Design of Beam Shifter Based on the Schwarz-Christoffel mapping

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***Abstract* – In this paper, a beam shifter is proposed based on the Schwarz-Christoffel (SC) mapping, which transforms a traditional Maxwell's Fish-eye (MFE) lens to a slab beam shifter. The required refractive index distribution of the transformed slab is derived numerically from Jacobin matrix. The beam shifting effect is simulated with the full-wave software COMSOL Multiphysics for the slab with two waveguides at 180GHz. The proposed design is expected to be extended to other bands easily and also to be used as a wave component without circuit inside.**

***Index Terms* — Beam shifter, Maxwell's Fish-eye (MFE) lens, Schwarz-Christoffel (SC) mapping**

I. INTRODUCTION

Since Transformation optics (TO) was reported in 2006 [1, 2], it has been an appreciated design approach for electromagnetic and optical devices such as invisible cloak, high-gain antennas, enhancing imaging systems and so on. However TO derived from coordinate transformation always brings about anisotropic and inhomogeneous metamaterials which it is tough to implement. In order to make the TO-based devices available, a quasi-conformal TO (QCTO) as an important type of optimization techniques was introduced by Li and Pendry [3] in 2008, which can reduce the constructive parameters' anisotropy of metamaterials dramatically. Then many devices including the carpet cloaks and flattened Luneberg lenses [4] were realized with isotropic metamaterials through QCTO method.

Recently, the Schwarz-Christoffel (SC) mapping as a conformal transformation has been adopted to construct the mapping between two geometries, which can reduce the residual error in QCTO optimization process.

In this paper, SC mapping is employed to transform a conventional 2D conventional Maxwell's fish-eye (MFE) lens into a slab beam shifter. The conformal mapping is constructed between a circle region (the virtual domain) and a square region (the physical domain). The required refractive index distribution is derived numerically from Jacobin matrix. The beam shifting effect is verified by the full-wave simulation result

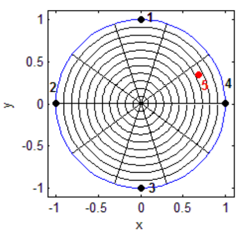
II. SC MAPPING AND REFRACTIVE INDEX DISTRIBUTION

In complex analysis, a SC mapping is a conformal transformation of the upper half-plane (Im z >0) onto the interior of a simple polygon, which can be used in minimal surfaces and fluid dynamics. Consider a polygon P in the complex plane w with vertices w_1, w_2, …, w_n. If the polygon has interior angles α_1, α_2, …, α_n, then SC mapping is given by

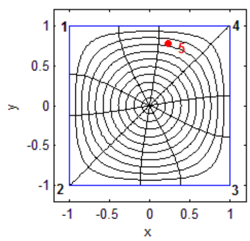
$$w = f(z) = B + A\int (z-z_1)^{\alpha_1/\pi-1}(z-z_2)^{\alpha_2/\pi-1}...(z-z_n)^{\alpha_n/\pi-1}dz \quad (1)$$

where A and B are suitably chosen constants determined by the size and position of P, and $z_1<z_2< \ldots <z_n$ are values, along the real axis of the z plane, of points corresponding to the vertices of the polygon P. In this paper, in order to transform the 2D circular MFE lens to a square one, the basic SC mapping has to be modified to construct a conformal mapping between a circular domain (the virtual domain) and a square domain (the physics domain).

Fig. 1 shows the modified SC mapping between the circular region and the square region. The interior of unit disk transformed from the upper half space displayed with ten uniformly-spaced concentric circles and ten uniformly-spaced radii in Fig. 1(a). And in Fig. 1(b), the transformed square is also centered at the origin with the mapped lines corresponding to the circles and radii in Fig. 1(a). The vertices 1, 2, 3 and 4 of the square marked counterclockwise are images of four points 1, 2, 3 and 4 of the unit disk respectively. When the origin and an arbitrary point 5 in the virtual domain are the reference points, there is an obviously space shift for the image point 5 in the physics domain due to the two different boundaries.



(a) Unit disk (the virtual domain)



(b) Square (the physical domain)

Fig. 1.The modified SC mapping with mesh lines.

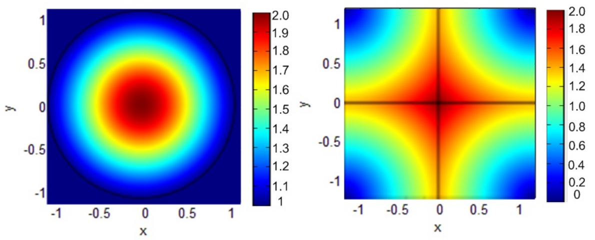
The conventional MFE lens as a gradient index (GRIN) lens, has a radial distribution of index which is expressed by

$$n(r) = \frac{n_0}{1 + (r/r_0)^2} \quad (2)$$

where n_0 is the index of refraction at the center of the lens and r_0 is the radius of the lens [5]. Since the wave impedance matching is required at the boundary of the lens $r = r_0$, the index n_0 is set as 2. Using the TO theory, the refractive index in the slab can be given by

$$n' = n/|dw/dz| \quad (3)$$

where n and n' denotes the refractive indexes of the virtual and physical spaces, respectively. The refractive index profiles of the virtual and physics domain are illustrated in Fig.2 (a) and (b). Compared with n ranging from 1 to 2, n' has a large value range from 0 to 2 and the indexes which are less than 1 are mainly located in the corner region of the square.

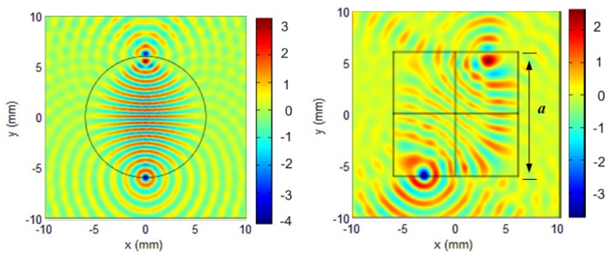


(a) Unit disk (the virtual domain) (b) Square (the physical domain)

Fig. 2.The refractive index distribution

III. Simulation and Analysis

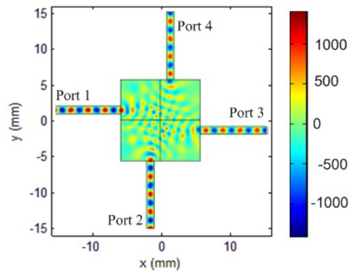
To explore the beam shift effect, a 2D conventional MFE lens with the radius r_0 = 6mm and the transformed slab with a = 12mm are designed. The electric field distributions of both them are simulated for 180GHz TE wave by full-wave software COMSOL Multiphysics in Fig. 3. For the virtual domain in Fig. 3(a), the index n inside the circle satisfies (1) while it equals to 1 outside the circle where it is free space. And for the physical domain in Fig. 3(b), the index inside the square n' is numerically calculated, and it is also free space outside the slab. There is an obviously space shift about 6mm between the object point and image point for the transformed slab.



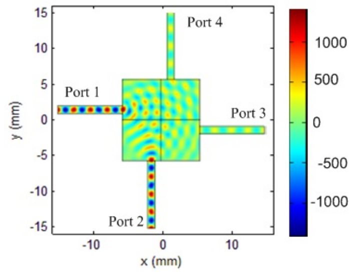
(a) Conventional MFE lens (b) Transformed slab

Fig. 3.The electric field distribution

When the waveguides WR-5 (1.30mm × 0.65mm) are connected to the transformed slab, there is a power transmission between the port 1 and port 3 with the space shift 3mm as same as the port 2 and port 4 while if the slab is not transformed which is only filled with air, then nearly all the wave is resonant in the slab and the power is almost hardly transferred. The electric field distributions in Fig.4 not only verify the beam shift effect but also indicate that the beam shifter based on SC mapping can be used in microwave or optic system as a component where the input and the output are not coaxial.



(a) Beam shifter with four ports



(b) Normal slab with four ports

Fig. 4.The electric field distribution of beam shifter and normal slab

IV. Conclusion

A beam shifter based on the SC mapping is proposed in this paper. The required refractive index is numerically derived for the conformal transformation from the circular region to the square region. The electric field distributions of the slab lens and the beam shifter with four ports verify the beam shift effect. The beam shifter can be believed to have a potential application in imaging system and transceiver system.

Acknowledgment

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