

An Efficient MHz Range Analysis by ARMA-FDTD Method with AIC

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Abstract— MHz frequency range is often used for wireless power transmission (WPT), antenna engineering and so on. The FDTD method is one of the powerful tools in EM field. In the FDTD method, small cell size is required to model complex geometry such as human body. Therefore, the FDTD method needs long calculation time in MHz range analysis, because the time step of the FDTD method is should be small with cell size reducing. Furthermore, time period of MHz range becomes long. On the other hand, ARMA (Autoregressive moving average model)-FDTD method is effective to reduce calculation time. In this research, ARMA-FDTD is applied to MHz range analysis to reduce calculation time. Furthermore, AIC (Akaike Information Criterion) is utilized to determine number of coefficients in ARMA-FDTD method.

Keywords—FDTD; ARMA; AIC; MHz

I. INTRODUCTION

In recent years, MHz range usages are widely spreading. The application of MHz range are wireless power transmission (WPT) engineering, antennas engineering and others. Therefore, analyzing antennas and electromagnetic fields in MHz range are needed. The FDTD method is widely used to analyze MHz frequency range. The FDTD method solves the Maxwell's equations within the unit cell in time domain. In the FDTD analysis, quite small cell size is needed with complexity of analysis object. However, time step becomes smaller with cell size[1] as follows.

$$\Delta t \leq 1/c \sqrt{\left(\frac{1}{\Delta x}\right)^2 + \left(\frac{1}{\Delta y}\right)^2 + \left(\frac{1}{\Delta z}\right)^2} \quad (1)$$

Where Δt is time step, Δx , Δy , Δz are cell sizes, and c is 3.0×10^8 [m/s]. Furthermore, long calculation time is needed due to long time period of MHz range. For example, to analyze the problem of 10MHz frequency, large number of time steps are required. In this case, time period of 10MHz is 0.1 μ sec, on the other hand, time step should be 20psec if 5mm cell size is used.

In this research, ARMA-FDTD is applied to MHz range analysis to reduce calculation time. On the other hand, the accuracy of the ARMA-FDTD method depends on number of coefficient of ARMA. In this paper, the AIC (Akaike Information Criterion) is utilized to determine number of coefficients in ARMA. The analysis model of this paper is human head size layered dialectic sphere.

II. ARMA-FDTD

In this paper, the ARMA is used to reduce calculation time of FDTD method for analysis of antenna impedance in MHz range. In ARMA analysis, function $H(z)$ that is linear model [2] is used. The transfer function $H(z)$ is expressed by Eq.(2).

$$H(z) = \frac{a_0 + a_1 Z^{-1} + a_2 Z^{-2} + \dots + a_q Z^{-q}}{1 + b_1 Z^{-1} + b_2 Z^{-2} + \dots + b_p Z^{-p}} \quad (2)$$

Where z is complex variable, a and b are unknown coefficients. To obtain transfer function $H(z)$, unknown coefficients a and b should be obtained. The transfer function $H(z)$ can also be expressed in the time domain as

$$y(n) = -\sum_{i=1}^p b_i y(n-i) + \sum_{j=0}^q a_j x(n-j) \quad (3)$$

Where $x(n)$ and $y(n)$ are input and output signals of the linear system. b_i and a_j are ARMA coefficients to be determined. In the FDTD method such as a dipole antenna impedance, an incident electric current is used as the input signal $x(n)$ and an incident voltages are used as output signal $y(n)$. In order to obtain b_i and a_j , N th samples are required in FDTD calculation. Eq.(3) can be expressed linear equation system as follows.

$$[Y]_{N \times 1} = [D]_{N \times (p+q+1)} [C]_{(p+q+1) \times 1} \quad (4)$$

Where $[Y]$ is the output vector, $[C]$ is the coefficient vector, and $[D]$ is a matrix generated by the input and output data as expressed bellows.

$$[Y] = [y(1), y(2), \dots, y(N)] \in R^{N \times 1} \quad (5)$$

$$[C] = [b_1, \dots, b_p, a_0, \dots, a_q] \in R^{(p+q+1) \times 1} \quad (6)$$

$$[D] = \begin{bmatrix} 0 & 0 & \dots & 0 & x(1) & 0 & \dots & 0 \\ -y(1) & 0 & \dots & 0 & x(2) & x(1) & \dots & 0 \\ \vdots & \ddots & \ddots & 0 & \vdots & \ddots & \ddots & 0 \\ -y(p) & -y(p-1) & \dots & -y(1) & x(p+1) & x(p) & \dots & x(p+1-q) \\ -y(p+1) & -y(p) & \dots & -y(2) & x(p+2) & x(p+1) & \dots & x(p+2-q) \\ \vdots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \ddots \\ -y(N-1) & -y(N-2) & \dots & -y(N-p) & x(N) & x(N-1) & \dots & x(N-q) \end{bmatrix} \in R^{N \times (p+q+1)} \quad (7)$$

$[C]$ is the unknown coefficient to be determined and the number of unknowns is $p+q+1$. When the sampling number N is larger than $p+q+1$, the coefficients can be determined using the least mean square error estimation.

$$[C] = ([D]^T [D])^{-1} [D]^T [Y] \quad (8)$$

Once $[C]$ is determined, the frequency domain transfer function $H(j\omega)$ can be obtained from Eq.(2) by replacing Z with $\exp(j\omega)$.

III. Determination of number of coefficients using AIC

Optimal number of coefficients of ARMA can be determined by using AIC[4]. Eq.(9) is AIC equation to determine number of coefficients of ARMA. In AIC model, the optimal number of coefficients is determined when the value of AIC becomes the minimum.

$$AIC = N \log \hat{\sigma}_e + 2(p + q + 1) \quad (9)$$

Where N is number of samples. $\hat{\sigma}_e$ is dispersion of prediction error which is calculated by (10).

$$\hat{\sigma}_e = \frac{1}{N - p} \frac{\sum_{n=p+1}^N \left(x_n + \sum_{i=1}^p a_i x_{n-i} \right)^2}{\left(1 + \sum_{j=1}^q b_j^2 \right)} \quad (10)$$

On the other hand, dipole antenna's equivalent circuit can be express Fig.1.

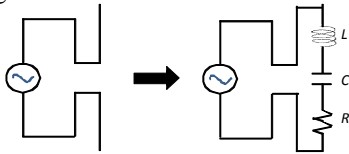


Fig.1 Equivalent circuit of dipole antenna

Equivalent circuit of input impedance of dipole antenna is expressed by (11). Therefore, the number of p , q should be $p=q-1$ to express impedance of dipole antenna, correctly.

$$Z_{dipole} = \frac{1 + j\omega R + (j\omega)^2 L}{j\omega C} \quad (11)$$

IV. ANALYSIS RESULT

The analysis models is shown in Fig.2. This model has 3 layers as skin, born and brain. At first, ordinary FDTD method is used to analyze input impedance of dipole antenna. In this calculation, 1500 time steps are required to obtain correctly result. Next, same model is calculated by using ARMA-FDTD method. In this calculation, number of p is set as $p=q-1=62$, this number is determined by AIC. Figure 3(a)(b) are shown calculated input impedance of dipole antenna. The number of time steps of ARMA-FDTD method is 300 steps. This number of time steps is correspond to 1/5 of ordinary FDTD method. To compare our proposed ARMA model ($p=q-1$) and usual ARMA model ($p=q$), we calculated same model by ARMA-FDTD with $p=q=57$. This number of p is determined by AIC. From results, our proposed method ($p=q-1$) is more correctly than ordinary ARMA-FDTD ($p=q$). Next, we calculated the same model by increasing number of p to improve the calculation accuracy, because AIC gives the minimum number of p to calculate appropriately. Figure 4(a)(b) are shown calculated input impedance of dipole antenna by using number of p is $p=q-1=93$. The number of

time steps is 300 steps in this calculation. The accuracy is improved by increasing number of p .

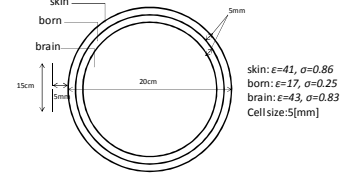
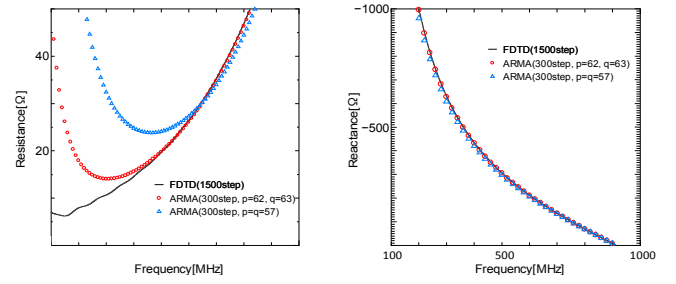


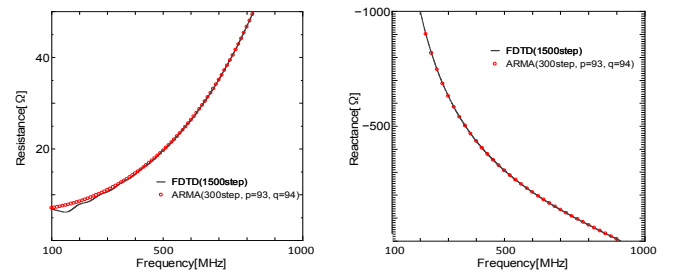
Fig.2 Analysis model



(a)Resistance

(b)Reactance

Fig.3 Dipole antenna impedance



(a)Resistance

(b)Reactance

Fig.4 Dipole antenna impedance($p=93$, $q=94$)

V. CONCLUSION

In this paper, we applied ARMA-FDTD to MHz frequency range analysis. Moreover, necessary number of coefficients of ARMA is estimated by AIC. Applying AIC to ARMA-FDTD method, the few time steps ARMA-FDTD results are good agreement with ordinary large time steps FDTD method result.

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