Iterative Determination of Complex Permittivity Profile of a Dielectric Cylindrical Object

Mitsuru Tanaka[1] and Kaoru Hiramatsu[1]
[1]Department of Electrical and Electronic Engineering, Oita University, 700 Dannoharu, Oita 870-1192, Japan

***Abstract* - An iterative reconstruction method for microwave imaging of a lossy dielectric cylinder from measurements of multi-frequency scattering data is presented. The multigrid optimization method is used to minimize a cost functional defined by the norm of a difference between the scattered electric fields measured and calculated. Numerical results for two dielectric circular cylinders located in free space demonstrate that the proposed method is effective in determining the complex permittivity profile of the scattering object even in the presence of a measurement error in the scattering data.**

***Index Terms* — Inverse scattering problem, multigrid optimization method, iterative determination, complex permittivity profile.**

I. INTRODUCTION

Electromagnetic image-reconstruction is very important in nondestructive testing, medical diagnosis, detection of underground objects, and radar target identification. A variety of imaging methods have been proposed to determine the information on the object such as the size, shape, and material properties [1]-[8].

It is the purpose of this paper to consider the frequency-domain inverse scattering problem of determininig the complex permittivity profile of a dielectric cylinder based on the multigrid optimization method (MGOM). The MGOM consists of the conjugate gradient method (CGM), the modified frequency-hopping technique with a weighting factor (MFHT), and the multigrid method with V-cycle (MGM-V). The scattering object situated in a homogeneous background medium is illuminated with multi-frequency cylindrical electromagnetic waves in microwave region. The method of moments with pulse-basis functions and point matching [9] is employed to discretize the inverse scattering problem. A cost functional is defined as the norm of the discrepancy between the scattered electric fields measured and calculated for an estimated permittivity profile. Then the inverse scattering problem may be treated as an optimization problem, where the contrast function is determined by minimizing the cost functional. Note that the contrast function is described by the complex relative permittivity of the object. Applying the MGOM to the optimization problem, one can obtain an iterative reconstruction scheme.

Numerical results for two lossy dielectric circular cylinders located in free space are given to show the effectiveness of the proposed method. It is seen from the results that the iterative method gives good reconstructions with the property of fast convergence even in the presence of a measurement error in the scattering data.

II. THEORY

Let us consider the reconstruction of the relative permittivity $\varepsilon_s(\boldsymbol{\rho})$ of a lossy dielectric cylinder from the measurements of multi-frequency scattering data in microwave region. The scattering object, which is located in a homogeneous background medium of the relative permittivity ε_b, is illuminated by TM electromagnetic waves generated from L line sources placed at the points with polar angles $\theta = \theta_\ell$ along a circle of radius a, where $\ell = 1, 2, \cdots, L$. For each angle of incidence, the measurements of the scattered waves are made at observation points with polar angles $\phi = \phi_m$ along the same circle, where $m = 1, 2, \cdots, M$. The information on the object is described by a contrast function given by $c(\boldsymbol{\rho}) = \varepsilon_s(\boldsymbol{\rho}) - \varepsilon_b$. We employ multi-frequency scattering data at the frequencies of $f = f_p$, where $p = 1, 2, \cdots, P$. The square investigation domain containing the object and the background medium is divided into small square cells. We employ the method of moments with pulse-basis functions and point matching to discretize the inverse scattering problem discussed here.

Now a cost functional for the p-th frequency is defined as

$$F_p(c) = \sum_{q=1}^{p} \sum_{\ell=1}^{L} \sum_{m=1}^{M} w^{p-q} \left| E_q^s(c; \theta_\ell; \phi_m) - \tilde{E}_q^s(\theta_\ell; \phi_m) \right|^2, \quad (1)$$

where $\tilde{E}_q^s(\theta_\ell; \phi_m)$ and $E_q^s(c; \theta_\ell; \phi_m)$ are, respectively, the scattered electric fields measured and calculated for an estimated contrast function, and w indicates a weighting factor. Then the inverse scattering problem is reduced to an optimization problem, where $c(\boldsymbol{\rho})$ may be determined by minimizing $F_p(c)$ at the corresponding frequency. Note that the measured scattering data are simulated by solving the direct scattering problem for the true contrast function.

The MGOM, which is composed of the CGM, the MFHT, and the MGM-V, is used to minimize $F_p(c)$. Then one can obtain an iterative formula for determining $c(\boldsymbol{\rho})$. The current frequency hopps to the next higher-frequency after two V-cycles are completed except for the highest frequency. The relaxation calculation based on the CGM is made at the fine and the coarse grid levels. The MGOM reduces the computational cost, and mitigates the effect of nonlinearity on the reconstructed images. The reconstruction scheme terminates if the convergence criterion for the relative residual error δ in the scattered electric field is less than a prescribed value at the second fine grid of the last V-cycle for the highest frequency.

III. Numerical Considerations

Numerical results are presented for two lossy dielectric circular cylinders located in free space. We use six frequencies of 1GHz, 2GHz, 3GHz, 4GHz, 5GHz, and 6GHz. The two objects with the same radius of $0.4\lambda_0$ have the relative permittivities of $7.0 - j0.4$ and $4.0 - j0.3$, where λ_0 is the free-space wavelength for the highest frequency. We use 18 line sources for one frequency, and make 18 measurements of the scattered waves for each illumination. The line sources and the measurement points are uniformly distributed along a circle of radius $a = 4\lambda_0$. The $3\lambda_0 \times 3\lambda_0$ investigation domain is divided into 80×80 or 40×40 small square cells corresponding to the fine or the coarse grid level. Now the investigation domain is expanded from the $2\lambda_0 \times 2\lambda_0$ domain used in [8] to the present one. The sampling rate for the fine grid level is kept at more than 10 points per the shortest wavelength in the investigation domain. The initial guess of the contrast function is assumed to be zero. Furthermore the random noise with uniform distribution is added to the real and the imaginary parts of the scattered electric field. The signal-to-noise ratio is SNR=15dB.

Fig. 1 illustrates the reconstructed results of the real part and the imaginary part of the complex relative permittivity. The solid, the dashed-dotted, and the dotted curves present the results for $w = 0.5, 1.0,$ and 1.5, respectively. For reference, the true profiles of the real and the imaginary parts are also shown by the thin solid lines. The results are obtained after 125, 130, and 90 iterations for the corresponding values of w. The convergence criterion for δ is taken as 3.0×10^{-2}. It is seen from Fig. 1 that one can obtain good reconstructions with the fast property of convergence even in the presence of a measurement error in the scattering data.

IV. Conclusion

An iterative reconstruction method of determining the complex permittivity profile of a dielectric cylindrical object from measurements of the multi-frequency scattering data has been presented. The MGOM is applied to the minimization of a cost functional defined by the norm of the difference between the scattered electric fields measured and calculated. Numerical results for two dielectric circular cylinders located in free space confirm that our method provides good reconstructions with the property of fast comnvergence even for noisy case. Research on the reconstruction of higher-contrast objects for different noise levels remains a topic for further study.

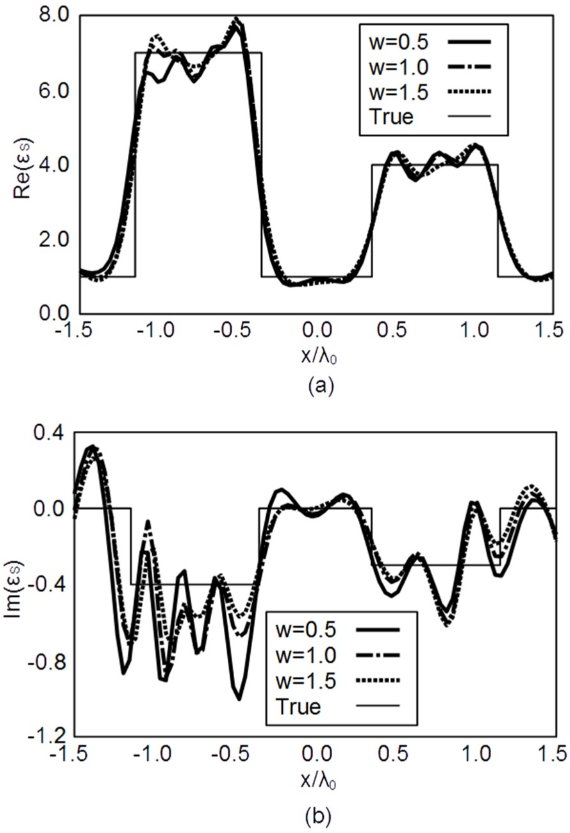

Fig. 1. Reconstructed results of (a) the real part and (b) the imaginary part of complex relative paermittivity.

References

[1] W. C. Chew and J. H. Lin, “A frequency-hopping approach for microwave imaging of large inhomogeneous bodies,” *IEEE Microwave Guided Wave Lett*., vol. 5, pp. 439-441, 1995.

[2] K. Belkebir, R. E. Kleinman, and C. Pichot, “Microwave imaging—Location and shape reconstruction from multifrequency scattering data,” *IEEE Trans. Microwave Theory Tech*., vol. 45, pp. 469-476, 1997.

[3] A. Qing, “Electromagnetic imaging of two-dimensional perfectly conducting cylinders with transverse electric scattered field,” *IEEE Trans. Antennas Propag*., vol. 50, pp. 1786-1794, 2002.

[4] S. Caorsi, M. Donelli, and A. Massa, “Detection, location, and imaging of multiple scatterers by means of the iterative multiscaling method,” *IEEE Trans. Microwave Theory Tech*., vol. 52, pp. 1217-1228, 2004.

[5] D. Franceschini, M. Donelli, G. Franceschini, and A. Massa, “Iterative image reconstruction of two-dimensional scatterers illuminated by TE waves,” *IEEE Trans. Microwave Theory Tech*., vol. 54, pp. 1484-1494, 2006.

[6] P. Mojabi and J. LoVerti, “Overview and classification of some regularization techniques for the Gauss-Newton inversion method applied to inverse scattering problem,” *IEEE Trans. Antennas Propag*., vol. 57, pp. 2658-2665, 2009.

[7] P.-A. Barriere, J. Idier, Y. Goussard, and J.-J. Laurin, “Fast solutions of the 2D inverse scattering problem based on a TSVD approximation of the internal field for the forward model,” *IEEE Trans. Antennas Propag*., vol. 58, pp. 4015-4024, 2010.

[8] M. Tanaka, “Performance improvement of an inversion algorithm for the reconstruction of lossy dielectric cylinder,” in *Proc. 2013 URSI-B International Symposium on Electromagnetic Theory*, 2013, Hiroshima, Japan, pp.938-941.

[9] R. F. Harrington, *Field computation by moment methods*, Macmillan: New York, 1968.