A Statistical Model for Delay Domain Radio Channel Parameter Affected with Extreme Values

Hassan El-Sallabi*, Mohamed Abdallah*, Jean-Francois Chamberland+ and Khalid Qaraqe*
*Electrical and Computer Engineering Department, Texas A&M University at Qatar, Qatar
+Electrical and Computer Engineering Department, Texas A&M University, College Station, USA

Abstract – Root mean square (RMS) delay spread characterizes power delay profile of a radio channel and is sensitive to extremely delayed multipath components. This work presents a generalized extreme value (GEV) distribution to model this radio channel parameter. The present work presents characterization results that cover three different indoor environments; lecture hall, corridor and banquet hall.

I. Introduction

Multipath radio channel is a function of distribution of scatterers in radio environment. Radio channel characterization in different domains [1],[2] have been studied in literature for long time for different indoor and outdoor environment with different propagation conditions at different frequency bands and different environment scenarios such urban, suburban, rural for outdoor RF propagation and office, shopping malls, factories, hotels, etc for indoor RF propagation. Channel characterization is important to design a communication system to tackle channel impairments with single antenna or multiple antenna systems [3-5]. Due to complexity of multipath channels, there are still many issues that are not addressed. In this work, we address the impact of extreme values of channel characteristics on channel parameters. We present a statistical distribution that can model the RMS delay spread (RDS) as a parameter that is affected heavily by extreme values of the channel in terms of extremely delayed components.

II. Delay Spread Impacted by Extreme Values

The dispersion in delay domain is measured with RDS, which is a second central moment of a power delay profile. Extremely delayed multipath components relative to early arrival ones contribute significantly in RDS, which can be computed in as follows

$$\sigma_\tau = \sqrt{\frac{\int(\tau-\mu_\tau)^2 P(\tau)d\tau}{\int_{\tau=0}^{\infty} P(\tau)d\tau}}, \qquad \text{where} \qquad \mu_\tau = \frac{\int \tau P(\tau)d\tau}{\int_{\tau=0}^{\infty} P(\tau)d\tau}.$$

where μ_τ is the center of gravity, first central moment, $P(\tau)$ is power delay profile (PDP) of the radio channel. Figure 1 shows two different samples of PDPs with and without extreme delayed multipath cluster.

II. Generalized Extreme Value Distribution

The extreme value distribution arises from extreme values (maxima or minima) in sample data, which is unlike normal distribution that arises from central limit theorem on sample averages of data. The GEV distribution is a family of statistical distributions that combines Gumbel, Fréchet and Weibull statistical distributions. Extreme value theory originally is used as a framework to analyze the tail behavior of statistical distributions in different applications. In this work we use it to model channel parameters that are impacted by extreme values of the channel such extremely delayed components. The probability distribution function of the GEV distribution can be expressed [6] as

$$F(x;\sigma,\mu,k) = exp\left[-\left\{1+k\left(x-\frac{\mu}{\sigma}\right)\right\}^{1/k}\right],$$

for $\left\{1+k\left(x-\frac{\mu}{\sigma}\right)\right\} > 0$,where $\mu \in R$ is location parameter, k is called shape factor, and scaling parameter $\sigma > 0$. Based on value of *k,* different statistical distributions result, $k \neq 0$. When $k \to 0$, $k > 0$, and $k < 0$, the GEV distribution becomes Gumbel, Weibull and Fréchet distributions, respectively.

III. Numerical Results

Numerical Results are based on physics based model, which is in essence is based on work presented in [7],[8] and extending it to indoor propagation scenarios. In order to study the parameters of interest, we simulated three different indoor environments. They are 1) lecture hall with dimensions, height (H) =4 m, width (W) = 8 m, length (L) = 10 m, 2) corridor with dimensions: H = 4 m, W = 2 m, and L = 30 m, and 3) banquet hall with dimensions: H = 10m, W = 15 m, and L = 50. We simulated radio channel of IEEE802.11ac system, which its frequency range is 5 GHz and its bandwidth is 80 MHz. The antenna height of receiver, *i.e.* mobile client, is 1.7 m, which has a speed of 0.8 m/s. The height of transmitter, *i.e.* access point, is on ceiling. In addition to line of sight component, multiple specular reflections are included, where number of images per surface is assumed 6. The multiple reflection rays that result from different combinations of bouncing between walls, ceiling and ground result in long paths of reflection order more than six. The electrical relative permittivity of every reflecting surface is assumed to be 5, while the conductivity is assumed 0.02. The simulated temporal range is for one second for every spatial location. The temporal sampling rate is 26,000 samples/sec. The simulation is for a client station travelled a path starting from a horizontal distance

of 2 m from AP till 9.5 m with spatial resolution of about 2.5 cm for the three indoor environments. It is assumed that transmitter and receiver antennas are vertically polarized. The antenna pattern is omnidirectional, which has the well- known donut shape in its three dimensional pattern.

The empirical probability density function has been tested for fitting against different statistical distribution to select the best fitting parametric model of statistical distributions. The tested statistical distributions are normal, lognormal, exponential, gamma, logistic, loglogistic, uniform, weibull, extreme values, generalized pareto, generalized extreme value, inverse Gaussian, beta, Nakagami, and Rayleigh. The decision of selection the statistical distribution is based on results of likelihood value of maximum likelihood estimator for 95% confidence interval. Table I presents fit parameters of the GEV distribution. In the three indoor environments, the shape parameter (k) and scale parameter (σ) are pretty similar but the location parameter (μ) is less for corridor environment. The power delay profiles for the routes of three tested indoor environments are obtained with noise floor around -95 dBm, which is due to the large bandwidth. The PDPs are characterized by the RDS, which is computed as described earlier. It was found that the GEV distribution has the best fitting model with parameters presented in Table I. The pdf of the GEV distribution model with RDS data are presented in Figure 2 for the three tested indoor environments.

II. Conclusion

This work has presented the radio channel parameter RDS that is affected with extreme values of their channel characteristics. The extreme values in extremely delayed components have dramatic impact on computation of RDS. It has been shown that they can be modeled with GEV distribution for different indoor environments; lecture hall, corridor, and banquet hall.

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Table I. Parameters of GEV RDS

	K	μ (nsec)	σ (nsec)
Lecture Hall	-0.2	33	7.6
Corridor	0.04	71	23
Banquet Hall	-0.10	160	12

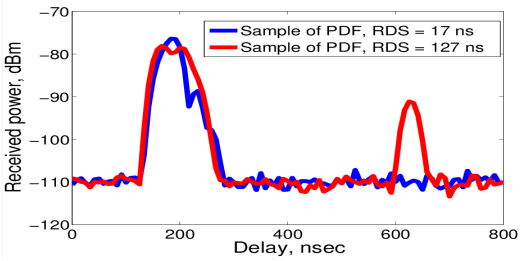

Figure 1. Samples of power delay profile and impact of extreme delayed multi-path components RDS.

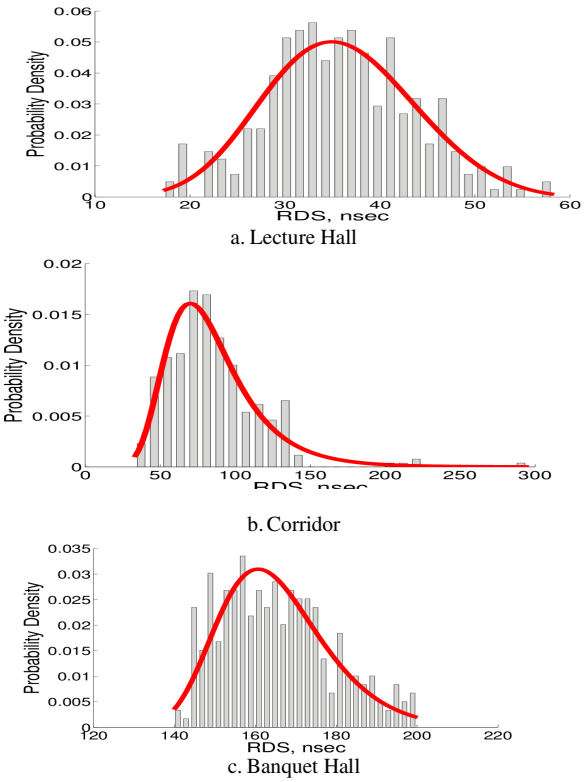

Figure 2. Probability density function of RDS for different indoor environments and their models with GEV distribution (red lines).