

An Approximate Method of Obtaining Surface-wave Band Gap from Reflection Phase Plots for Mushroom Structures

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Abstract – This paper proposes an approximate method of obtaining the surface-wave band gap from reflection phase plots for mushroom structures. The upper bound of the band gap can be found by observing the first in-phase frequency on the reflection phase plot of TE plane waves under grazing incidence, and the lower bound of the band gap can be approximated by the three specific frequencies on reflection phase plots of TM plane waves: the first in-phase frequencies under normal and grazing incidence and the frequency where the first $+90^\circ$ reflection phase occurs. By comparing surface-wave band gaps obtained from dispersion plots and from reflection phase plots through the approximate method in the commercial software CST, results show that the errors of estimated values are less than 10% in different cases of mushroom structures.

Index Terms — Periodic structures, artificial magnetic conductors (AMCs), electromagnetic band gap (EBG) structures, EM wave theory and modeling, antennas.

I. INTRODUCTION

In antenna areas, taking periodic structures as ground planes has two possible advantages for designing low-profile antennas. The first advantage is the AMC's characteristic which can resolve the limitation of one-quarter wavelength from ground planes in in-phase bands. The second advantage is the EBG's characteristic which can suppress surface waves in surface-wave band gaps.

In [1], Sievenpiper et al. introduce a periodic structure so-called mushroom structure which possesses both AMC's and EBG's properties. Additionally, they experimentally reported that the in-phase band overlapped with the surface-wave band gap for mushroom structures. Although later research, such as [2, 3, 4], shows that these two bands do not necessarily overlap, the thought of connecting the two bands gives us an idea to think whether it is possible to obtain information of in-phase bands and surface-wave band gaps on only one plot. If possible, we can reduce the time on simulation or on calculation through circuit models of mushroom structures [5, 6] for obtaining the two-band information.

Hence, the purpose of this report is to modify a method to find the surface-wave band gap from reflection phase plots for mushroom structures. The results indicate that we can take the first in-phase frequency of TE grazing incidence as the upper bound and use the approximate formula with three specific frequencies on reflection phase plots of TM plane waves to find the lower bound. This method may not only

help us spend less time looking for the AMC's and EBG's properties for mushroom structures but also find possible relationship of the two bands in the future.

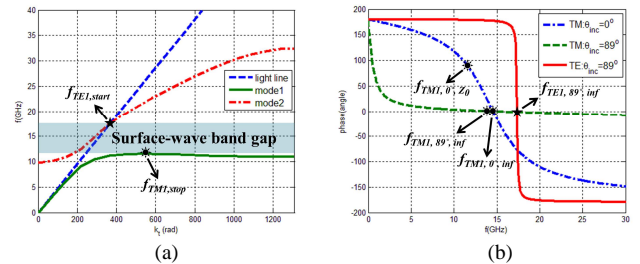


Fig. 1 (a) The surface-wave band gap on a dispersion plot (O-X part only) and (b) the required data on reflection phase plots under normal incidence ($\theta_{inc} = 0^\circ$) and grazing incidence ($\theta_{inc} = 89^\circ$) for the approximate method.

II. THE UPPER BOUND OF SURFACE-WAVE BAND GAP

The upper edge of the surface-wave band gap is usually the cutoff frequency of the first TE surface wave, $f_{TE1,start}$, which occurs near the right side of the light line on dispersion plots (see Fig. 1 (a)). The light line ($k_t = k_0$) is the interface between the plane-wave region ($k_t < k_0$) and the surface-wave region ($k_t > k_0$). k_0 is the wave number in free space, and k_t is the tangential wave number to periodic surfaces.

According to the transverse resonance condition [5, 8], for surface waves ($k_t > k_0$), the TE surface impedance of mushroom structures, $Z_{inp,TE}(k_t, k_0)$, near the light line should approach infinity; that is, $Z_{inp,TE}(k_t \rightarrow k_0^+, k_0 = k_{TE1,inf}) \rightarrow +j\infty$. $k_{TE1,inf}$ is the wave number where TE surface impedance approaches infinity as $k_t \rightarrow k_0^+$. If we assume that the TE surface impedance is continuous at $k_t = k_0$, $Z_{inp,TE}(k_t \rightarrow k_0^-, k_{TE1,inf})$ can be equal to $Z_{inp,TE}(k_t \rightarrow k_0^+, k_{TE1,inf})$. Since $Z_{inp,TE}(k_t \rightarrow k_0^-, k_{TE1,inf})$ is able to be observed in the plane-wave region, we can obtain the cutoff frequency of the first TE surface wave by finding the in-phase frequency ($Z_{inp,TE} \rightarrow +j\infty$) on the reflection plot of TE incident waves under grazing incidence ($k_t \rightarrow k_0^-$) (see $f_{TE1,89}, inf$ in Fig. 1 (b))

III. THE LOWER BOUND OF SURFACE-WAVE BAND GAP

The lower edge of the surface-wave band gap is the stop frequency, $f_{TM1,stop}$, where the first TM surface wave stops propagating (see Fig. 1(a)). The reason that TM surface

waves stops is that TM surface waves should exist on an inductive surface [1]. Therefore, when frequencies increase, and the TM surface impedance of mushroom structures become capacitive, TM surface waves will stop propagating. The interface where surface impedances change from inductance to capacitance is an in-phase frequency, so the first in-phase frequency becomes an important key to find $f_{TM1,stop}$. However, in-phase frequencies may be changed by k_t , and we only can observe movements of in-phase frequencies as $k_t < k_0$ on reflection phase plots. Hence, we have to use approximation to describe the TM surface impedance in the surface-wave region ($k_t > k_0$) from the plane-wave region. The idea from pole-zero methods [7] and circuit models of mushroom structures [5, 6] is used to derive an approximate formula for finding lower edge. The first step in obtaining the formula is to introduce a TM simplified surface impedance, $Z_{imp,TM}(k, k_t)$ observed from TM reflection phase plots.

$$Z_{imp,TM}(k, k_t) = jZ_{imp0,TM}k / (k - k_{TM1,inf}(k_t)) \quad (1)$$

where $k_{TM1,inf}(k_t) = mk_t + k_{TM1,0^{\circ},inf}$, and $k_{TM1,0^{\circ},inf}$ is the wave number of the first in-phase frequency for TM normal incidence. k can represent the wave number in air or in mushroom structures because effects of the different regions (ex: dielectric) will be cancelled in the final expression. Since the in-phase frequencies may be changed by k_t , we use linear approximation with the first in-phase frequencies of normal ($k_t = 0$) and grazing incidence ($k_t \rightarrow k_0$); that is, $f_{TM1,0^{\circ},inf}$ and $f_{TM1,89^{\circ},inf}$ in Fig. 1(b). m is the slope of $k_{TM1,inf}(k_t)$, and $Z_{imp0,TM}$ is a scaling factor which can be obtained by substituting the frequency where reflection phase is 90° under normal incidence, $f_{TM1,0^{\circ},Z_0}$ (see Fig. 1(b)), into (1) because $f_{TM1,0^{\circ},Z_0}$ is also the frequency where the TM surface impedance equals the intrinsic impedance in air, Z_0 .

Using the transverse resonance condition with the simplified TM wave impedance $Z_{TM,air} \approx -jZ_0k_t / k$ (assume $k_t \gg k$), we can derive the formula for the lower edge of the band gap, $f_{lower,app}$, shown below.

$$f_{lower,app} = \frac{f_{TM1,0^{\circ},inf} (2\sqrt{(1-\alpha)(1-\beta)} - 1)}{4(1-\alpha)(1-\beta) - 1} \quad (2)$$

where $\alpha = f_{TM1,0^{\circ},inf} / f_{TM1,89^{\circ},inf}$, $\beta = f_{TM1,0^{\circ},inf} / f_{TM1,0^{\circ},Z_0}$. The three frequencies for the formula are shown in Fig. 1 (b).

IV. NUMERICAL RESULT

Table I shows the parameters (unit: mm) of mushroom structures for different cases, and Table II shows surface-wave band gaps (unit: GHz) obtained from dispersion plots (f_{upper} and f_{lower}) and reflection phase plots with the approximate formula ($f_{TE1,89^{\circ},inf}$ and $f_{lower,app}$) in CST. For the upper edge, we observe the grazing incident angle 89° ($k_t \approx k_0$) on reflection phase plots, and the results indicate that errors

are small, which supports the assumption made in II. For the lower edge, errors of the formula are smaller than 10%.

TABLE I
PARAMETERS OF MUSHROOM STRUCTURES

Case	Period	Patch width	Height	Pin radius	ϵ_r
1	2.4 mm	2.25 mm	1.6 mm	0.18 mm	2.2
2	2 mm	1.8 mm	1 mm	0.05 mm	10.2
3	6 mm	5.9 mm	1.6 mm	0.2 mm	4.4
4	7.3 mm	7.0 mm	1.5 mm	0.5 mm	2.65
5	10 mm	9.6 mm	3.08 mm	0.125 mm	2.33
6	4.8 mm	4.5 mm	1.6 mm	0.2 mm	2.2

TABLE II
COMPARISON TABLE FOR SURFACE-WAVE BAND GAPS

Case	f_{upper}	$f_{TE1,89^{\circ},inf}$	Error [*]	f_{lower}	$f_{lower,app}$	Error
1	17.69	17.36	-1.7 %	11.49	12.15	6.9 %
2	12.30	12.24	-0.5 %	9.21	9.64	5.0 %
3	6.25	6.34	1.4 %	4.37	4.77	8.8 %
4	8.82	8.71	-1.3 %	5.80	6.24	7.4 %
5	5.40	5.40	0 %	3.10	3.33	7.7 %
6	12.44	12.09	-2.8 %	7.51	7.94	6.2 %

*Error(%) = $(f_{TE1,89^{\circ},inf} - f_{upper}) / f_{upper} \times 100\%$

V. CONCLUSION

In this report, an approximate method is provided for obtaining the surface-wave band gap from reflection phase plots for mushroom structures. By observing the first in-phase frequency of TE plane waves at grazing angles, the upper bound can be obtained, and by using the approximate formula, the lower bound can be estimated. The results show that errors are lower than 10 %; thus, if errors are allowed for designs, we can find two bands faster, and give further thought about two-band relationship for mushroom structures.

REFERENCES

- [1] D. Sievenpiper, L. Zhang, R. F. J. Broas, N. G. Alexopoulos, and E. Yablonovich, "High-impedance electromagnetic surfaces with a forbidden frequency band," *IEEE Trans. Microw. Theory Tech.*, vol. 47, pp. 2059–2074, 1999.
- [2] S. Clavijo, R. E. Diaz, and W. E. McKinzie, "Design methodology for Sievenpiper high-impedance surfaces: an artificial magnetic conductor for positive gain electrically small antennas," *IEEE Trans. Antennas Propag.*, vol. 51, no. 10, pp. 2678 – 2690, Oct. 2003.
- [3] S. Raza, "Characterization of the Reflection and Dispersion Properties of 'Mushroom'-Related Structures and Their Application to Antennas." M.A.S thesis, University of Toronto, Ontario, Canada, 2012.
- [4] L. Li, Q. Chen, Q. Yuan, C. Liang, and K. Sawaya, "Surface-wave suppression bandgap and plane-wave reflection phase band of mushroomlike photonic bandgap structures," *J. Appl. Phys.*, vol. 103, no. 2, Jan.2008.
- [5] O. Luukkonen, C. Simovski, G. Granet, G. Goussetis, D. Lioubtchenko, A. V. Raisanen, and S. A. Tretyakov, "Simple and accurate analytical model of planar grids and high-impedance surfaces comprising metal strips or patches," *IEEE Trans. Antennas Propag.*, vol. 56, no. 6, pp. 1624–1632, Jun. 2008.
- [6] O. Luukkonen, A. B. Yakovlev, C. R. Simovski, and S. A. Tretyakov, "Comparative study of surface waves on high-impedance surfaces with and without vias," in *Proc. IEEE AP-S Int. Symp.*, Jul. 2008.
- [7] S. Maci, M. Caiazzo, A. Cucini, and M. Casaletti, "A pole-zero matching method for EBG surfaces composed of a dipole FSS printed on a grounded dielectric slab," *IEEE Trans. Antennas Propag.*, vol. 53, no. 1, pp. 70–81, Jan. 2005.
- [8] R. E. Collin, *Field Theory of Guided Waves*, IEEE Press, New York, 1991.