagrange equation is given by

$$M(\theta)\ddot{\theta} + W(\theta, \dot{\theta}) = u(\theta, u \in R^2) \qquad (6)$$

where $M(\theta)$ represents the inertia matrix $M(\theta) = M^T(\theta) = \begin{bmatrix} M_{11}(\theta) & M_{12}(\theta) \\ M_{21}(\theta) & M_{22} \end{bmatrix} > 0$ with components $M_{11}(\theta) = (m_1 + m_2)a_1^2 + m_2a_2^2 + 2m_2a_1a_2c_2$, $M_{12}(\theta) = m_2a_2^2 + m_2a_1a_2c_2$, $M_{22} = m_2a_2^2$, $M_{21}(\theta) = M_{12}(\theta)$, $a_i = d_i$, $c_i = cos\theta_i$, $s_i = sin\theta_i$ and $c_{12} = cos(\theta_1 + \theta_2)$. Here $\theta_i = \theta_i(t) \ \ (i = 1, 2)$ denotes the independent joint angular position defining two degrees of mobility, m_i and d_i are the mass and the length of the corresponding links.

$W(\theta, \dot{\theta})$ represents the Coriolis and centrifugal matrix $W(\theta, \dot{\theta}) = W_1(\theta, \dot{\theta}) + W_2(\dot{\theta})$ with components $W_1(\theta, \dot{\theta}) = \begin{bmatrix} W_{1a} \\ W_{1b} \end{bmatrix}$, $W_{1a} = -m_1a_1a_2(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2)s_2 + (m_1 + m_2)ga_1c_1 + m_2ga_2c_{12}$, $W_{1b} = m_2a_1a_2\dot{\theta}_1^2s_2 + m_2ga_2c_{12}$ and $W_2(\dot{\theta}) = k\nu$. k can be expressed as $k = k_0 + \Delta k_t$ where k_0 is the friction parameter and Δk_t is the time-varying uncertainty. $\nu(t)$ is $\nu^T = [\dot{\theta}_1, sgn\dot{\theta}_1, \dot{\theta}_2, sgn\dot{\theta}_2]$.

To represent this system in the standard form, we introduce the extended vector

$$x^T = [\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2]$$

and , in view of this definition, the dynamic equation can be written as follows [6]: $[\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4] =$

$$\begin{bmatrix} x_3 \\ x_4 \\ (-M^{-1}(x)W_1(x) - M^{-1}(x)k\nu(x) + M^{-1}(x)u)_1 \\ (-M^{-1}(x)W_1(x) - M^{-1}(x)k\nu(x) + M^{-1}(x)u)_2 \end{bmatrix} \qquad (7)$$

where $()_1$ and $()_2$ denote the 1st component and the 2nd component in the vector, respectively [5].

It is assumed that only the angular positions can be measured. Since the system (7) is an interconnected system, the combination of observers for separate subsystems is needed. We use similar ideas for the observer design of interconnected systems which treats the velocity reconstruction problem for a robot manipulator application.

3.2 The Observer for the Robot

If the joint velocities x_3 and x_4 are not measurable and the dynamics of manipulator are unknown, the cascade observer can be used to estimate them for the system (7). As for the observer design, two separate observers can be designed where we assign the proposed observer for the robot manipulator dynamics. One is composed by the sub-dynamics of $\dot{\hat{x}}_1, \dot{\hat{x}}_3$ and $\dot{\hat{x}}_5$. The other is composed by the sub-dynamics of $\dot{\hat{x}}_2, \dot{\hat{x}}_4$ and $\dot{\hat{x}}_6$.

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_3 + l_1(x_1 - \hat{x}_1) + \hat{\rho}_1 sgn(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 &= \hat{x}_4 + l_2(x_2 - \hat{x}_2) + \hat{\rho}_2 sgn(x_2 - \hat{x}_2) \\ \dot{\hat{x}}_3 &= \hat{x}_5 + l_3(\dot{\hat{x}}_1 - \hat{x}_3) + \hat{\rho}_3 sgn(\dot{\hat{x}}_1 - \hat{x}_3) \\ \dot{\hat{x}}_4 &= \hat{x}_6 + l_4(\dot{\hat{x}}_2 - \hat{x}_4) + \hat{\rho}_4 sgn(\dot{\hat{x}}_2 - \hat{x}_4) \\ \dot{\hat{x}}_5 &= l_5(\dot{\hat{x}}_3 - \hat{x}_5) + \hat{\rho}_5 sgn(\dot{\hat{x}}_3 - \hat{x}_5) \\ \dot{\hat{x}}_6 &= l_6(\dot{\hat{x}}_4 - \hat{x}_6) + \hat{\rho}_6 sgn(\dot{\hat{x}}_4 - \hat{x}_6). \qquad (8) \end{aligned}$$

Adaptive law of $\hat{\rho}_i$ is designed as

$$\begin{aligned} \dot{\hat{\rho}}_i &= \gamma_i|\dot{\hat{x}}_{i-2} - \hat{x}_i| \qquad if\ \hat{\rho}_i \leq \bar{\rho}_i,\ i = 1, \cdots, 6 \\ &= \gamma_i[1 + \frac{\bar{\rho}_i - \hat{\rho}_i}{\delta_i}]|\dot{\hat{x}}_{i-2} - \hat{x}_i| \quad otherwise \qquad (9) \end{aligned}$$

where $\dot{\hat{x}}_{-1}$ is replaced by x_1 in the case of $i = 1$ and $\dot{\hat{x}}_0$ is replaced by x_2 in the case of $i = 2$.

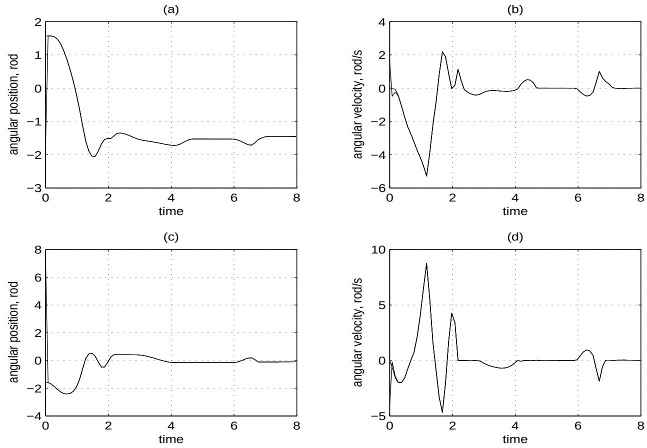

Figure 2. The convergence of the estimated states in the presence of uncertainties: (a) x_1 and $\hat{x}_1$, (b) x_3 and $\hat{x}_3$, (c) x_2 and $\hat{x}_2$, (d) x_4 and $\hat{x}_4$.
figure

4. Simulation

The system parameters of (6) are $a_1 = a_2 = 0.365m$, $m_1 = m_2 = 1.53kg$,
$k_0 = \begin{bmatrix} 0.35 & 1.25 & 0 & 0 \\ 0 & 0 & 0.35 & 1.25 \end{bmatrix}$ and $\Delta k_t =$
$\begin{bmatrix} 0.5\omega sin(\omega t) & 0.9\omega cos(\omega t) & 0 & 0 \\ 0 & 0 & 0.2\omega sin(\omega t) & 0.6\omega cos(\omega t) \end{bmatrix}$.

The initial conditions for the system and the observer are taken as $x_1(0) = \frac{\pi}{2}$, $x_2(0) = -\frac{\pi}{2}$, $x_3(0) = 0$, $x_4(0) = 0$, $\hat{x}_1(0) = -\frac{\pi}{2}$, $\hat{x}_2(0) = 7$, $\hat{x}_3(0) = \frac{\pi}{2}$, $\hat{x}_4(0) = -4$, $\hat{x}_5(0) = 0$, and $\hat{x}_6(0) = 0$. Design parameters are set to $l_{1-6} = 100, 100, 50, 50, 10, 10$, $\gamma_{1-6} = 2, 2, 2, 2, 1, 1$, $\bar{\rho}_{1-6} = 10, 10, 5, 5, 1, 1$, $\hat{\rho}_{1-6}(0) = 0$ and $\delta_{1-6} = 0.1$. We set the control input as $u^T = (sin(t), sin(t))$.

In the presence of uncertainties $\omega = 2.5$, the convergence of estimated states for the proposed observer are shown in Fig.2. Fig.2 indicates the satisfied estimation performance without peaking phenomenon. For the sake of comparison, we have also tried to solve this problem using a high-gain observer but it showed peaking phenomenon where $-30 \leq \hat{x}_3 \leq 180$ and $-180 \leq \hat{x}_4 \leq 30$ in the interval of t=[0,0.1] when $\epsilon = 0.01$ in the case without or with uncertainties. Even if compared with the results of [6] using nonlinear high-gain observer, the proposed method gets much better performance. The estimation in Fig.2 is simply to show the result in the presence of uncertainties and compare the result of [6]. Because the use of a high-gain observer for a robot manipulator shows peaking phenomenon either without or with uncertainties. In the paper, the analysis of the proposed observer in the existence of uncertainties is not dealt with. Note that our attention is restricted to overcome peaking phenomenon by using smaller observer gains than a high-gain observer through a cascade structure.

5. Conclusion

In this paper, we present a new observer, namely a cascade observer, for a general class of nonlinear SISO systems. Based on the cascade observer, velocity estimation for a robot manipulator with two degrees of freedom has been presented. The proposed observer features a cascade structure and adaptive observer gains. In doing so the cascade observer attempts to overcome some of the typical problems that may pose to a high-gain observer. As in the case of a high-gain observer, the cascade observer structure is simple and universal in the sense that it is independent of the system dynamics and parameters. The proposed observer doesn't require a information about the manipulator model at all. Also, the proposed observer doesn't cause a peaking phenomenon, differently from the high-gain observer. It is shown that the proposed cascade observer guarantees asymptotic stability of the estimation error. Research is ongoing to address robustness issues for the proposed observer in the presence of bounded disturbances and uncertainties.

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