

# A new study on cascading failures in power networks

Kei Shimizu<sup>†</sup>, *Non-member*, Masahiro Hayashi<sup>†</sup>, *Senior member*

**SUMMARY** This paper proposes a scheme to evaluate the impact of strategies against cascading failures in power networks. Specifically, we evaluate the impact of a model strategy for stopping power to users before a cascading failure reaches its final stage. We call such a strategy that takes action during the failure an “ongoing strategy” to distinguish it from preset strategies in place before the failure. Numerical examples demonstrate the effectiveness of our scheme.

**key words:** Cascading failure, Power networks, Earthquake, Bipartite graph, Survival power ratio.

## 1. Introduction

Power outages cause serious disruptions in our lives by causing stoppages in various services, including communications. While various mechanisms underlie power network outages, this paper focuses on a specific kind called a cascading failure that involves successive stoppages of equipment in the affected network.

As an example, a real cascading failure was experienced in Hokkaido, Japan in 2018 [1], whose mechanism is sketched below.

1. An earthquake caused an outage (trigger failure) at a power plant (trigger plant), which led to a shortage in power supplied to certain areas.
2. Other power plants in Hokkaido tried to back up the trigger plant’s functions.
3. These efforts led to stoppages at those plants as they became overloaded.
4. The above steps repeatedly occurred and resulted in a blackout of the whole Hokkaido area.

Ref. [1] proposed a model of this mechanism, a measure of impact, and a way to evaluate it. It also presented numerical examples to clarify strategies to help the affected area.

While the scheme proposed in ref. [1] can be applied to real cascading failures, it focuses on only “preset”

strategies that involve taking no action in the middle stages of a cascading failure. We should be able to apply “ongoing” strategies that do actions in its middle stages. Specifically, it might be effective to stop supplying power to users to avoid burdening power plants before the final stage is reached.

This paper proposes a new way to evaluate the effectiveness of such ongoing strategies against cascading failures similar to the Hokkaido case.

Numerical results suggest the following.

1. We should stop supplying power to an area whose power was supplied by the triggered power plant before the cascading failure started.
2. We should execute such stoppages very soon after the occurrence of a trigger failure.

## 2. Previous research

While there have been many researches [1]-[6] on cascading failures, ref. [1] claimed that the models of refs. [2]-[6] are not suitable for representing the cascading failure that occurred in Hokkaido in 2018, because the congestion described in ref. [2] does not occur in power networks and the models in refs. [3]-[4] are too complicated for the failure in Hokkaido, which was relatively simple. As well, the proposal in ref. [5] is an interaction model, whereas our target is a single network, and the one in ref. [6] assumes a discrete flow while we must consider a continuous one.

Accordingly, ref. [1] proposed a new model for representing the mechanism of cascading failures in power networks that is applicable to the real experience in Hokkaido, as well as a measure of impact and how to evaluate it. In this section, we explain the research of ref. [1] as preparation.

### 2.1 Key idea

The basic idea of the model in ref. [1] is as follows.

- (1) Users are classified into several areas (Fig. 1).

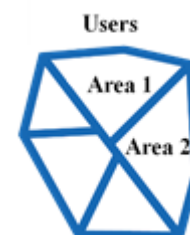


Fig. 1 Users and areas

<sup>†</sup>The author is with Tokyo City University, 1-28-1 Tamazutsumi, Setagaya-ku, Tokyo, 158-8557 Japan.

- (2) Each area connects to two or more power plants and these plants supply power to that area.
- (3) If a power plant has an outage, then the area this power plant power services sends requests for help to other connected power plants (Fig. 2).

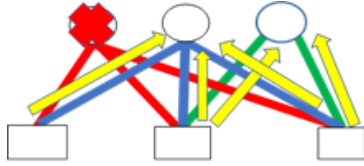


Fig. 2 Power-plant outage and requests for help.

In the figures of this paper, ellipses show power plants and rectangles show areas. Cross symbols show the outage and arrows show requests for help sent from the damaged areas to other power plants.

- (4) Each power plant that has received a request increases the rotational speed of its motor to supply power to the affected areas. However, too rapid a rotation causes the motor to stop to avoid a breakdown.
- (5) The iteration of the above stops results in a cascading failure.

2.2 Model

The model of ref. [1] is as follows. (Readers may refer the review on graph theory in ref. [7].)

1. The model is a bipartite graph consisting of two sets of nodes  $V_1$  and  $V_2$  and links between them. Nodes in  $V_1$  represent power plants, and nodes in  $V_2$  represent areas. Links represent power transmission lines between power plants and areas, where one end node for each link is in  $V_1$  and the other is in  $V_2$ .
2. A non-negative value (called a ‘rating’) is given to each node in  $V_1$ ; this value represents the maximum power that the corresponding power plant can supply.
3. A non-negative value (demand) is given to each node in  $V_2$ ; this value represents the power volume that the corresponding area must keep to avoid adversely affecting its users.
4. A non-negative number (conveyed power volume) is given to each link; this value is the power volume each link conveys.
5. The sum of conveyed power volumes of links connecting to a node in  $V_2$  equals the demand of the node.
6. The sum of conveyed power volumes of links connecting to a node in  $V_1$  must be equal to or less than the rating of the corresponding node.

An example of the model is illustrated in Fig 3. Ellipses show nodes in  $V_1$  and rectangles show nodes in  $V_2$ . Numbers denote ratings, conveyed power volumes, and demands.

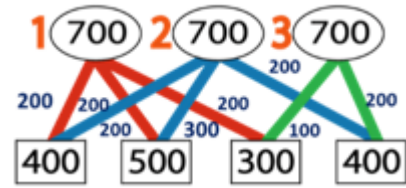


Fig. 3 Example model.

Mechanism of cascading failure

We delete a node in  $V_1$  (representing a trigger failure or another outage of a power plant) and repeat the following steps until we find no node to be deleted.

Step 1. We find or calculate the following two values for every node in  $V_2$ .

$A$  = Demand of this node

$B$  = Sum of conveyed power volumes of links connecting to this node

We call  $A - B$  the ‘power shortage’.

Step 2. For each node  $a$  in  $V_2$ :

If  $a$  connects to at least one node in  $V_1$ , give non-negative values (requested volume) to links connecting to  $a$  in  $V_1$  satisfying that the sum of the requested volumes of these links equals the power shortage of  $a$  (see Subsection 2.4).

else,

set the demand of  $a$  to 0.

Step 3. Do Substeps 3-1 to 3-2 for every node  $b$  in  $V_1$ .

Substep 3-1. Calculate  $C + D$  for each link connecting to  $b$ , where

$C$  = requested volume of this link

$D$  = conveyed power volume of this link

Execute  $D = C + D$ .

Substep 3-2. If the sum of  $D$ s of all links connecting to  $b$  > the rating of  $b$ , delete  $b$  from  $V_1$ .

We use word ‘stage’ to denote the time of a deletion of node at the top of this cascading failure mechanism.

The cascading failure mechanism is demonstrated in Fig. 3, 4, and 5. Fig. 3 shows the model before the cascading failure. Fig. 4 shows the stage of a trigger failure at power plant 2.

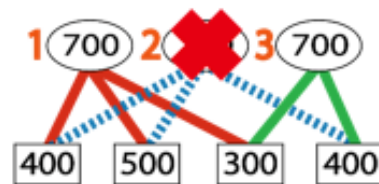


Fig. 4 Occurrence of a trigger failure.

Because of this trigger failure, 700 power is lost and the first area from the left sends a request of volume  $C = 200$  to power plant 1 and the second area from the left sends a request of volume  $C = 300$  to power plant 1. In Fig. 3,  $D$  of the link connecting to the first area from the left is 200 and  $D$  of the link connecting to the second area from the left is also 200. Accordingly, after the stage in Fig. 4,  $D$  of the

former link becomes  $200 + 200 = 400$  and  $D$  of the latter link becomes  $300 + 200 = 500$ .

The sum of the  $D$ s of the links connecting to power plant 1 becomes  $400 + 500 + 200 = 1100$ , and it is more than 700 rating of power plant 1. This overload causes a stoppage of power plant 1; the situation is depicted in Fig. 5.

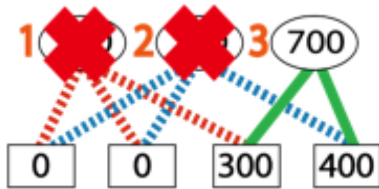


Fig. 5 Stoppage of power plants in a cascading failure.

The  $D$ s of links connecting to surviving power plant 3 become 300 and 400 such that  $300 + 400 \leq 700$ , which is the rating of plant 3. Therefore, no more nodes are deleted and Fig. 5 depicts the situation just after the final stage of this cascading failure.

### 2.3 Measure of impact

Ref. [1] defined a measure, denoted by  $S$ , of the impact of cascading failure in the entire power network:

$$S = \frac{\text{(Sum of demands of all areas after the cascading failure)}}{\text{(Sum of demands of all areas before the cascading failure)}}$$

Ref. [1] called this measure the ‘survival power ratio’. The survival power ratio for Fig. 5 is estimated as follows.

$$S = (300 + 400) / (400 + 500 + 300 + 400) = 0.4375$$

### 2.4 How to determine the requested volume

When an area sends a request for help to a power plant through links, requested volume of each link must be determined in Step 2 above. For example, Fig. 6 shows requested volumes ‘150 and 150’ sent on two links to power plants to recover a 300 power shortage in area  $a$ . While we have various alternatives such as ‘200 and 100’ or ‘300 and 0’. The experimental results in ref. [1] indicated that the best strategy is for the most powerful power plant to maximally help the affected area.

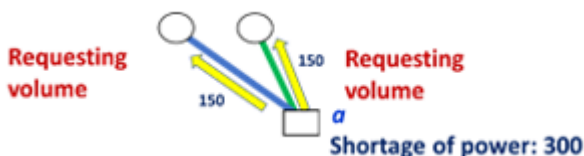


Fig. 6 Example of requested volumes.

### 3. The problem with the existing research

Ref. [1] assumed that the power-plant manager takes no action in the middle of a cascading failure. However, some actions to reduce the impact can be taken in the middle stages before the final stage. We call such actions ‘ongoing strategies’, while we call the strategies of ref. [1] ‘pre-set strategies’. Specifically, we focus on a strategy that stops supplying power to users before the final stage of a cascading failure. This strategy is similar to the one of cutting and moving trees (establishing a firebreak) around a wildfire before it spreads, as illustrated in Fig. 7.

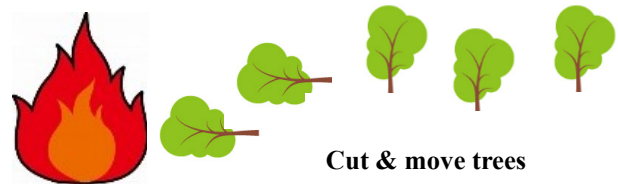


Fig. 7 Example of ongoing strategy to tackle a wildfire.

For example, if we stop supplying power to the second area from left in Fig. 4 just after the stage of this figure as in Fig. 8, the cascading failure stops before reaching the stage in Fig. 5.

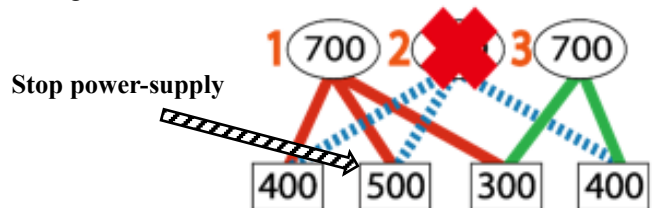


Fig. 8 Stopping the power of a link.

Ref. [1] does not consider such ongoing strategies.

### 4. Proposal

Let us change the way of dealing with the cascading failure explained in Subsection 2.2 to the one described below.

#### Change of mechanism.

At just after each stage, we select areas and delete all links connecting to them.

These deletions represent stoppages of power to the corresponding areas. If this selection and deletion is performed just after the  $n$ -th (chronological order) stage, we call it the ‘ $n$ -th cut’.

### 5. Numerical examples

We incorporated the mechanism described in the previous section in the software of ref. [1]. This section shows numerical examples obtained by this software.

The environment was as follows.

OS: Windows 10 Pro 64bit, Language: MATLAB 2019a,  
CPU: Intel® Core™ i7-6700, Memory: DDR4-2133 8GB

The target model was the same as in ref. [1], with eleven power plants denoted by  $A, B, C, D, E, F, G, H, I, J,$  and  $K, F$  being the trigger plant, and fifteen areas denoted by  $a, b, c, d, e, f, g, h, i, j, k, l, m, n,$  and  $o$ .

How to determine the requested volumes discussed in Subsection 2.4 is fixed to the best one clarified in ref. [1].

Fig. 9 shows the results of executing the 1<sup>st</sup> cut in each area. Vertical line denotes the survival power ratio, indicates the impact to the entire power network. Horizontal line denotes the area where the cut was executed. Fig. 10 shows the results of the 2<sup>nd</sup> cut.

Note that while our software can evaluate the impact of power stoppages in multiple areas, we execute a stoppage in a single area in these numerical examples for simplicity.

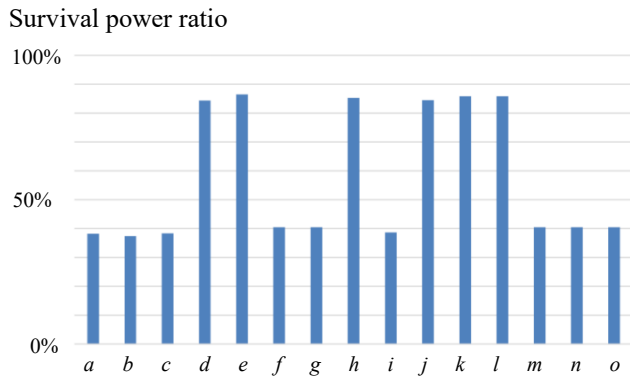


Fig. 9 Numerical results for the 1<sup>st</sup> cut.

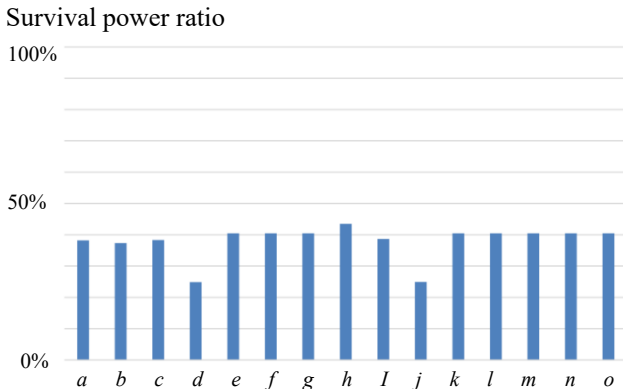


Fig. 10 Numerical results for the 2<sup>nd</sup> cut.

We obtained results very similar to Fig. 10 for the  $n$ -th cut with  $n = 3, 4, \dots$ .

The survival power ratio in areas  $c, e, h, j, k,$  and  $l$  are rather larger than in the other areas in Fig. 9, while not such large values are in Fig. 10. The power to these areas before the cascading failure was supplied by the trigger plant  $F$ .

These results present us with the following suggestions.

1. We should stop supplying power to an area supplied by the trigger power plant before the cascading failure starts.
2. We should stop supplying power in the very early stages after the trigger failure.

## 6. Conclusion

This paper proposed a new scheme to evaluate the impact of ongoing strategies against cascading failures in power networks. Specifically, we focused on strategies to stop supplying power to users in the middle stages of a cascading failure while the previous studies focused on pre-set strategies that take no action during the outage itself. Numerical results suggest that it is effective during the very early stages of a cascading failure to stop supplying power to an area whose power was supplied by the trigger plant before the failure started.

In the future, we will perform further numerical studies, including of power stoppages in multiple areas. Moreover, we will improve the model, such as by considering time-varying fluctuations in the demand for power in each area.

## Acknowledgement

We sincerely appreciate Mr. Tetsuhiro Kano for his support of our practices of the software used in this study.

## References

- [1] M. Hayashi, "A Study of cascading failure on power networks", ICETC, D4-2, 2020.
- [2] J. Hazra et al., "A network congestion management approach considering the risk of cascading failures", PEDES, INSPEC Accession Number: 11823839, 2010.
- [3] I. Simonsen et al., "Transient dynamics increasing network vulnerability to cascading failures", Physical Review Letters, vol. 100, pp. 218701-218705, 2008.
- [4] P. Dey et al., "Impact of topology on the propagation of cascading failure in power grid", IEEE Trans. Smart Grid, vol. 7, no. 4, pp. 1970-1978, 2016.
- [5] S. V. Buldyrev et al., "Catastrophic cascade of failures in interdependent networks", Nature, vol. 464, pp. 1025-1028, 2010.
- [6] M. A. Di Muro et al., "Cascading failures in interdependent networks with multiple supply-demand links and functionality thresholds", Nature Scientific Reports, vol. 7, 15059, 2017.
- [7] A. Bondy et al., Graph Theory, Springer, 2010.