Proceedings of ISAP 2014, Kaohsiung, Taiwan, Dec. 2-5, 2014

A new effective SC-PML Implementation for WLP-FDTD Method in Plasma medium

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***Abstract*—An unsplit-field and stretched coordinate (SC) based perfectly matched layer (PML) is presented for Weighted-Laguerre-based finite-difference time-domain (WLP-FDTD) method. The proposed SC-PML is used to truncate plasma medium. A numerical example is included to demonstrate the performance of these proposed formulations.**

***Index Terms* —Weighted Laguerre polynomials (WLPs), finite difference time-domain (FDTD), Plasma.**

I. INTRODUCTION

The finite-difference time-domain (FDTD) method has been widely used to simulate the wave propagation in dispersion medium since its easy implementation. However, the well-known Courant-Friedrichs-Levy (CFL) stability condition constrains the application of conventional FDTD method for the simulation of structure with fine geometries. To overcome these limitations, unconditionally stable FDTD method has been developed, such as alternating direction implicit (ADI) FDTD [1], locally one dimensional (LOD) FDTD [2], WLP-FDTD [3] method. Among them, the WLP-FDTD method not only removes the CFL stability restriction, but also avoids the numerical dispersion error of the ADI-FDTD with the increase of the time-step [4].

The perfectly matched layer, introduced by Berenger, has been widely used for truncating FDTD computational domains[5]. The original formulation is based on splitting field. From then on, different unsplit-field PML implementations have been presented for FDTD method, such as uniaxial PML (UPML) [6] and SC-PML [7, 8]. Among the various implementations of the PML, the SC-PML has the advantage of simple implementation in the corners and edges of the PML regions and is independent of background medium. Recently, a split-field PML [9] based on Berenger's original formulation was employed within the WLP-FDTD formulation and so as the UPML [10].

In this letter, a perfectly matched layer boundary condition based on the stretched coordinate is presented for WLP-FDTD method. The proposed PML avoids field splitting and is easy to be implemented for dispersion medium. Then the SC-PML is used to truncate the plasma lattices. Numerical results show the effectiveness of the proposed PML algorithm.

II. FORMULATION

Using the stretched coordinate PML formulation and considering the kinetic equations for cold electron plasma, the field equations for a TEMz wave propagation in one-dimensional plasma medium can be written as

$$-\frac{1}{S_z}\frac{\partial H_y}{\partial z}=\varepsilon_0\frac{\partial E_x}{\partial t}-en_e u_{ex}+J_D \tag{1.a}$$

$$-\frac{1}{S_z}\frac{\partial E_x}{\partial z}=\mu_0\frac{\partial H_y}{\partial t} \tag{1.b}$$

$$\frac{\partial u_{ex}}{\partial t}=-\frac{e}{m}E-vu_{ex} \tag{1.c}$$

where n_e is the density of the electron, v is the collision frequency, u_{ex} is the electron velocity, m is the electron mass, e is the electron charge, J_D is excitation source, S_z is the coordinate-stretching variable defined as

$$S_z=1+\frac{\sigma_z}{j\omega\varepsilon_0} \tag{2}$$

where σ_z is the PML conductivity profile along the z-direction. In this letter, the PML parameter are scaled following the expression [6].

Introducing the following auxiliary variables

$$H_{yz}=1/S_z\cdot\partial H_y/\partial z\quad E_{xz}=\frac{1}{S_z}\frac{\partial E_x}{\partial z} \tag{3}$$

and noting the $j\omega\rightarrow\partial/\partial t$, (3) can be written in the time domain as

$$\partial H_{yz}/\partial t+\sigma_z H_{yz}/\varepsilon_0=\partial\left(\partial H_y/\partial t\right)/\partial z \tag{4}$$

$$\partial E_{xz}/\partial t+\sigma E_{xz}/\varepsilon_0=\partial\left(\partial E_x/\partial t\right)/\partial z \tag{5}$$

With reference to Chung [3], the equations (1), (4) and (5) are expanded using a entire-domain temporal basis function . For example, (4) can be expanded as

$$\begin{aligned}&s\sum_{p=0}^{\infty}\left(0.5H_{yz}^{p}(z)+\sum_{m=0,p>0}^{p-1}H_{yz}^{m}(z)\right)\varphi_p(\bar{t})+\frac{\sigma}{\varepsilon_0}\sum_{p=0}^{\infty}H_{yz}^{p}(z)\varphi_p(\bar{t})\\&=s\sum_{p=0}^{\infty}\left(0.5\frac{\partial H_y{}^{p}(z)}{\partial z}+\sum_{m=0,p>0}^{p-1}\frac{\partial H_y{}^{m}(z)}{\partial z}\right)\varphi_p(\bar{t})\end{aligned} \tag{6}$$

Eliminating basis function $\varphi_p(\bar{t})$ by Garlerkin temporal testing procedure, we can get

$$H_y^q|_{i+1/2}=-2\sum_{k=0}^{q-1}H_y^k|_{i+1/2}-C_{2z,i+1/2}\left[\frac{\left(E_x^q|_{i+1}-E_x^q|_i\right)}{\Delta z_{i+1/2}}+2\hbar^{q-1}\left(E_{xz}|_{i+1/2}\right)\right] \tag{7}$$

$$u_{ex}{}^{q}\Big|_i=-c_{3z,i}E_x{}^{q}\Big|_i-c_{4z,i}\sum_{m=0,q>0}^{q-1}u_{ex}{}^{m}\Big|_i \tag{8}$$

$$H_{yz}^{q}\big|_{i}=s\varepsilon C_{1z,i}\left[\frac{H_{y}^{q}\big|_{i+1/2}-H_{y}^{q}\big|_{i-1/2}}{2\Delta z_{i}}+\hbar^{q-1}\left(H_{yz}\big|_{i}\right)\right] \quad (9)$$

$$E_{xz}^{q}\big|_{i+1/2}=s\varepsilon_{0}C_{1z,i+1/2}\left[\frac{E_{x}^{q}\big|_{i+1}-E_{x}^{q}\big|_{i}}{2\Delta z_{i+1/2}}+\hbar^{q-1}\left(E_{xz}\big|_{i+1/2}\right)\right] \quad (10)$$

where q is the order of the weighted Laguerre polynomials,

$$\hbar^{q-1}\left(H_{yz}\big|_{i}\right)=\sum_{k=0}^{q-1}\frac{\partial H_{y}^{k}}{\partial z}\big|_{i}-\sum_{k=0}^{q-1}H_{yz}^{k}\big|_{i},$$

$$\hbar^{q-1}\left(E_{xz}\big|_{i+1/2}\right)=\sum_{k=0}^{q-1}\frac{\partial E_{x}^{k}}{\partial z}\big|_{i+1/2}-\sum_{k=0}^{q-1}E_{xz}^{k}\big|_{i+1/2},$$

$$C_{1z}=1/\left(s\varepsilon_{0}0.5+\sigma_{z}\right),C_{2z}=1/\left(s\mu_{0}0.5+\sigma_{z}\mu/\varepsilon\right),$$

$$C_{3z}=\frac{2e}{m\left(s+2v\right)},C_{4z}=\frac{2s}{s+2v}.$$

After some manipulation, the SC-PML update equation for E_x can be obtained as:

$$-\frac{C_{1z,i}}{\Delta z_{i}}\frac{C_{2z,i-1/2}}{\Delta z_{i-1/2}}\left(E_{x}^{q}\big|_{i-1}\right)-\frac{C_{1z,i}}{\Delta z_{i}}\frac{C_{2z,i+1/2}}{\Delta z_{i+1/2}}\left(E_{x}^{q}\big|_{i+1}\right)$$
$$+\left(1+\frac{C_{1z,i}}{\Delta z_{i}}\frac{C_{2z,i+1/2}}{\Delta z_{i+1/2}}+\frac{C_{1z,i}}{\Delta z_{i}}\frac{C_{2z,i-1/2}}{\Delta z_{i-1/2}}+\frac{2en_{e}c_{4,i}}{\varepsilon_{0}s}\right)E_{x}^{q}\big|_{i}$$
$$=-2\sum_{k=0}^{q-1}E_{x}^{k}\big|_{i}-\frac{C_{1z,i}}{\Delta z_{i}}\left(-2\sum_{k=0}^{q-1}H_{y}^{k}\big|_{i+1/2}-2C_{2z,i+1/2}\hbar^{q-1}\left(E_{xz}\big|_{i+1/2}\right)\right)$$
$$-\frac{C_{1z,i}}{\Delta z_{i}}\left(+2\sum_{k=0}^{q-1}H_{y}^{k}\big|_{i-1/2}+2C_{2z,i-1/2}\hbar^{q-1}\left(E_{xz}\big|_{i-1/2}\right)\right)$$
$$-2C_{1z,i}\hbar^{q-1}\left(H_{yz}\big|_{i}\right)+\frac{2J_{D}^{q}}{\varepsilon_{0}s} \quad (11)$$

It is clear that the left hand side in (11) forms a tri-diagonal matrix, which can be solved efficiently with the approach presented in [11]. Once E_x is obtained, (7)~(10) can be updated explicitly.

III. NUMERICAL STUDY

A one dimension numerical example is used to verify proposed PML absorber. The total WLP-FDTD domain contains 200 grids with each size $\Delta z=150\,\mu m$. Both ends of the computational domain are terminated with 15 grids PML. The plasma occupied 169-199 grid, with n_e=1.0231×10^{19} m^{-3}, v=2×10^{10} rad/s. The other grids are free space.

The excitation source located at grid 100 is defined as a differential Gaussian pulse given by

$$J_{x}(t)=\left(t-t_{0}\right)/\tau\times\exp\left(-\left(t-t_{0}\right)^{2}/\tau^{2}\right) \quad (11)$$

where $t_0=0.01ns$ $\tau=0.05ns$. We chooses the order of the weighted Laguerre polynomials $q=300$, and the timescale factor $s=2\times10^{11}$. The WLP-FDTD takes a time step of Δt = 2.5 ps, such that the CFL number is 5.The total time duration is $T_f=2.4\,ns$. Fig. 1 illustrates the plasma reflection coefficient calculated by WLP-FDTD and analytical method. It can be seen that the result of the proposed algorithm agree with that of the analytical method very well.

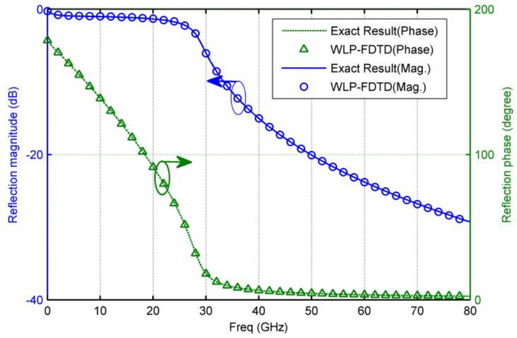

Fig. 1 Reflection coefficients calculated by WLP-FDTD and analytical method

IV. CONCLUSION

We presented an unsplit-field and stretched coordinate based perfectly matched layer for WLP-FDTD in plasma medium. Numerical results show the effectiveness of the proposed PML algorithm. In a similar manner, the formulation can be extended to two dimensions and other types of dispersive medium.

ACKNOWLEDGMENT

This work was supported in part by the foundation of State Key Laboratory under Grant 9140C530404130C53193 and the Ph.D. research startup foundation of Xi'an University of Technology under Grant 105-211410.

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